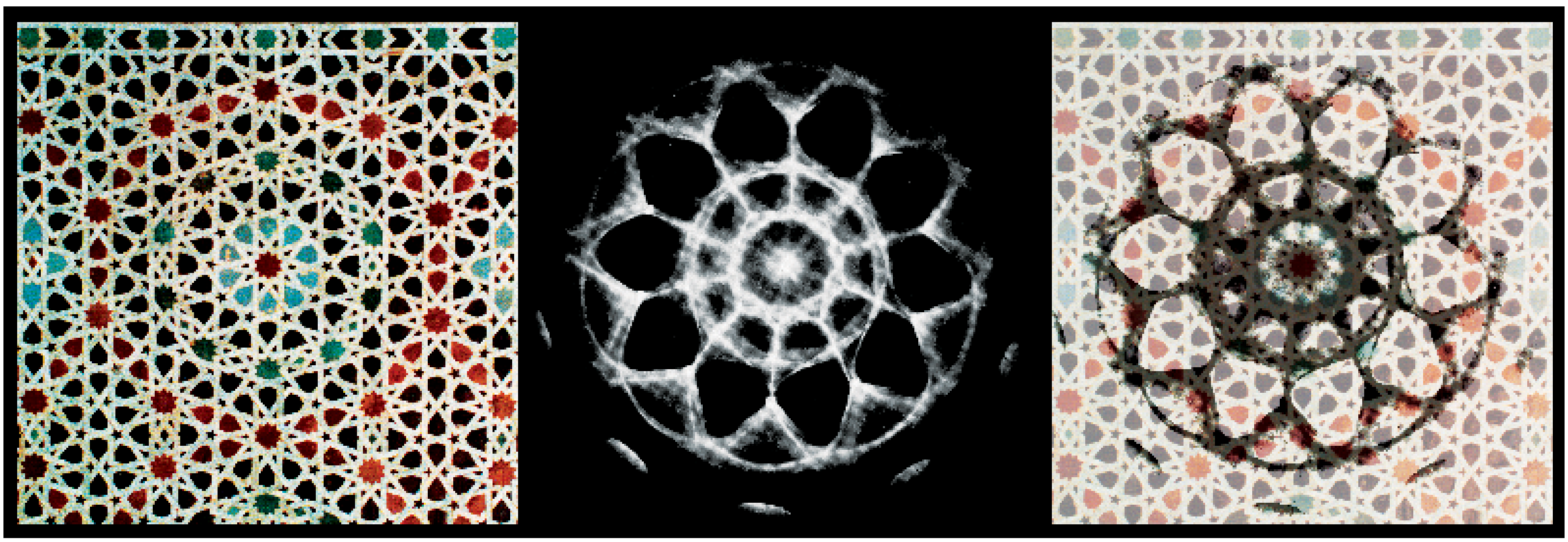
Images of Sound

Symmetry hidden and manifest in physics and art

Stephen Morris and Lydia Sharman



idea&s vol. 2: no. 2

Inspired by the experiments of Hans Jenny and medieval Moroccan mosaics, Lydia Sharman uses small icons in the form of a silver pendant, an intaglio plate or a silk screen image based on these fundamental geometries, and then connects and expands them over the surface in closed or open repeating forms.

Triptych by Lydia Sharman (2004), illustrating the relationship between a 10-fold zillij pattern and a 10-fold sound pattern.

In the creation myths of many cultures, a sound or a song calls forth the Earth and universe.

In Western tradition, the idea that a mathematical harmony, or pattern of vibration, underlies the visible world descends from Pythagoras; it finds its contemporary expression in the esoteric vibrations of string theory, now very remote from sensory experience. Concentric or periodic patterns, the visual manifestation of sound and rhythm, are the basis of traditional mandalas and tessellations, which, in many cultures, constitute a symbolic representation of the origin and nature of the universe.

Pythagoras was inspired by the vibrations of lute strings, as he observed their successive modes of vibration produce notes whose frequencies have a simple mathematical relationship to one another. Each mode, or pattern of vibration, is composed of nodes (points on the string which remain stationary) and antinodes (points of maximum vibrational amplitude). Thus, sound is joined to a spatial pattern.

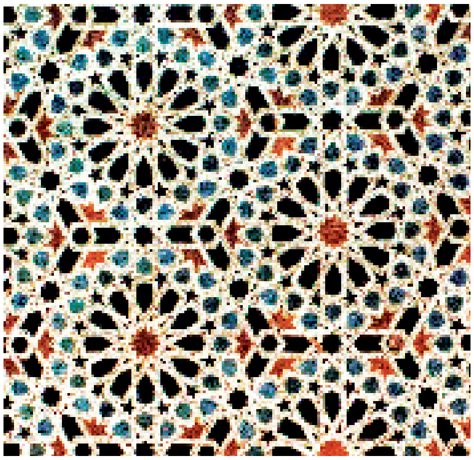
In 1802, Ernst Chladni published *Die Akustik*, his descriptive treatise on the emergent science of sound. Following on the 17th-century observations of Robert Hooke, Chladni described a simple method to make sound visible. He was concerned with the vibration of thin plates, extended two-dimensional objects on which the nodes formed a network of lines, rather than single points. He invented a musical instrument consisting of glass disks of various shapes and sizes that were stroked at their edges with a violin bow. The pattern of vibration was made visible by sprinkling the plate with sand. The sand was thrown about by the vibration at the antinodes and collected at the stationary nodes, making the overall mode pattern visible. Remarkable and strikingly symmetric patterns emerged from this delightful musical experiment, which Chladni demonstrated on his extensive speaking tours of Europe. *Die Akustik* contains beautiful and exhaustive figures of the patterns he observed (the entire book is available online in the digital collection of the University of Toronto Library at http://link.library.utoronto.ca/booksonline).

A modernized version of Chladni's experiment uses a powerful electromagnetic shaker to excite the modes of an aluminum plate: by tuning precise frequencies, very high modes of vibration can be visualized.

Chladni's patterns drew the attention of leading French scientists and, on 7 February 1808, Chladni demonstrated his experiment in a two-hour audience with Napoleon. Napoleon, who considered himself something of a mathematician, was sufficiently impressed that, not only did he fund a French edition of *Die Akustik*, he also instituted a prize for the mathematical explanation of the patterns. The value of the prize was one kilogram of gold.

The prize was won eventually in 1815 by Sophie Germain, on her third attempt. Germain, who, in addition to her work on Chladni patterns, made important contributions to number theory and Fermat's Last Theorem, is famous today as the greatest female mathematician of the modern era. In her day, she had an heroic and ultimately tragic struggle for acceptance as a mathematician, working in isolation and by correspondence with well-established male mathematicians. She sometimes hid her gender using the pseudonym "M. LeBlanc," a deceased member of the French Academy. The prize was to be awarded at a public ceremony, but Germain chose not to appear to collect it. She died of breast cancer in 1831 at the age of 55.

Sophie Germain's solution did not end the controversy over the mathematical theory of the patterns. Its ultimate solution had to wait for the development of more powerful "variational" methods, which are based on the form of the energy stored in the plate by small amounts of bending. The theory was finally considered complete after the work of German physicist Gustav Kirchoff, who, in 1850, derived the correct expression for the bending energy. Germain's last paper on the subject, submitted in 1825 to a commission of the Institut de France, was simply ignored by the referees and published only posthumously in 1880 after it was rediscovered among the papers of a member of the commission, French mathematician and engineer Gaspard de Prony, who had died in 1839.



A six-fold, tessellating zillij pattern.
Photo by Lydia Sharman (1994), Fez, Morocco.

The definitive treatise on the linear theory of sound, containing a concise and complete treatment of Chladni patterns, is that of John William Strutt, better known as Lord Rayleigh. His *Theory of Sound* (1894), begun on a Nile cruise in 1872, ran to two volumes and several editions. Using the elegant methods of variational calculus, Rayleigh could solve for the frequencies and modal patterns on round plates and for some of the simpler patterns on square plates. A complete solution of his equations, today easily obtained with a computer, is possible because these motions are "linear": they involve only very small deviations away from a flat plate, which can be treated mathematically as infinitesimal. Such infinitesimal deviations do not interact or mix with one another and the problem can be attacked with very general methods based on superposition. Today, such linear problems are considered as part of the canon of classical physics.

One gateway into "nonlinear" pattern formation, a very active research topic today, was opened in the 1830s by the great experimentalist Michael Faraday. While reproducing Chladni's experiments, he was able to explain his puzzling observation that very light particles, such as hairs detached from the violin bow, moved to the antinodes, rather than to the nodes. Faraday showed that they were moved by air currents driven by the vibration in a nonlinear, "acoustic streaming" effect.

Faraday investigated the motion of liquid layers lying on vibrating plates and observed several new nonlinear effects. A horizontal liquid layer shaken vertically becomes unstable to the formation of surface waves, an effect now called the Faraday instability. These surface waves are not related to the modal pattern of the plate and, indeed, occur over thick plates which do not flex significantly. The waves are not of small amplitude and appear suddenly above a threshold acceleration. Similar waves are seen often on the surface of coffee in a Styrofoam cup that is made to vibrate by dragging it across a table.

Faraday waves have become, as of the late 1980s, one of the laboratory systems often studied as a paradigm of nonlinear pattern formation. The surface waves exhibit several forms—straight waves, square patterns, hexagons and even spirals and chaotic states—brought about by the strongly nonlinear interactions between waves of different orientations.

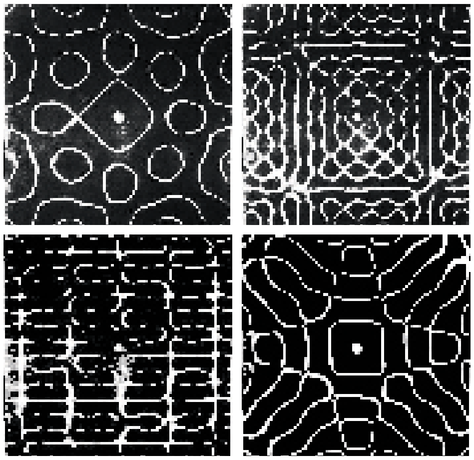
The strongly nonlinear nature of the governing equations makes Faraday patterns much more complex than those of Chladni. They are also surprising and beautiful. Their symmetry is "emergent" in the sense that it is not apparent in the underlying forces or the featureless fluid. Rather, the patterns are "self-organized."

Faraday also studied the action of vibration on heaps of grains and more complex fluids like egg whites. Recently, attention has been drawn to the bizarre behaviour of shaken cornstarch suspensions. The cornstarch layer stiffens under vibration and eventually breaks into a mass of writhing fingers, animated by the shaking. It is hard not to see these fingers as something alive. Here we find inanimate matter imitating biology in an uncanny way. These motions are not driven by events at the molecular scale so beloved by modern biology, but rather macroscopically and in response to nonequilibrium, nonlinear effects that spontaneously build structure out of a continuum (the movie, by Robert D. Deegan *et al*., can be viewed at http://www.maths.bris.ac.uk/~mardd/).

These self-organized patterns have caught the attention of physicists, mainly in the last few decades, as well as artists and others interested in symmetry and form.

In the 1960s and '70s, Hans Jenny, a Swiss medical doctor, amateur scientist and artist, undertook a series of experiments on the visual characteristics of vibrational effects and wave phenomena. He named his area of research Cymatics, and wrote a book with that title. Like Chladni in an earlier era, Jenny's book is bereft of mathematics; however, it is richly illustrated with his remarkable photographs.

Hans Jenny's experiments both postdated and predated a large body of experimental work in physics. He includes references to Chladni and some of his experiments, such as the square patterns created in fluids spread on a plate, are similar to Faraday's. Jenny also



Examples of Chladni patterns on vibrated square plates, created by Stephen Morris.

undertook and documented experiments with "writhing and leaping" cornstarch suspensions. This strange phenomenon was rediscovered only recently by physicists. At the time Jenny was conducting his experiments, physicists had little interest in strongly nonlinear effects, which were out of fashion. Jenny seems to have worked in isolation, separated from the scientific community. His studies of vibrations were focused on a limited range of phenomena and conducted with scientific rigour. However, in his book, he considered the results in a comprehensive framework which touches on such diverse fields of science as mineralogy, solar physics and cosmology, and he refers to the vibrational effects as a comprehensively ordered harmonic. This may explain why he appeals more to artists than to scientists.

The most aesthetically beautiful images in *Cymatics* (1974), in both colour and black-and-white, are those created by animating a fluid drop with a simple audio frequency. Forms emerge and are documented in photographic images of the transmitted light. The animation of the drop starts with a series of concentric rings, which are radially symmetric waves: the higher the tone, the greater the number of rings.

When the amplitude of the vibration is increased, the radial pattern changes, in a series of sudden transitions, to waves with fascinating rotational symmetries. These have a pleasing degree of mathematical order usually involving the division of the circle-based pattern into six-, eight- or 10-fold symmetry. When the amplitude is increased again, the pattern breaks into a highly dynamic, chaotic state. With a further increase in amplitude, a new symmetric pattern emerges. This process is repeated as the amplification is increased. The patterns created by Faraday and Chladni, often with single or tessellating squares or hexagons, are simple in comparison.

The geometric structures and progressions created in a fluid drop have similarities to the circular symmetries of tessellations and mandalas found in the art of diverse cultures. Islam, for example, which discourages figurative art, has developed complex geometric art forms in its place. The works often constitute a pictorial representation of the universe or creation as conceived by a given culture. The very universality in the structure of the circular patterns across cultures may be related to the ease of creating certain geometries (the radius divides a circle into six equal parts) and to observation of the natural world (flower petals) and the universe (the solstice and equinox).

Of the many traditional patterns based on geometric structure, the Moroccan zillij mosaics, which in the 15th century were used in the Medrassh (religious schools) for decoration and as a focus for contemplation, are among the most sophisticated. In fact, the geometric patterns of the zillij mosaics provide an interesting artistic counterpart to the Jenny patterns created in a fluid droplet, both of which are usually based on six-, eight- or 10-fold symmetry of considerable complexity. Overlapping the two images reveals that the geometric progression of the mosaic patterns corresponds very precisely with that of the sound patterns.

The intimate connection between sound and spatial form has provided a rich arena of collaboration between physicist and artist/designer. When the authors of this paper were asked in an interview what they had learnt from the collaboration, the physicist answered, "More about aesthetics," and the artist/designer, whose work represents a contemporary expression of the generative capacity of geometry, answered, "More about the process creating the aesthetic forms." It became apparent that there is an aesthetic appreciation, viewed from two different positions, that connects science and art. &

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