Sniffing out new laws...

How can dimensional analysis help us figure out what new laws might be?

(Why is math important not just for calculating, but even just for understanding?)

(And a roundabout way of seeing where quantum mechanics came from 100 years ago...)

Question

I am reading about a country in which the speed limit is 500 km/h.

How long are typical car trips in this country?

(a) My fictional country may be 100 km across or may be 10,000 km across.
I probably would have heard of it if it were that big.
So, 1,000 km / (500 km/h) = 2 hours max?

(b) What if it were on a distant planet, and I don't even know the size of the planet?

Limited dimensions

If I have no idea of the size (km), and the only number I have is speed (km/hour), nothing I do to that speed will ever give me a time.

[If I *knew* a time ("1 hour"), that would yield a special distance ("500 km")... but what could be special about one distance rather than another, if you don't know anything about the place?]

SYMMETRY:

if we have no information to make 1 metre a worse guess than 100 km, then the situation should look the same at 1m and at 100km...

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Another possible answer: people don't like driving more than about 8 hours straight...
On the other hand, maybe these aliens move very slowly and live for millions of years...

Reasonable assumptions, and order of magnitude estimates

How many piano tuners are there in Toronto?

Maybe there are 4 million people in Toronto.

Maybe there are 3 million, maybe 5; do I care if I'm off by 25%? So far, I don't know if there are 5 or 500,000.

Maybe that's 2 million homes.
Maybe 200,000 have pianos.
Maybe the average piano is tuned once a year.
Maybe a piano tuner tunes 3 pianos a day, 15 a week, 750 a year?
(200,000 / about 1000) = about 200 piano tuners.

Maybe there are 20... maybe there are 2000...
I doubt there are only 5, or as many as 10,000!

How big are aliens?

Suppose we receive a radio signal from distant aliens, and wish to tell them what we're like...

Let's say "we're about 2 metres tall."

We may have worked out how to make them understand "2"...
but how do we tell them what a metre is?

Since 1983, it has been defined as "the length of the path travelled by light in vacuum during a time interval of $1/299,792,458$ of a second."[2]

Originally intended to be one ten-millionth of the distance from the Earth's equator to the North Pole (at sea level)

Well, maybe if we tell them what a second is...

Between 1000 (when al-Biruni used seconds) and 1960 the second was defined as 1/86,400 of a mean solar day

the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

Dimensionless numbers

Everything we measure is *in terms of something else.*

This stick is a meter; how many times taller are you?

"Humans are roughly 40 billion times taller than Hydrogen atoms."

But what if they didn't know how big Hydrogen atoms were?! It turns out unless you know the charge and mass of the electron, there'd be nothing special about this size either, so it just needed to be discovered (or think of "charge and mass of electron" as extra laws of physics... but why are they what they are?).

"Interesting" numbers have no dimensions (π).

The muon is called "heavy" not because it weighs a trillionth of a trillionth of a gram, but because it weighs 100 times more than an electron....

Aside: special scales

The speed of light is known (velocity).

Newton's gravitational constant lets us convert mass and distance to force... but that's sort of like defining the kg.

Coulomb's electrical force law lets us convert electrical charge and distance to force... but that's sort of like defining the unit of charge.

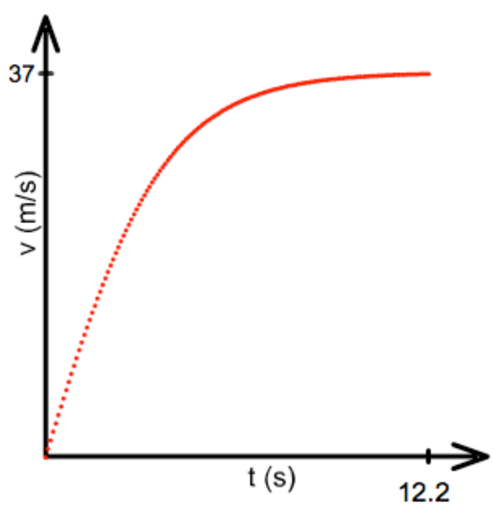
There was no "special scale" one would expect the universe, or planets, or us, to be built on [unless you add in the fact that for some reason electrons weigh what they do and therefore atoms have the sizes they do], until quantum mechanics.

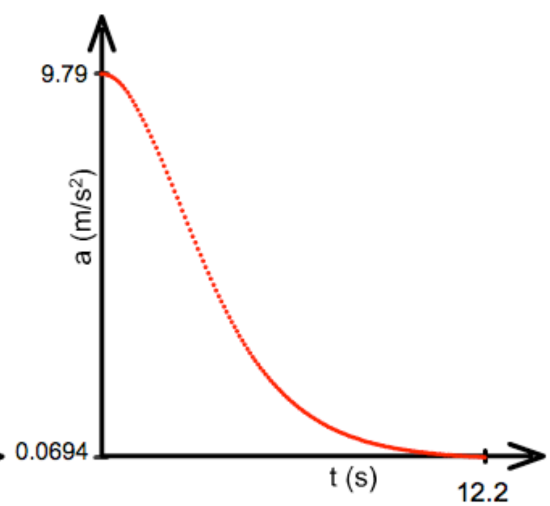
Now, there is the "Planck scale," a special length (& mass & energy & ...) fixed by the laws of physics – but too small for any of us to have studied so far!

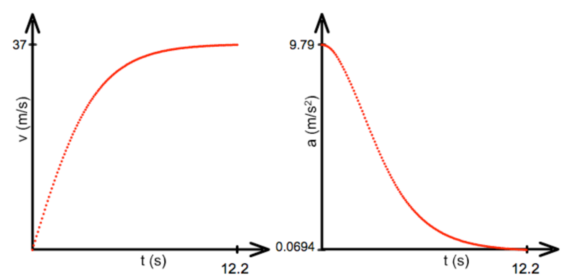
How fast does a raindrop fall?

Shouldn't it depend on how long it's been falling??

Velocity versus time
(and acceleration versus time)







Why "37 m/s" and not 3, or 300?
The "law of physics" is "9.8 m/s²"...

but nothing you can do to a "m/s²" will get you a "m/s", any more than a speed without a distance can give a time

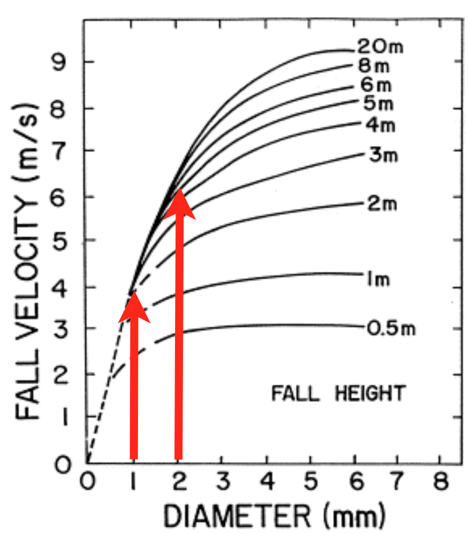
There's something special about "4 seconds"... for some reason, that's how long raindrops accelerate...
(Or: there's something special about 37 m/s, and that's why raindrops only accelerate for 4 seconds...)

There must be another law of physics that specifies a particular time (or a particular velocity, or a particular distance)...

Oh – we already *had* a particular distance: the size of the drop!

Maybe the distance over which you accelerate depends on the size of the drop? (Because otherwise, we're still stuck...)

Well, bigger drops do accelerate longer, but they seem to go about 2000 times the size of the drop before they stop accelerating.



“Naturalness”

Why 2000?
Laws of physics tend to have 2’s and π’s, not 2000’s...
But maybe this one is different.

Or maybe we’re still missing something...

Does a 2mm droplet of lead fall at the same speed as a 2mm raindrop?

Air resistance

Galileo: all objects fall w̄ same accel.
Newton: Force ∝ accel, but also to mass.

If lead drop is 10× as heavy as water drop, F_{Grav} is 10× bigger.
But the air "only knows" about the shape; same force.

So: bigger net force on lead than on rain; lead falls faster.

But wait... lead would *accelerate* faster, but both would still just accelerate forever.

If the "air resistance force" is also some m/s^2, there's still no special velocity / time / distance; you still accelerate forever.

If the "air resistance force" is also some m/s², there's still no special velocity / time / distance; you still accelerate forever.

Intuition: you feel more force from air resistance if you go faster.

So if air resistance is *proportional to* velocity, it's described by an "acceleration per velocity": $\frac{m/s^2}{m/s} = 1/s$

I get some "special number of seconds," as desired.

On the other hand, how do I know it's not proportional to v^2? That would give me "accel per v^2": $\frac{m/s^2}{(m/s)^2} = \frac{m/s^2}{m^2/s^2} = \frac{1}{m}$

Just as good.

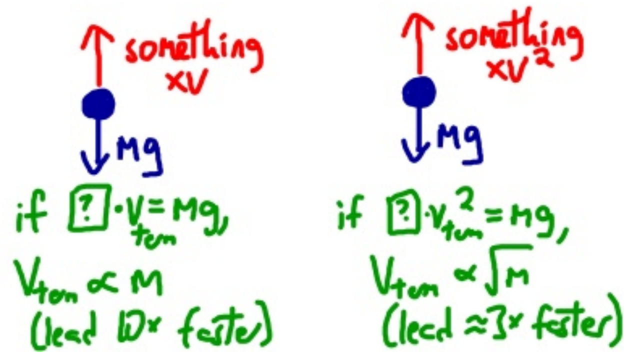
Which is true?

We could try to understand the microscopic theory of air resistance, which might give us an answer... but (a) it's actually really complicated and there isn't always just one answer; and (b) what about before people even knew air was made of molecules?

We could carefully track one raindrop and measure its acceleration at every instant (hard for Galileo)

We could also just ask "how does terminal velocity *scale* with the density of the object"? (Does lead fall 10 times faller than water, or 3 times, or 100 times?)

Air resistance (“drag”): 2 options



Physics is largely about looking for these relationships.

Does the force depend on the length? the area? the mass? the density?

Does lead fall 3 times faster, 10 times faster, 100 times faster?

(It doesn’t end)

Even after we find that, why do water droplets go about 2000 times their size before they reach “terminal velocity”?

Are they 2000 (or 45, or 4,000,000) times less dense than some “special density”?

Then what is it about air resistance that makes that density special?

Why the digression?

The discovery of the fundamental length scale of the universe:

1900 or so:

We finally understand electricity, magnetism, & optics
(and that they're the same thing)
Ballistic motion, planetary motion, all that is old hat.
We understand thermodynamics, and build steam engines.
Huge progress on hydrodynamics, aerodynamics, et cetera.

Just a few puzzles:

Do atoms actually exist?
(Why do different elements have different "spectra"?)
If so, what are they made of? (electron discovered 1899)
Why & how does light make current flow out of metals?
Why do things glow the colours they do when they get hot?

"Black"-body radiation

The discovery of the fundamental length scale of the universe:

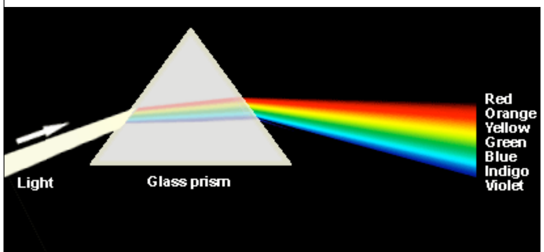
Why do things glow as they heat up, and glow red at first but then whiter when they get hotter?

Light = electricity and magnetism
Heat is a form of energy

The electrons in hotter objects move around faster, and give off more light... okay... but

(a) how much more?

(b) and what colour (what frequency)?



Remarkably, no matter what an object is made of, what its shape, et cetera, its "spectrum" (intensity of each of the colours it emits) follows a simple curve that seems to depend only on temperature *(aside from some details...)*

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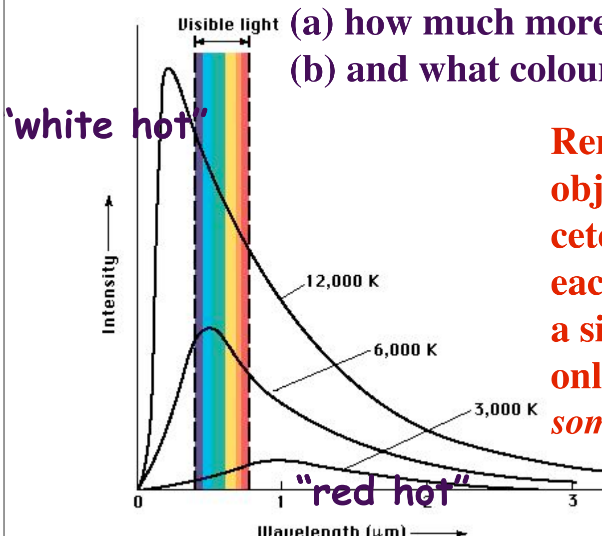

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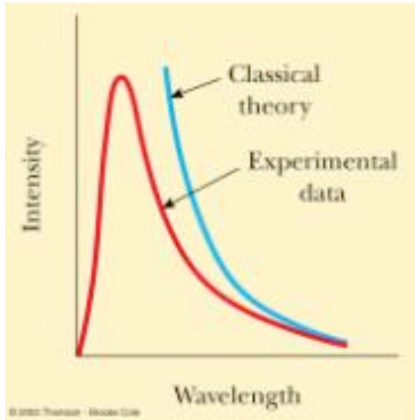
"Ultraviolet catastrophe"



(1) Every object should give off an infinite amount of ultraviolet energy (obviously impossible).

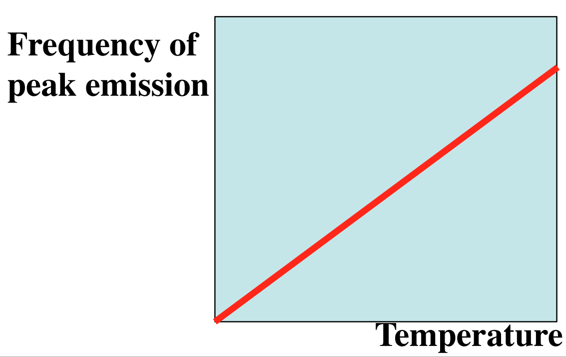
(2) More importantly, it means our theory is broken!

"Ultraviolet catastrophe"

A given object radiates more of its energy around one particular frequency (colour) than any other.

It turns out that this frequency gets higher (colour gets "bluer") when the temperature (average energy per electron) goes up.

In fact, the frequency is proportional to the temperature!



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Look for a law with some constant that looks like "frequency per energy," or energy * time.

There was simply no such law.
No combination of constants known to physics could give you this "energy * time" or "energy per frequency."

What can you do?

We have no idea what the missing theory is, but we *know* it must contain some constant with units of "energy per frequency."
Planck's constant: $h = 6.6 \cdot 10^{-34}$ J s [or J / Hz, since Hz = 1/s]

Planck's "quantum hypothesis":
maybe when a body emits or absorbs light of frequency f, it can't emit any old amount of energy it likes, because there is some "special energy": it emits energy in "steps" of $E=hf$.

Miracle: take all the 19th century knowledge of electricity, magnetism, and thermodynamics, and add this one (unjustified) assumption -- and you can predict exactly what the "energy spectrum" of glowing objects (like our Sun) should be: and get it exactly right.

But no idea in the world why it should be like that!