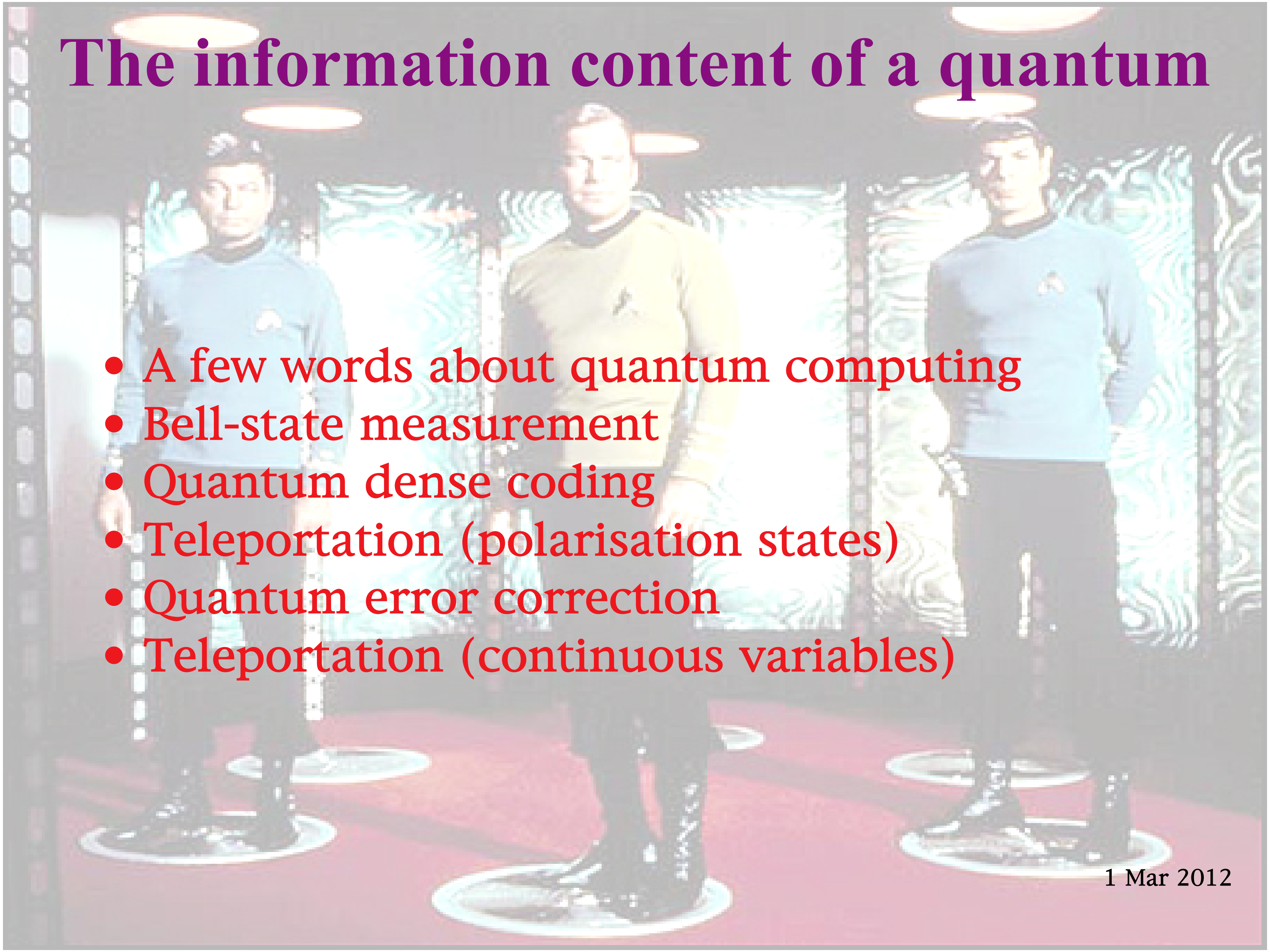
The information content of a quantum

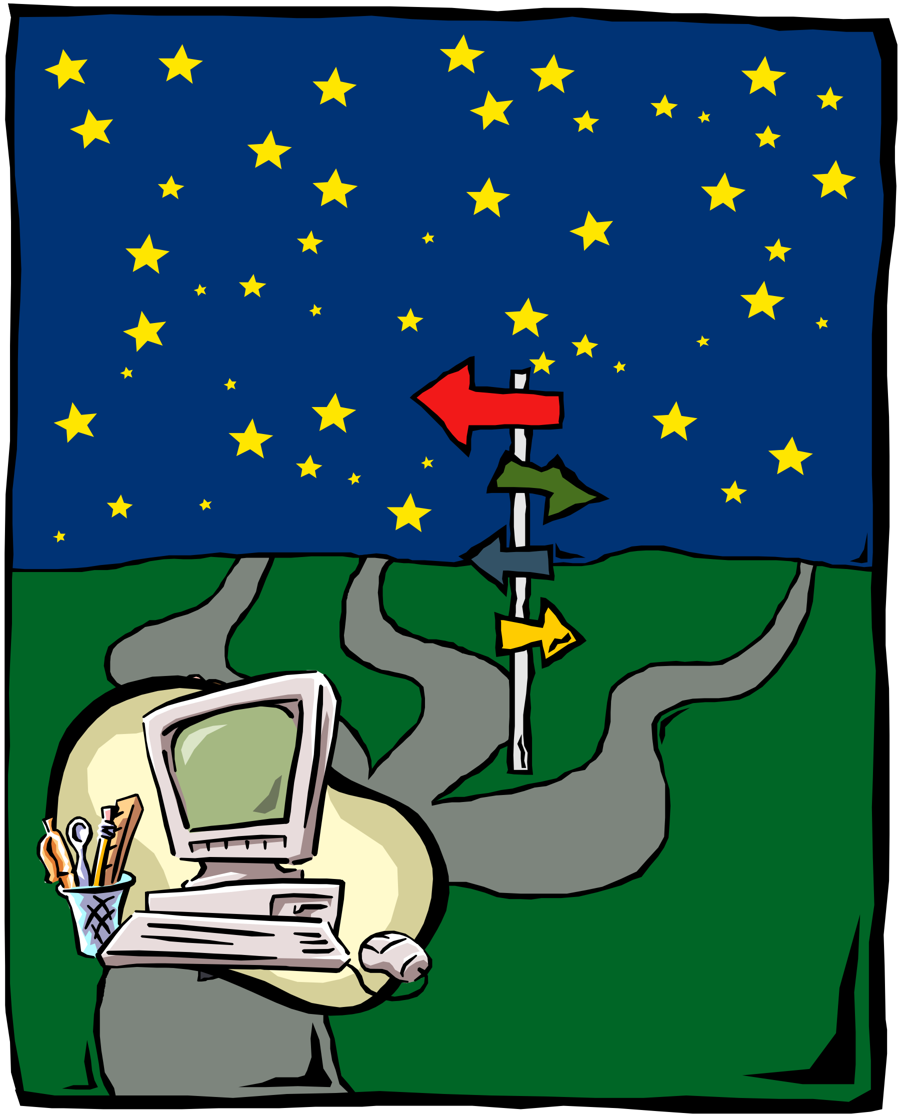


- A few words about quantum computing
- Bell-state measurement
- Quantum dense coding
- Teleportation (polarisation states)
- Quantum error correction
- Teleportation (continuous variables)

1 Mar 2012

Quantum Information

What's so great about it?



Quantum Information

What's so great about it?

If a classical computer takes input |n> to output |f(n)>, an analogous quantum computer takes a state |n>|0> and maps it to |n>|f(n)> (unitary, reversible).

By superposition, such a computer takes Σ_n |n>|0> to Σ_n |n>|f(n)>; it calculates f(n) for every possible input simultaneously.

A clever measurement may determine some global property of f(n) even though the computer has only run once...

A not-clever measurement "collapses" n to some random value, and yields f(that value).

The rub: any interaction with the environment leads to "decoherence," which can be thought of as continual unintentional measurement of n.

What makes a computer quantum?

(One partial answer...)

If a quantum "bit" is described by two numbers:

$$|\Psi\rangle = c_0|0\rangle + c_1|1\rangle,$$

then n quantum bits are described by 2^n coeff's:

$$|\Psi\rangle = c_{00..0}|00..0\rangle + c_{00..1}|00..1\rangle + ... c_{11..1}|11..1\rangle;$$

this is exponentially more information than the 2n coefficients it would take to describe n independent (e.g., classical) bits.

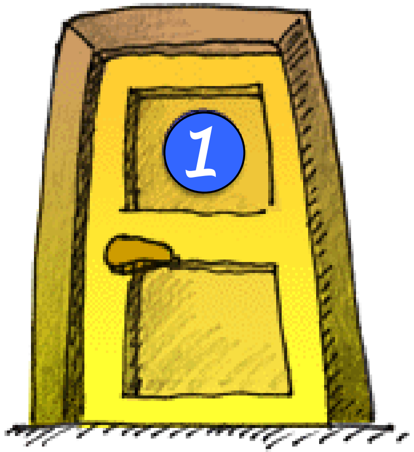
We need to understand the nature of quantum information itself.

How to characterize and compare quantum states?
How to most fully describe their evolution in a given system?
How to manipulate them?

The danger of errors & decoherence grows exponentially with system size.

The only hope for QI is quantum error correction.

We must learn how to *measure* what the system is doing, and then correct it.



Measuring and manipulating entanglement

Information and measurement

Any measurement on a qubit (two-level system) yields at most 1 bit of info.

On the other hand, a full specification of the state (density matrix) of a qubit involves 3 independent real parameters (coordinates on Bloch/Poincaré sphere); this is in principle an infinite amount of information.

How much information can be stored or transferred using qubits?

Measure & reproduce – only one classical bit results from the measurement, and this is all which can be reproduced.

"No cloning": cannot make faithful copies of unknown, non-orthogonal quantum states, because $\{\langle a|\langle a|\}\{|b\rangle|b\rangle\} = \{\langle a|b\rangle\}^2$ and unitary evolution preserves the inner product.

[Wooters & Zurek, Nature 299, 802 (1982).]

(N.B.: Applies to *unitary* evolution. With *projection*, one can for instance distinguish 0 from 45 *sometimes*, and then reproduce the exact state – but notice, still only one classical bit's worth of information.)

Dense coding & Teleportation

Bennett & Wiesner, PRL 69, 2881 (1992)

Observation: a pair of entangled photons has four orthogonal basis states – the Bell states – but they can be connected by operations on a single photon.

Thus sending that single photon to a partner who *already possesses* the other entangled photon allows one to convey 2 classical bits using a single photon.

single-photon operations: **flip phase** **flip pol.**

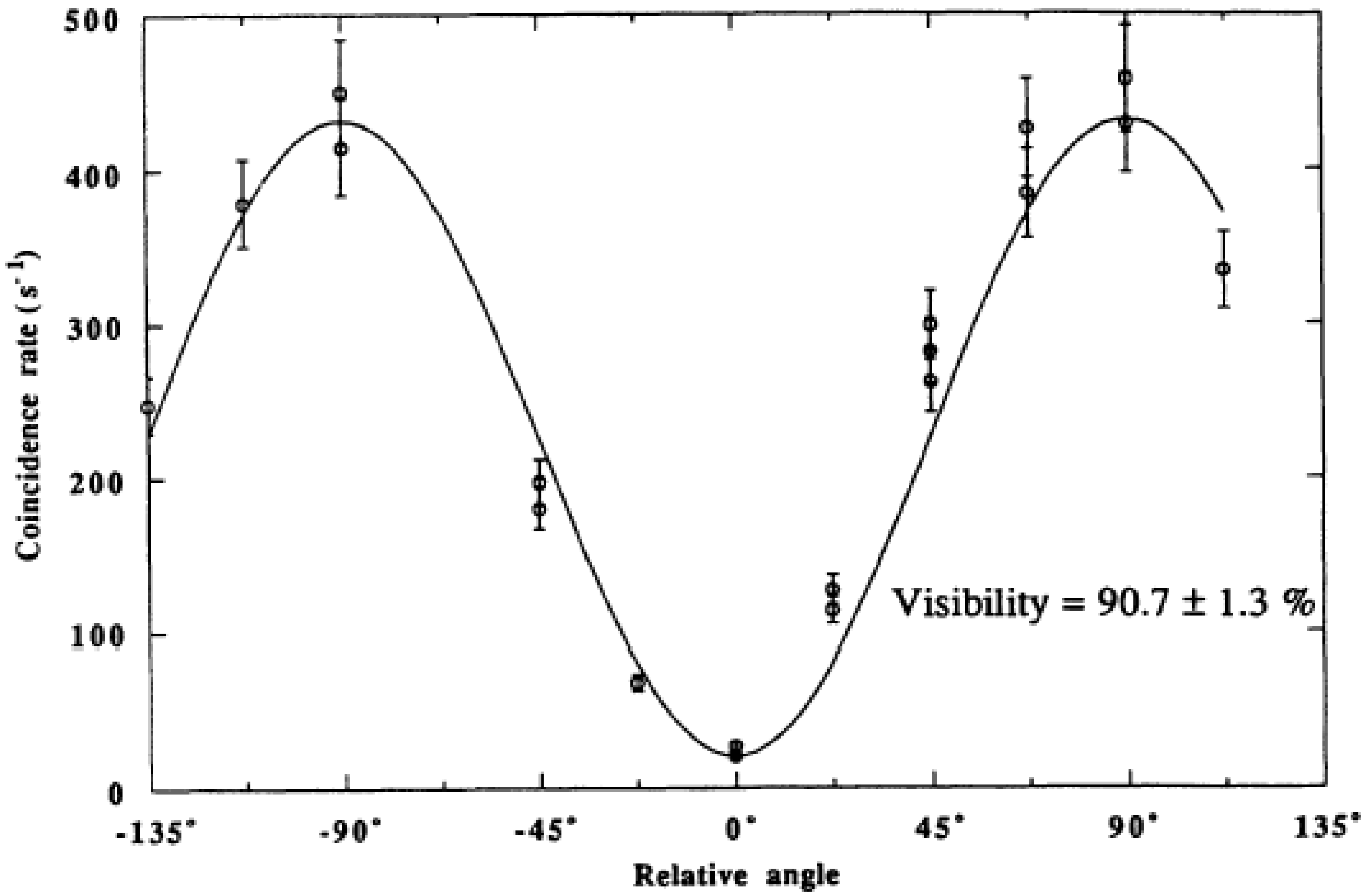
The Bell state basis:

$$|\Psi^+\rangle = (|H\rangle|V\rangle + |V\rangle|H\rangle)/\sqrt{2},$$
$$|\Psi^-\rangle = (|H\rangle|V\rangle - |V\rangle|H\rangle)/\sqrt{2},$$
$$|\Phi^+\rangle = (|H\rangle|H\rangle + |V\rangle|V\rangle)/\sqrt{2},$$
$$|\Phi^-\rangle = (|H\rangle|H\rangle - |V\rangle|V\rangle)/\sqrt{2}.$$

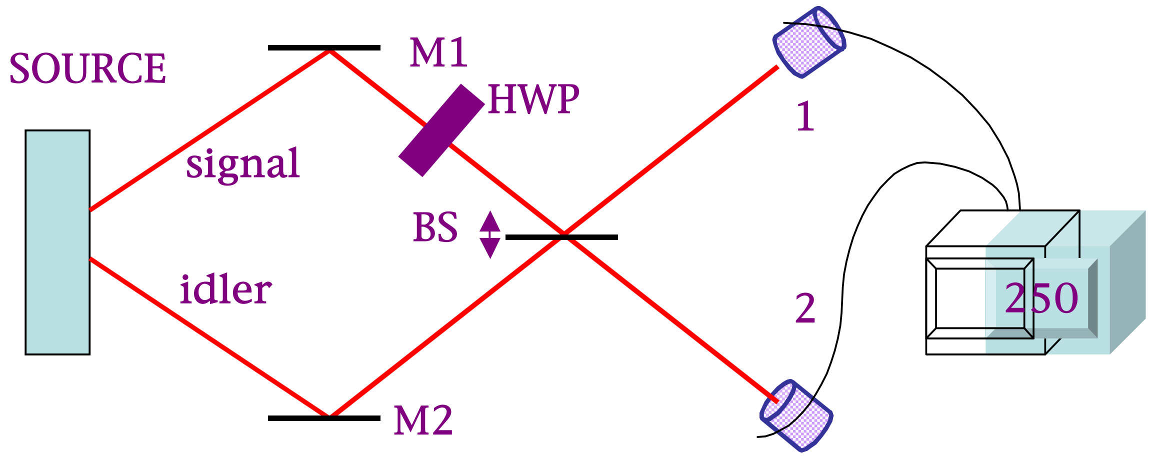
("σ_z") ("σ_x")

Note: even with more photons, you never get more than 2 bits per photon; see if that starts to make sense as the term goes on, or remember to bring it up later!

Remember the polarisation-dependence of rate at centre of H-O-M dip...



What does the HOM do to polarized photons?

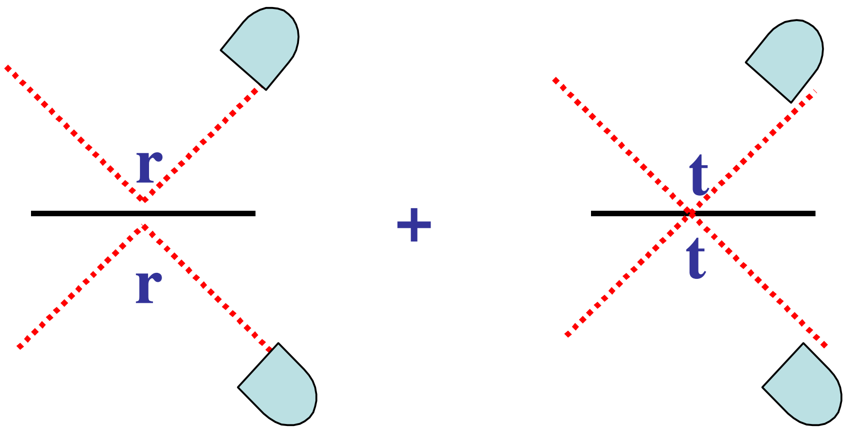


Vs Hi ⟶ (V2 + i V1) (H1 + i H2)

= 1H 2V - 1V 2H + i [1H 1V + 2H 2V]

In coincidence, only see |HV> - |VH> .... that famous EPR-entangled state.
Of course we see nonlocal correlations between the polarisations.

Hong-Ou-Mandel Interference as a Bell-state filter



$r^2+t^2 = 0$; total destructive interf. (if photons indistinguishable).

If the photons begin in a symmetric state, no coincidences.

{Exchange effect; cf. behaviour of fermions in analogous setup!}

The only *antisymmetric* state is the singlet state |HV> – |VH>, in which each photon is unpolarized but the two are orthogonal. Nothing else gets transmitted.

This interferometer is a "Bell-state filter," used for quantum teleportation and other applications.

$Log_2(3)$ bits in a single photon

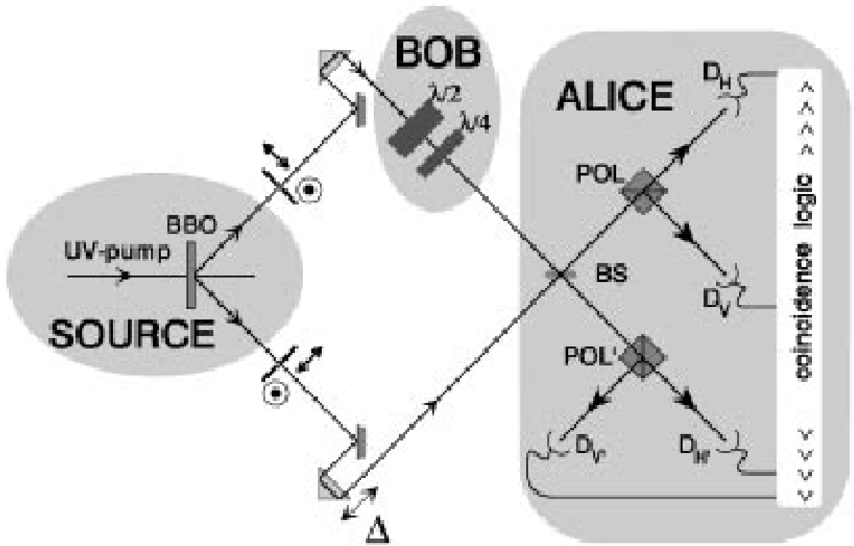


FIG. 1. Experimental setup for quantum dense coding. Because of the nature of the Si-avalanche photodiodes, the extension shown in the inset of Fig. 4 is necessary for identifying two-photon states in one of the outputs.

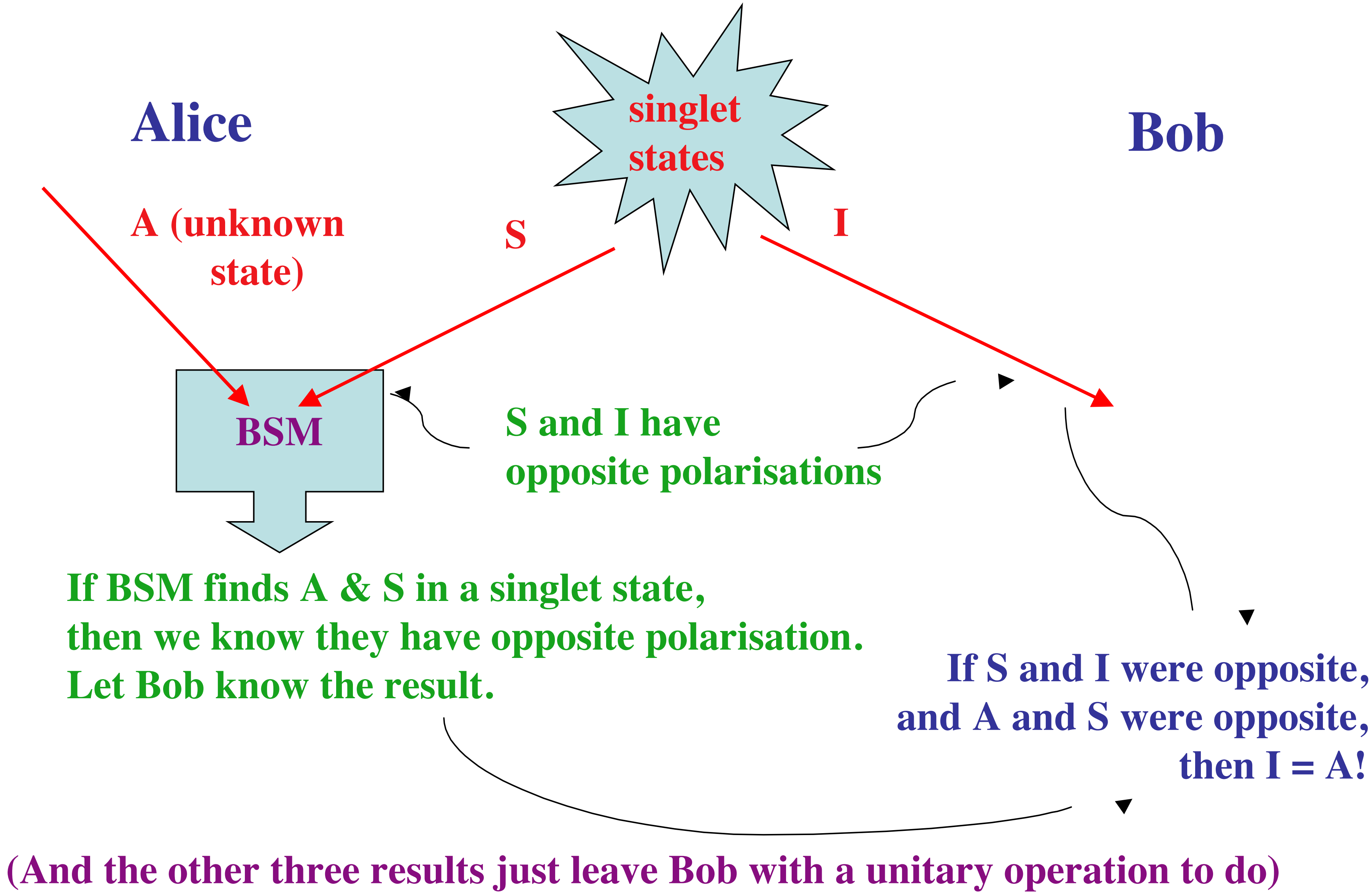
To extract both bits, one would need to distinguish all 4 Bell states – this can't be done with linear optics, but 2 of the 4 can, leaving a third (ambiguous) possibility.

Bob's setting $\lambda/2$	$\lambda/4$	State sent	State at output of Bell-state analyzer	Alice's registration events
0°	0°	$\|\Psi^+\rangle$	$\{hv + h'v' + vh + v'h'\}/2$	Coincidence between D_H *and* D_V or $D_{H'}$ *and* $D_{V'}$
0°	90°	$\|\Psi^-\rangle$	$\{hv' - h'v' + v'h - vh'\}/2$	Coincidence between D_H *and* $D_{V'}$ or $D_{H'}$ *and* D_V
45°	0°	$\|\Phi^+\rangle$	$\{hh + vv + h'h' + v'v'\}/2$	2 photons in either D_H, D_V, $D_{H'}$, or $D_{V'}$
45°	90°	$\|\Phi^-\rangle$	$\{hh - vv + h'h' + v'v'\}/2$	2 photons in either D_H, D_V, $D_{H'}$, or $D_{V'}$

Mattle *et al*., PRL 76, 4656 (1996)

Quantum Teleportation

Bennett *et al*., Phys. Rev. Lett. 70, 1895 (1993)



Teleportation as projections

$$|\psi_A\rangle = a|H\rangle + b|V\rangle$$

$$|\psi_{SI}\rangle = |HV\rangle - |VH\rangle$$

$$|\psi\rangle_{ASI} = \underline{a|HHV\rangle} + \underline{b|VHV\rangle} - \underline{a|HVH\rangle} - \underline{b|VVH\rangle}$$

$$\{\langle HV|_{AS} - \langle VH|_{AS}\}|\psi\rangle = -a|H\rangle - b|V\rangle \sim |\psi_A\rangle_i$$

$$\{\langle HV| + \langle VH|\}|\psi\rangle = -a|H\rangle + b|V\rangle \sim \sigma_z|\psi_A\rangle_i$$

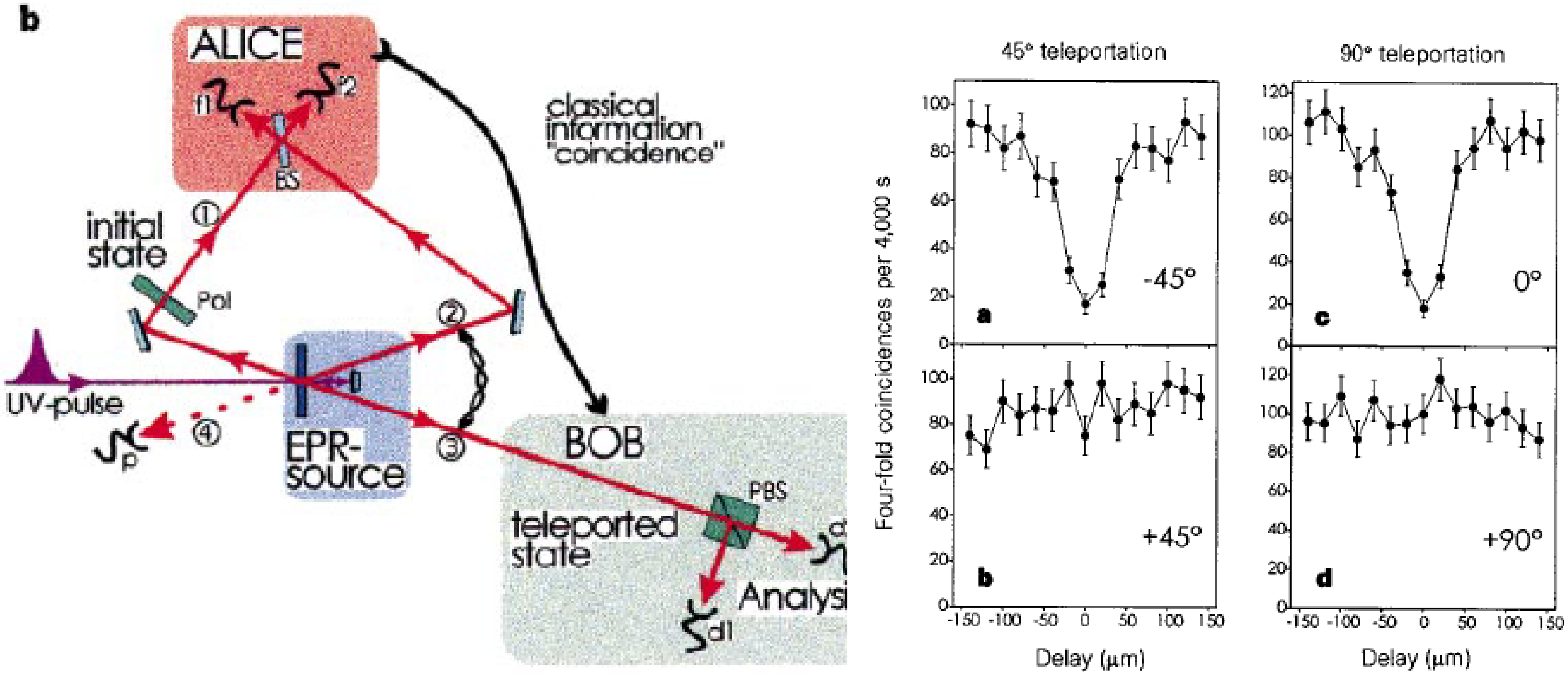
$$\{\langle HH| - \langle VV|\}|\psi\rangle = a|V\rangle + b|H\rangle \sim \sigma_x|\psi_A\rangle_i$$

$$\{\langle HH| + \langle VV|\}|\psi\rangle = a|V\rangle - b|H\rangle$$

$$\sim \sigma_z\sigma_x|\psi_A\rangle_i$$

$$\sim \sigma_y|\psi_A\rangle_i$$

Quantum Teleportation (expt)



Bouwmeester *et al*., Nature 390, 575 (1997)

One striking aspect of teleportation

- **Alice's photon and Bob's have no initial relationship – Bob's could be in any of an infinite positions on the Poincaré sphere.**
- **The Bell-state measurement collapses photon S (and hence Bob's photon I) into one of four particular states – states with well-defined relationships to Alice's initial photon.**
- **Thus this measurement transforms a continuous, infinite range of possibilities (which we couldn't detect, let alone communicate to Bob) into a small discrete set.**
- ***All* possible states can be teleported, by projecting the continuum onto this complete set.**

Quantum Error Correction

In classical computers, small errors are continuously corrected –
built-in dissipation pulls everything back towards a "1" or a "0".

Recall that quantum computers must avoid dissipation and irreversibility.

How, then, can errors be avoided?

A bit could be anywhere on the Poincaré sphere – and an error could in principle move it anywhere else. Can we use measurement to reduce the error symptoms to a discrete set, à la teleportation?

Yes: if you measure whether or not a bit flipped, you get either a "YES" or a "NO", and can correct it in the case of "YES".
As in dense coding, the phase degree of freedom is also important, but you can similary measure whether or not the phase was flipped, and then correct that.
Any possible error can be collapsed onto a "YES" or "NO" for each of these.

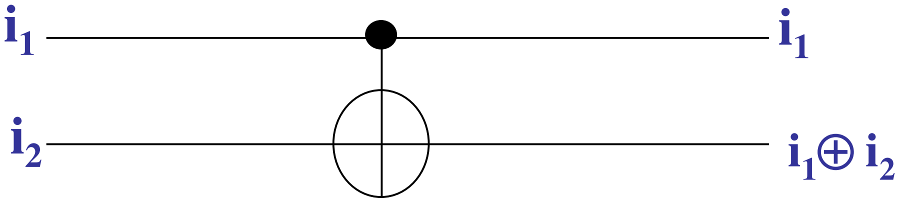
The four linearly independent errors

Identity	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$I\|a\rangle = \|a\rangle$
Bit Flip	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$X\|a\rangle = \|a \oplus 1\rangle$
Phase Flip	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$Z\|a\rangle = (-1)^a\|a\rangle$
Bit & Phase	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = iXZ$	$Y\|a\rangle = i(-1)^a\|a \oplus 1\rangle$

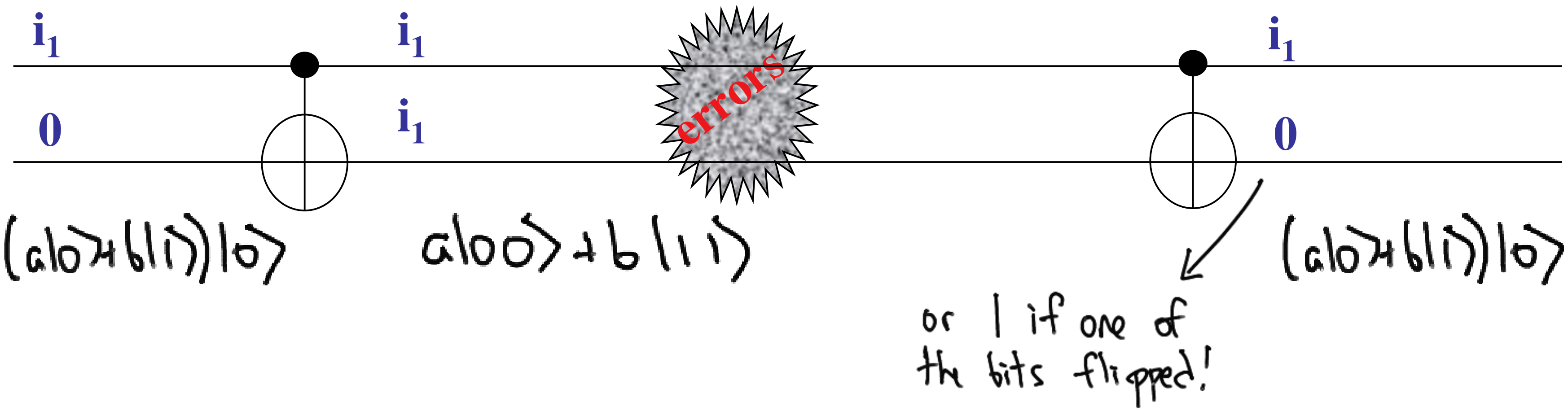
TABLE 1. The Pauli matrices

Encoding & decoding

Notation - the controlled NOT (CNOT):



In (ct)	**Out (ct)**
00	**00**
01	**01**
10	**11**
11	**10**



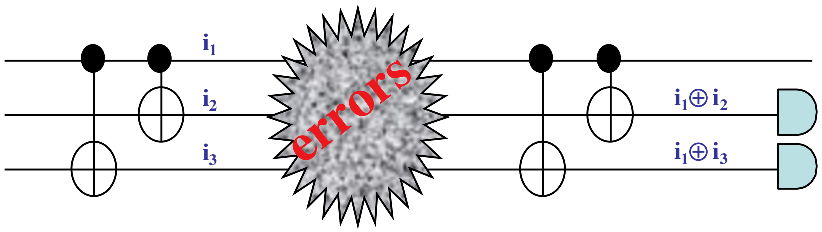
Error *detection*!

Q. error correction: Shor's 3-bit code

In case of bit flips, use redundancy – it's unlikely that more than 1 bit will flip at once, so we can use "majority rule"...

BUT: we must not actually measure the value of the bits!

Encode: $a|0\rangle + b|1\rangle \Rightarrow a|000\rangle + b|111\rangle$



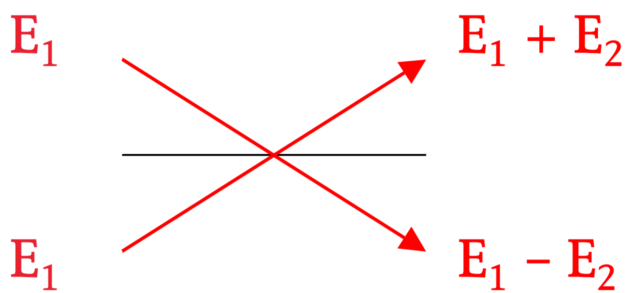
Symptom	State	$i_1 \oplus i_2$	$i_1 \oplus i_3$
Nothing happens	$a|000\rangle + b|111\rangle$	0	0
i_1 flips	$a|100\rangle + b|011\rangle$	1	1
i_2 flips	$a|010\rangle + b|101\rangle$	1	0
i_3 flips	$a|001\rangle + b|110\rangle$	0	1

And now just flip i_1 back if you found that it was flipped – note that when you measure which of these four error syndromes occurred, you exhaust all the information in the two extra bits, and no record is left of the value of i_1!

NOTE: you could have phase errors as well as bit flips; more copies required.

How to measure the continuous analog of Bell states ?

We wish to learn about the "relative" state of two systems, without measuring the exact state of either...



$X_1 + X_2$;

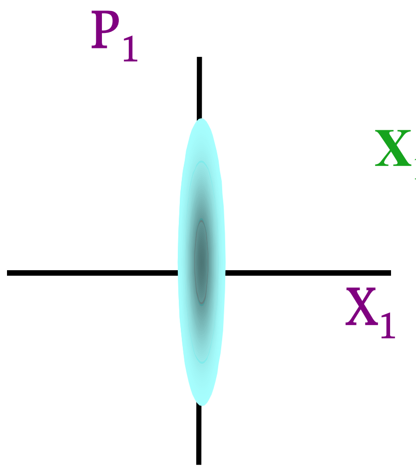
$P_1 + P_2$

$X_1 - X_2$;

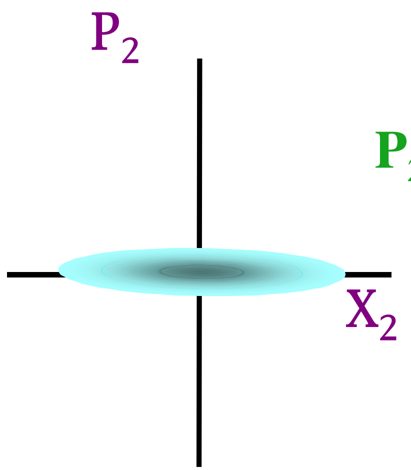
$P_1 - P_2$

Do homodyne measurement on the outcomes, to measure differences or sums of the chosen quadratures.
(At best, one difference and one sum.)

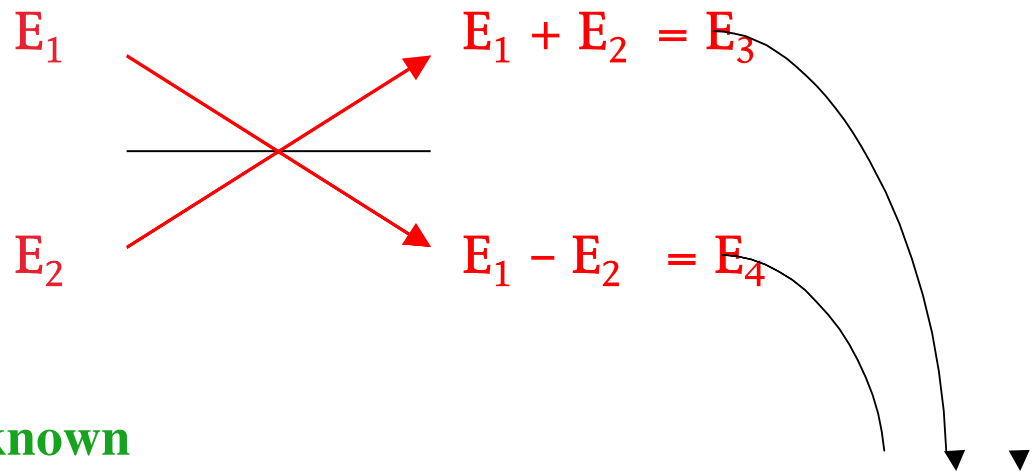
How to generate the continuous analog of Bell pairs?



X_1 is well known



P_2 is well known



$X_3 + X_4 = X_1$ is well known.
$P_3 - P_4 = P_2$ is well known.

E_3 & E_4 are entangled (EPR)

Some references

FOR QI in general...

Best to start with
Nielsen & Chuang's *Quantum Computation and Quantum Information* (Cambridge U.P., 2000)

Technical papers...

Dense coding and teleportation:
Bennett & Wiesner, PRL 69, 2881 (1992)
Mattle et al., PRL 76, 4656 (1996)
Benett et al., PRL 70, 1895 (1993)
Bouwmeester et al., Nature 390, 575 (1997)
Furusawa et al., Science 282, 706 (1998)

Error-correcting codes:
Steane, Proc. Roy. Soc. Lond. A 452, 2551 (1996)
Shor, PRA 52, 2493 (1995)
Knill et al, quant-ph/020717

UPCOMING TOPICS...

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Knill, Laflamme, & Milburn, Nature 409, 46 (2001)
Gottesmann & Chuang, Nature 402, 390 (1999)
Ralph, Langford, Bell, & White, PRA 65, 062324 (2002)
O'Brien, Pryde, White, Ralph, & Branning, Nature 426, 264 (2003)
Langford et al., PRL 95, 210504 (2005)

Cluster-state quantum computation:
Nielsen, "Universal quantum computation using only...", quant-ph/0108020
Raussendorf & Briegel, "A one-way quantum computer", PRL 86, 5188 (2001)
Raussendorf & Briegel, PRA 68, 022312 (2003)
Aliferis & Leung, "Computation by measurements: a unifying picture", quant-ph/0404082
Nielsen, "Cluster-state Quantum Computation", quant-ph/0504097
Walther et al, Nature 434, 169 (2005)

Weak-nonlinearity optical quantum computation:
Nemoto & Munro, PRL 93, 250502 (04)