FIRST: BRIEF RECAP OF ANG. MOM. IN 2D

Cartesian: if $U(x,y) = U_1(x) + U_2(y) \Rightarrow \mathcal{H} = \underbrace{\left(\frac{p_x^2}{2m} + U_1\right)}_{\mathcal{H}_x} + \underbrace{\left(\frac{p_y^2}{2m} + U_2\right)}_{\mathcal{H}_y}$

then for any $\left\{\begin{array}{l} \varphi(x) \text{ s.t. } \mathcal{H}_x \varphi(x) = E_x \varphi(x) \\ \psi(y) \text{ s.t. } \mathcal{H}_y \psi(y) = E_y \psi(y) \end{array}\right\}$

we have a 2D eigenstate $\Psi(x,y) = \varphi(x)\psi(y)$

because $\mathcal{H}\varphi\psi = (\mathcal{H}_x + \mathcal{H}_y)\varphi\psi$
$= (\mathcal{H}_x \varphi)\psi + \varphi(\mathcal{H}_y \psi)$
$= E_x \varphi\psi + E_y \varphi\psi = (E_x + E_y)\varphi\psi$

SIMILARLY: if $U(r,\theta) = U_1(r) + U_2(\theta)$ we can play a similar trick.
SPECIFICALLY, CENTRAL POTENTIALS HAVE $U = U(r)$; $\frac{\partial U}{\partial \theta} = 0 \Rightarrow [L_z, \mathcal{H}] = 0$

L_z conserved.

IN THESE POTENTIALS, WE CAN FIND A BASIS OF SIMULTANEOUS EIGENSTATES OF L_z & $\mathcal{H}$.

$|m\rangle \propto e^{im\varphi}$ ARE EIGENSTATES OF L_z;
$\Psi(r,\theta) = F(r)e^{im\varphi}$ ARE ALL m-eigenstates OF L_z... BUT WHICH $F(r)$ ARE $\mathcal{H}$-EIGENSTATES? DEPENDS ON $U(r)$.

LAST TIME I MERELY SHOWED YOU BY CONSTRUCTION
THAT SINCE $rG(r)e^{i\varphi} = xG(r) + iyG(r)$
AND $xG(r) = |10\rangle$ AND $yG(r) = |01\rangle$ AS YOU SAW ON 15 Nov,
EACH WITH ENERGY $2\hbar\omega$, $\mathcal{H}\Psi = \mathcal{H}(xG + iyG) = (2\hbar\omega xG + 2\hbar\omega iyG) = 2\hbar\omega\Psi$.

$xG(r)$ represents an oscillator with KE in the x-direction. (No ang mom?)

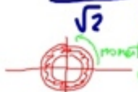
(classically, $x(t) = x_0 \cos \omega t$)

$yG(r)$ represents an oscillator with KE in the y-direction. (No ang. mom.?)

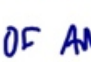
(classically, $y(t) = y_0 \cos \omega t$)

$\frac{x+y}{\sqrt{2}}G(r)$ has the same energy; it's like a marble bouncing back and forth at 45° in this isotropic bowl.
(classically, $x = y = \frac{r_0}{\sqrt{2}} \cos \omega t$)

$\frac{x+iy}{\sqrt{2}}G(r)$ also has the same energy: the quantum analog



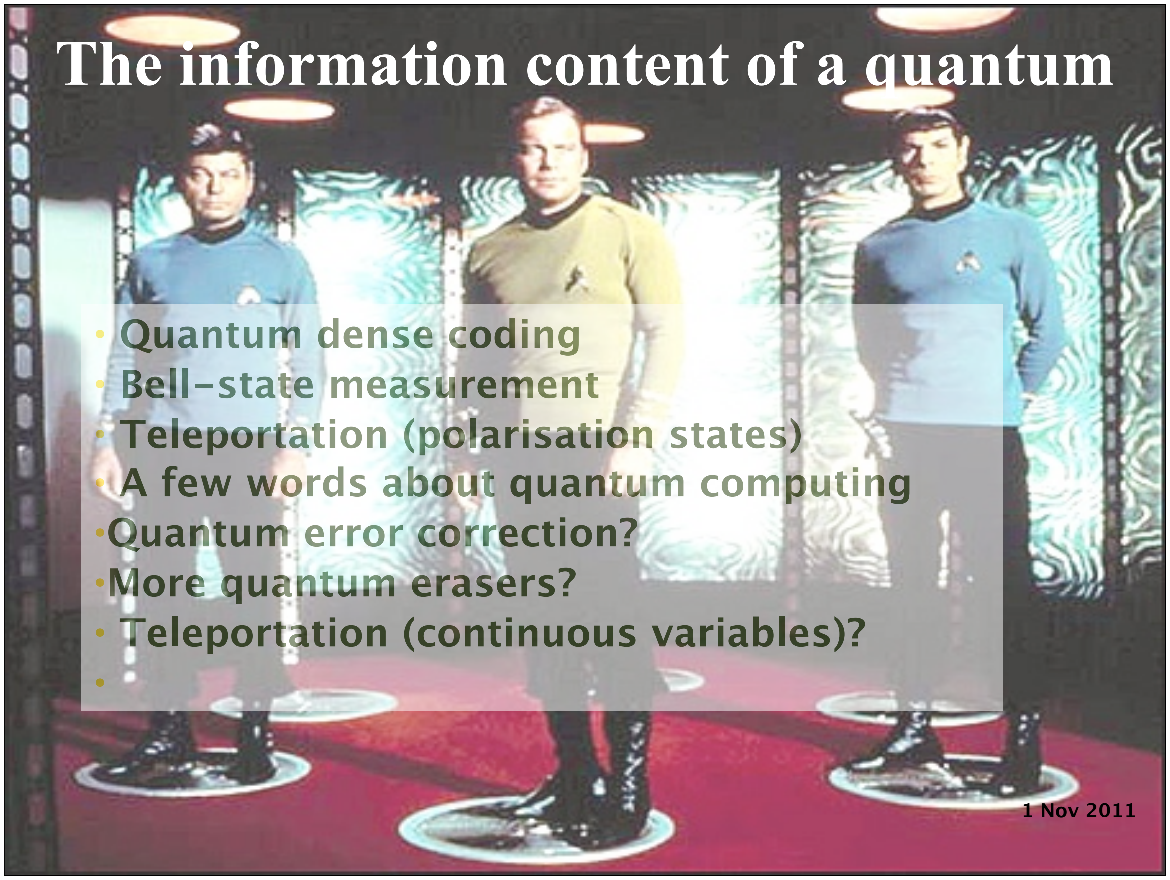
of an orbiting marble $\left\{\begin{array}{l} x(t) = \frac{r_0}{\sqrt{2}} \cos \omega t \\ y(t) = \frac{r_0}{\sqrt{2}} \sin \omega t \end{array}\right\} \rightarrow$ +1 unit of ang. mom.



These are all merely bases, like HV, DA, & RL for the photon.

IN FACT, PHOTON POLARISATION IS A FORM OF ANG. MOM. (S=1)

AND NOTE: WHILE $xG(r) = |10\rangle$ HAS NO AVERAGE ANG. MOM.,
IT $= \frac{|\ell=+1\rangle + |\ell=-1\rangle}{\sqrt{2}} \Rightarrow$ A SUPERPOSITION (50/50) OF POS & NEG ANG. MOM.
[EXACTLY AS X-POL'D LIGHT = SUP. OF R & L]

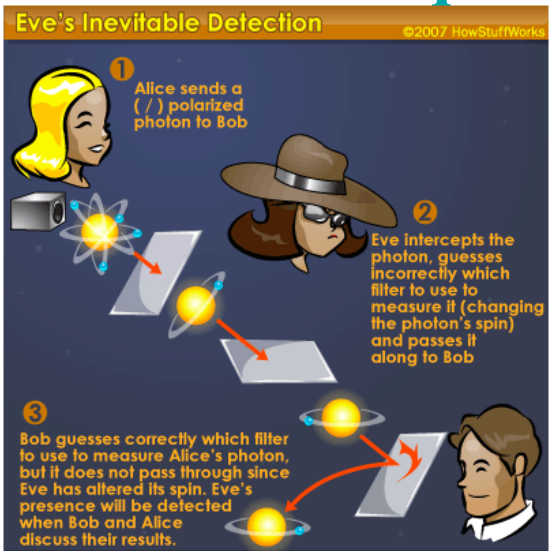


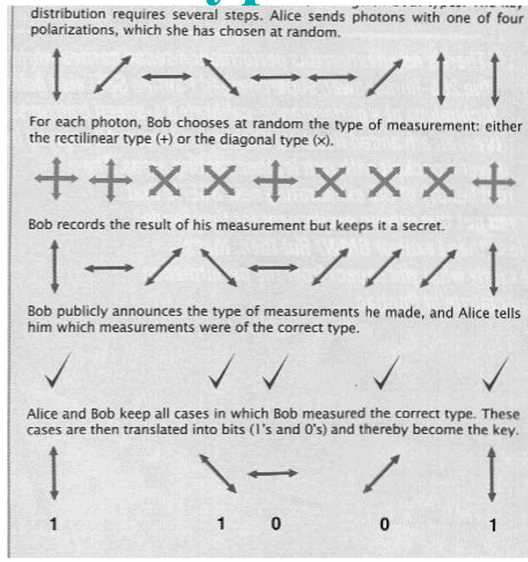
Who can remind me what the violation of Bell's inequalities really proves?

> The predictions of QM (and observations of experiment) cannot be explained by any model which assumes that what happens at one detector may be independent of what happens at another.

Yet we saw we can't use this ourselves to communicate FTL.
Why? ("No cloning" was the loophole that killed one particular scheme)
Because the outcome at B doesn't depend on *what I do* at A,
but merely on *what randomly happens* at A...

Recall quantum crypto

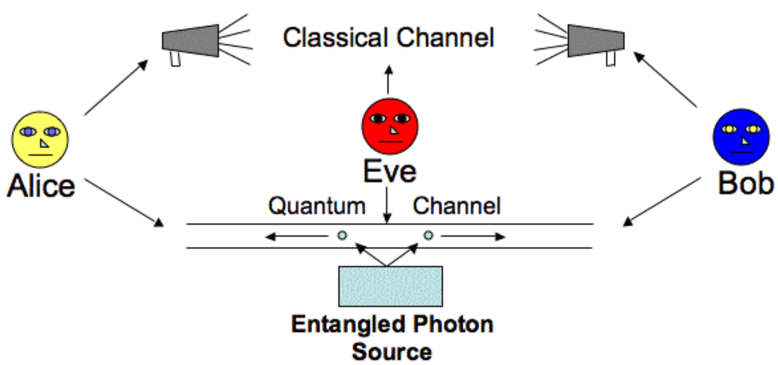




- Measurement disturbance -> if Eve measures the unknown quantity, there is a detectable effect
- Incompatible observables -> there is always at least one unknown quantity
- No-cloning -> she can't just make a second copy and wait to measure later

PROBLEM: side channels (what if you *hear* me switch from H to D?)

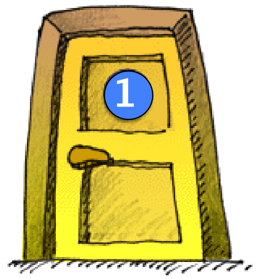
Solution: Ekert Protocol



In any basis, Alice and Bob see correlated (opposite) results.
They can construct a key just as in the Bennett-Brassard '84 protocol

They can use Bell Inequalities to test that their photons are entangled

Recall what BI's really test: could you explain your results with local hidden variables?
Well, if Eve had measured the photons, each would have "collapsed" to the outcome of some measurement. Eve has the hidden variable.
If BI's are violated, you know *Eve has no information, side channels or not!*



Measuring and manipulating entanglement

Information and measurement

Any measurement on a qubit (two-level system) yields at most 1 bit of info.

On the other hand, a full specification of the state (density matrix) of a qubit involves 3 independent real parameters (coordinates on Bloch/Poincaré sphere); this is in principle an infinite amount of information.

How much information can be stored or transferred using qubits?

Measure & reproduce – only one classical bit results from the measurement, and this is all which can be reproduced.

"No cloning": cannot make faithful copies of unknown, non-orthogonal quantum states, because $\{\langle a|\langle a|\}\{|b\rangle|b\rangle\} = \{\langle a|b\rangle\}^2$ and unitary evolution preserves the inner product.

[Wooters & Zurek, Nature 299, 802 (1982).]

(N.B.: Applies to *unitary* evolution. With *projection*, one can for instance distinguish 0 from 45 *sometimes*, and then reproduce the exact state – but notice, still only one classical bit's worth of information.)

Dense coding & Teleportation

Bennett & Wiesner, PRL 69, 2881 (1992)

Observation: a pair of entangled photons has four orthogonal basis states – the Bell states – but they can be connected by operations on a single photon.

Thus sending that single photon to a partner who *already possesses* the other entangled photon allows one to convey 2 classical bits using a single photon.

	single-photon operations:	flip phase	flip pol.

The Bell state basis:

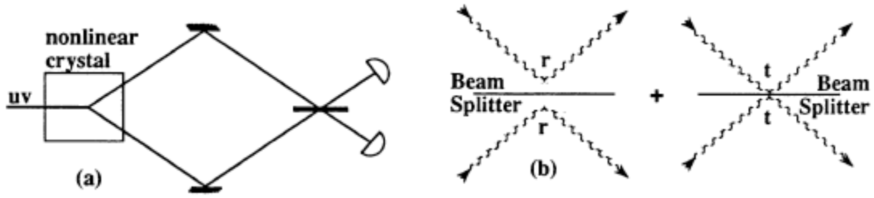
$$
\begin{aligned}
|\Psi^{+}\rangle &= (|H\rangle|V\rangle + |V\rangle|H\rangle)/\sqrt{2},\\
|\Psi^{-}\rangle &= (|H\rangle|V\rangle - |V\rangle|H\rangle)/\sqrt{2},\\
|\Phi^{+}\rangle &= (|H\rangle|H\rangle + |V\rangle|V\rangle)/\sqrt{2},\\
|\Phi^{-}\rangle &= (|H\rangle|H\rangle - |V\rangle|V\rangle)/\sqrt{2}.
\end{aligned}
$$

("σ_z") ("σ_x")

But how would the receiver know which state he/she had?.....

the prototypical two-photon interference effect:

the Hong-Ou-Mandel interferometer



$t^2 + r^2 =$
$1/2 - 1/2 = 0$;
no coincidence counts.

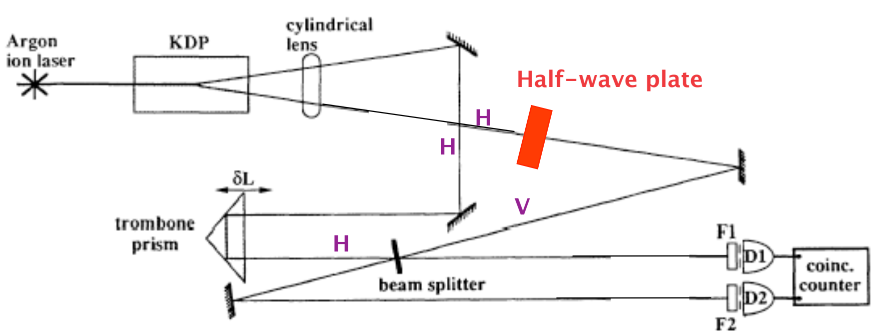
Remember: if you detect only one photon, the other photon "knows" where yours came from. Hence there is no interference (each detector sees 1/2 of the photons, irrespective of any phases or path-length differences).

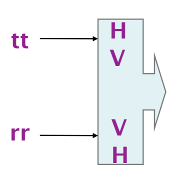
But: if you detect both photons, there is no way to tell whether both were reflected or both were transmitted. $r^2+t^2 = (i^2 + 1^2)/2 = 0$.
(any lossless symmetric beam splitter has a $\pi/2$ phase shift between r and t.)

CAVEAT: there must be no way to tell which occurred.
If the paths aren't aligned right, no interference occurs.
If one photon reaches the beam splitter before the other, no interference occurs.

Dirac: two photons never interfere with each other; each photon interferes only with itself.
Mandel (after Feynman): or, one photon pair can interfere with itself!

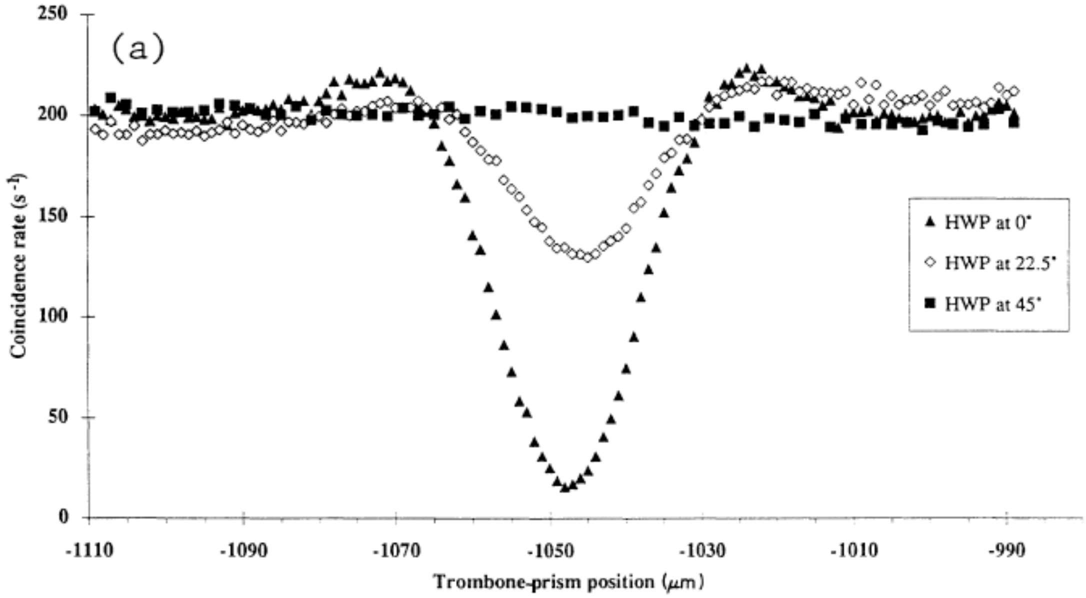
The polarisation quantum eraser



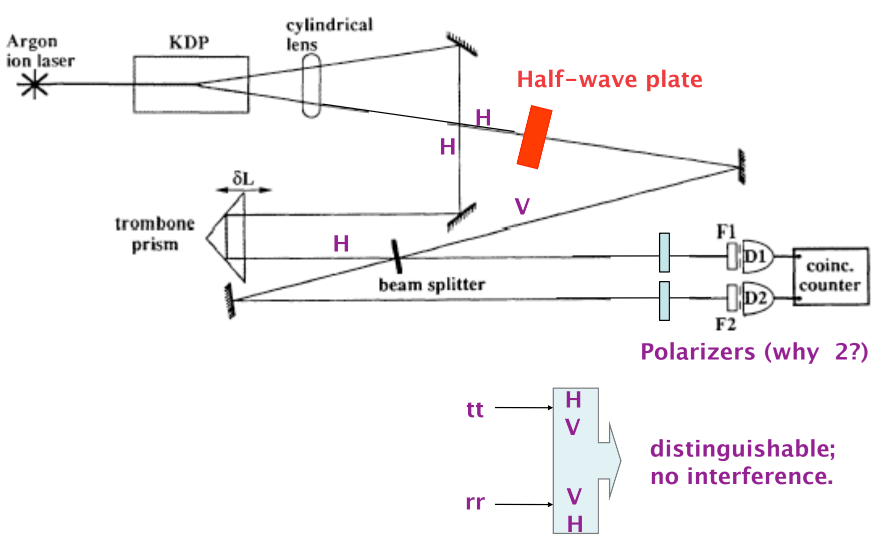


distinguishable;
no interference.

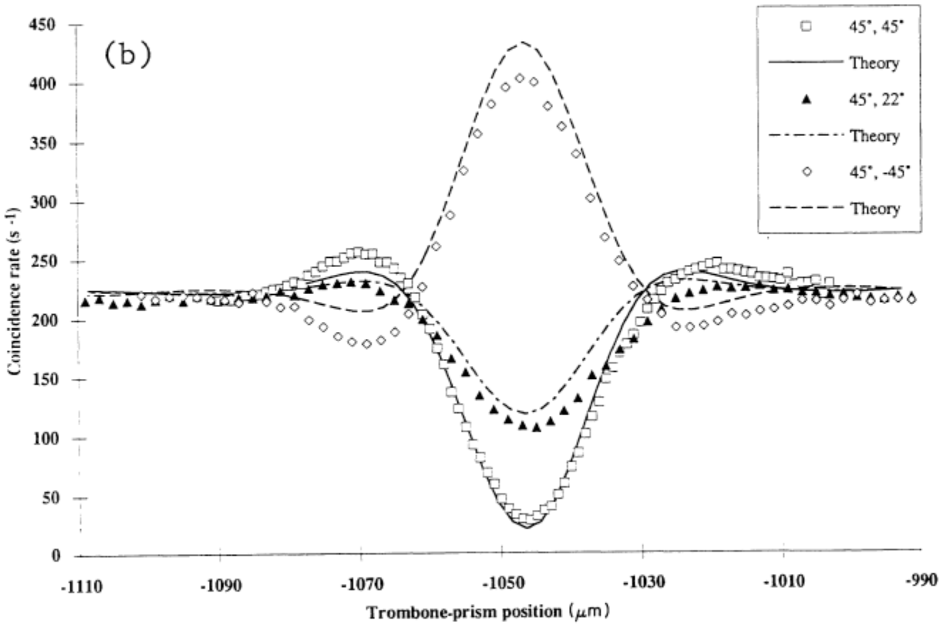
Interference going away...



The polarisation quantum eraser



And coming back again!



How complicated you have to make it sound if you want to get it published

We turn now to the general case with two polarizers set at arbitrary angles θ_1 and θ_2.

$$P_c(0)\approx\langle\psi|\hat{P}_{\mathrm{pol},1}(\theta_1)\hat{P}_{\mathrm{pol},2}(\theta_2)\hat{P}_{c,\mathrm{red}}{}'\hat{P}_{\mathrm{pol},2}(\theta_2)\hat{P}_{\mathrm{pol},1}(\theta_1)|\psi\rangle_{\Delta x=0}$$

$$=\tfrac{1}{2}[\langle 1_1^H 1_2^{H+\phi}|-\langle 1_1^{H+\phi}1_2^H|](|1_1^{H+\theta_1}\rangle\langle 1_1^{H+\theta_1}|)(|1_2^{H+\theta_2}\rangle\langle 1_2^{H+\theta_2}|)(\hat{a}^\dagger_{1,H}\hat{a}_{1,H}+\hat{a}^\dagger_{1,V}\hat{a}_{1,V})$$

$$\times(\hat{a}^\dagger_{2,H}\hat{a}_{2,H}+\hat{a}^\dagger_{2,V}\hat{a}_{2,V})(|1_2^{H+\theta_2}\rangle\langle 1_2^{H+\theta_2}|)(|1_1^{H+\theta_1}\rangle\langle 1_1^{H+\theta_1}|)\tfrac{1}{2}[|1_1^H 1_2^{H+\phi}\rangle-|1_1^{H+\phi}1_2^H\rangle]\ .$$

Using Eq. (A2), one can expand $|\tilde{\psi}\rangle_{\Delta x=0}=\hat{P}_{\mathrm{pol},2}(\theta_2)\hat{P}_{\mathrm{pol},1}(\theta_1)|\psi\rangle_{\Delta x=0}$. After simplifying algebra one finds

$$|\tilde{\psi}\rangle_{\Delta x=0}=|1_1^H1_2^H\rangle\cos\theta_1\cos\theta_2\sin(\theta_2-\theta_1)\sin\phi+||1_1^V1_2^V\rangle\sin\theta_1\sin\theta_2\sin(\theta_2-\theta_1)\sin\phi$$

$$+|1_1^H1_2^V\rangle\cos\theta_1\sin\theta_2\sin(\theta_2-\theta_1)\sin\phi+|1_1^V1_2^H\rangle\sin\theta_1\cos\theta_2\sin(\theta_2-\theta_1)\sin\phi\ .$$

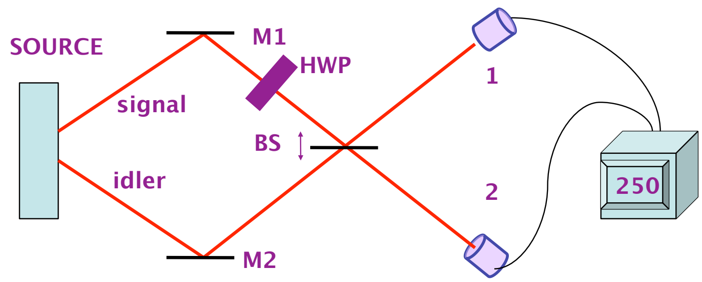
It then follows that

$$P_c(0)\approx\langle\tilde{\psi}|\hat{P}_{c,\mathrm{red}}{}'|\tilde{\psi}\rangle_{\Delta x=0}=\sin^2\phi\sin^2(\theta_2-\theta_1)\ ,$$

which is the more general case of Eq. (13).

"Calculations are for those who don't trust their intuition."

A simpler(?) way of seeing what happens...



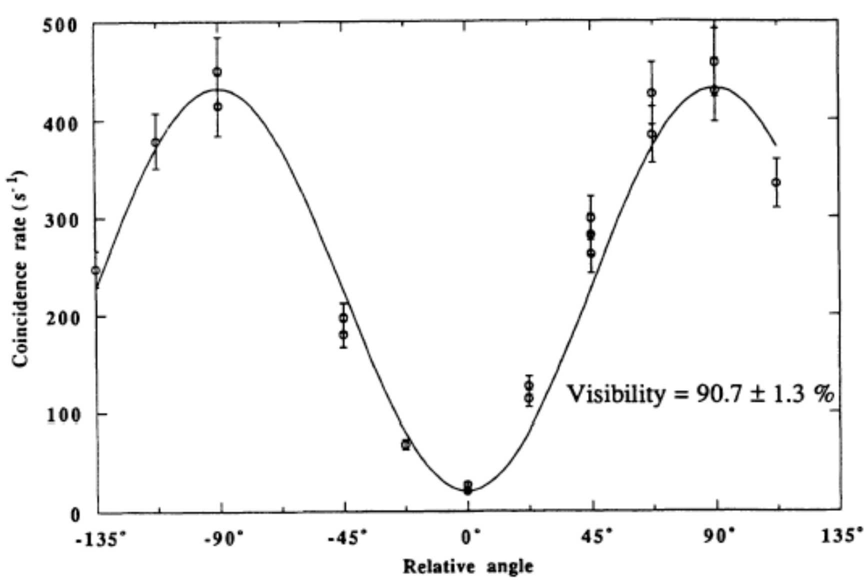
Vs Hi ⟶ (V2 + i V1) (H1 + i H2)
= 1H 2V – 1V 2H + i [1H 1V + 2H 2V]

In coincidence, only see |HV> – |VH> .... that famous EPR-entangled state.
Of course we see nonlocal correlations between the polarisations.

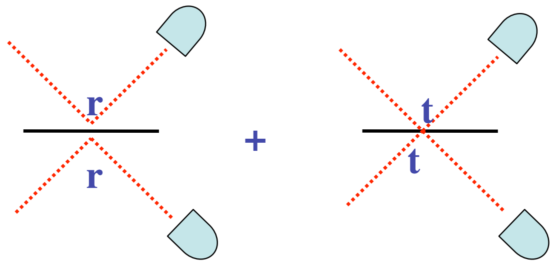
These joint-detection probabilities can be calculated directly, without collapse; add the amplitudes from HV and VH: $P(\theta_1,\theta_2) = |\cos(\theta_1)\sin(\theta_2)-\sin(\theta_1)\cos(\theta_2)|^2$
$= \sin^2(\theta_1 - \theta_2)$.

This is the Bell-Inequality experiment done by Shih&Alley and Ou&Mandel.

Polarisation-dependence of rate at centre of H-O-M dip...



Hong-Ou-Mandel Interference as a Bell-state filter



$r^2+t^2 = 0$; total destructive interf. (if photons indistinguishable).
If the photons begin in a symmetric state, no coincidences.

The only *antisymmetric* state is the singlet state |HV> – |VH>, in which each photon is unpolarized but the two are orthogonal. Nothing else gets transmitted.

This interferometer is a "Bell-state filter," used for quantum teleportation and other applications.

$\text{Log}_2(3)$ bits in a single photon

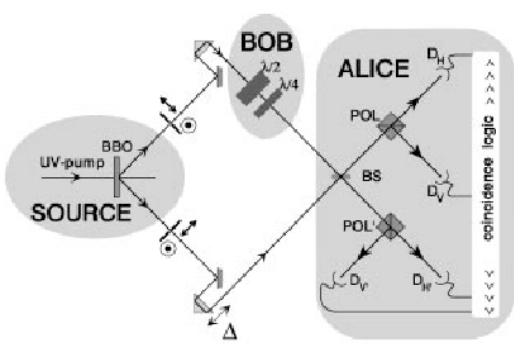


FIG. 1. Experimental setup for quantum dense coding. Because of the nature of the Si-avalanche photodiodes, the extension shown in the inset of Fig. 4 is necessary for identifying two-photon states in one of the outputs.

To extract both bits, one would need to distinguish all 4 Bell states – this can't be done with linear optics, but 2 of the 4 can, leaving a third (ambiguous) possiblity.

Bob's setting				
$\lambda/2$	$\lambda/4$	State sent	State at output of Bell-state analyzer	Alice's registration events
0°	0°	$\|\Psi^+\rangle$	$\{hv + h'v' + vh + v'h'\}/2$	Coincidence between D_H *and* D_V or $D_{H'}$ *and* $D_{V'}$
0°	90°	$\|\Psi^-\rangle$	$\{hv' - h'v' + v'h - vh'\}/2$	Coincidence between D_H *and* $D_{V'}$ or $D_{H'}$ *and* D_V
45°	0°	$\|\Phi^+\rangle$	$\{hh + vv + h'h' + v'v'\}/2$	2 photons in either D_H, D_V, $D_{H'}$, or $D_{V'}$
45°	90°	$\|\Phi^-\rangle$	$\{hh - vv + h'h' + v'v'\}/2$	2 photons in either D_H, D_V, $D_{H'}$, or $D_{V'}$

Mattle *et al.*, PRL 76, 4656 (1996)