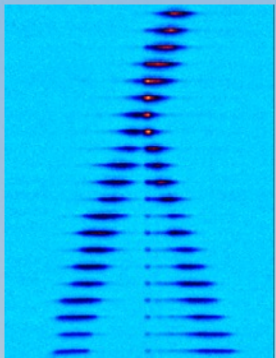
How to count one photon and get a(n average) result of 1000



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FPQP
ICTS, Bangalore
December 2016

Outline

Appetizer:

Intro to measurement tradeoffs

Weak-measurement

Measuring the measurement disturbance

Main Course:

How to count a single photon and get a result of 1000

- Giant optical nonlinearities
- NL phase shift driven by a single post-selected photon
- Weak-value amplification of the phase shift of a single photon
- (Questions about SNR)

Dessert:

Progress towards cold-atom tunneling experiments

(Digestif ?

Imaging as a Quantum State Discrimination problem
(better resolution through not discarding phase information)

?)

DRAMATIS PERSONÆ

Toronto quantum optics & cold atoms group:

Photons: **Hugo Ferretti** **Edwin Tham** Noah Lupu-Gladstein

Atoms: Ramon Ramos **David Spierings** Isabelle Racicot

Atom-Photon Interfaces: Josiah Sinclair Shaun Pepper Alex Bruening

Theory: Aharon Brodutch **TBD:** Arthur Pang

Some past contributors: Matin Hallaji, Greg Dmochowski, Shreyas Potnis, Dylan Mahler, Amir Feizpour, Alex Hayat, Ginelle Johnston, Xingxing Xing, **Lee Rozema**, Kevin Resch, Jeff Lundeen, Krister Shalm, Rob Adamson, Stefan Myrskog, Jalani Kanem, Ana Jofre, Arun Vellat Sadashivan, Chris Ellenor, Samansa Maneshi, Chris Paul, Reza Mir, Sacha Kocsis, Masoud Mohseni, Zachari Medendorp, Fabian Torres-Ruiz, Ardavan Darabi, Yasaman Soudagar, Boris Braverman, Sylvain Ravets, Nick Chisholm, Rockson Chang, Chao Zhuang, Max Touzel, Julian Schmidt, Xiaoxian Liu, Lee Liu, James Bateman, Luciano Cruz, Zachary Vernon, Timur Rvachov, Marcelo Martinelli, Morgan Mitchell,…

Some helpful theorists:

Daniel James, Pete Turner, Robin Blume-Kohout, Chris Fuchs, Howard Wiseman, János Bergou, John Sipe, Paul Brumer, Michael Spanner...

25 years cifar Canadian Institute for Advanced Research
NSERC CRSNG
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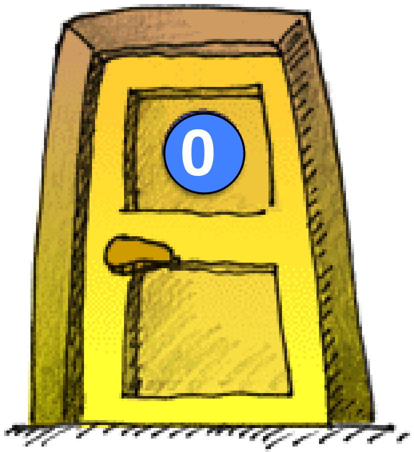
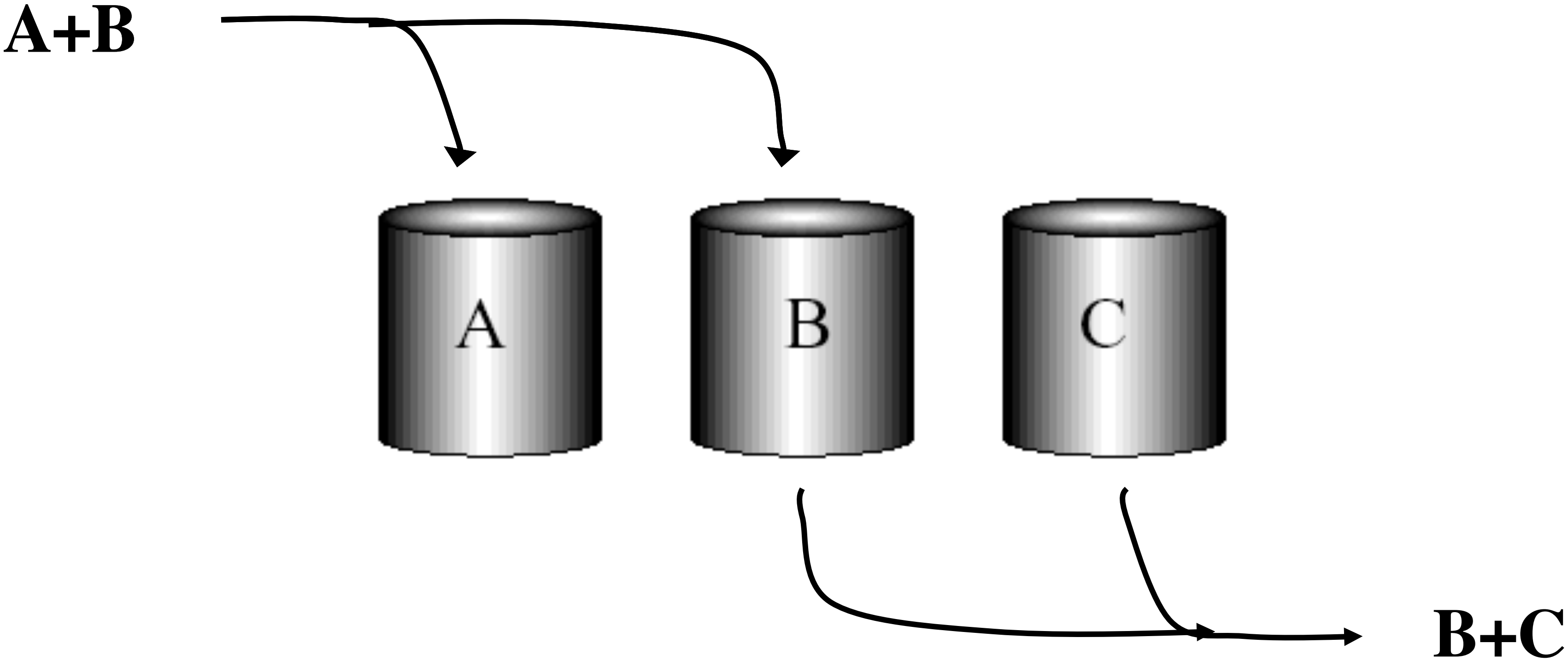
Quantum archaeology



Predicting the past...



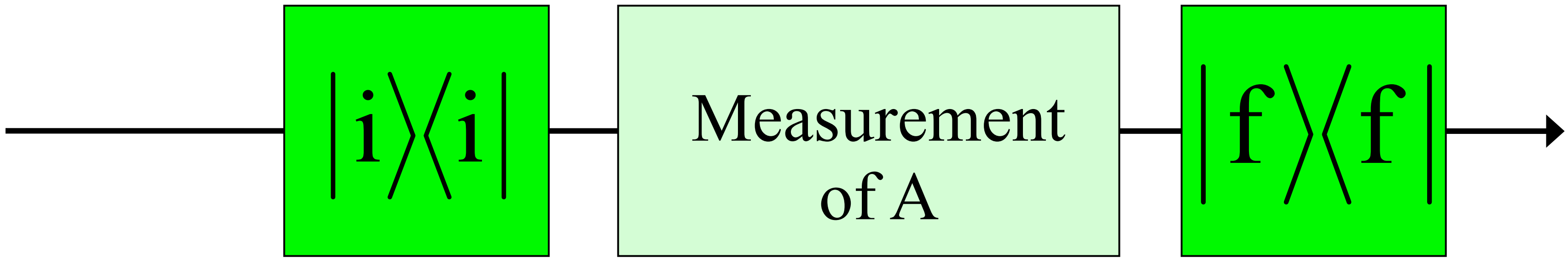
What are the odds that the particle was in a given box (e.g., box B)?

It had to be in B, with 100% certainty.

Conditional measurements (Aharonov, Albert, and Vaidman)

AAV, PRL 60, 1351 ('88)

Prepare a particle in |i> …try to "measure" some observable A…
***postselect* the particle to be in |f>**

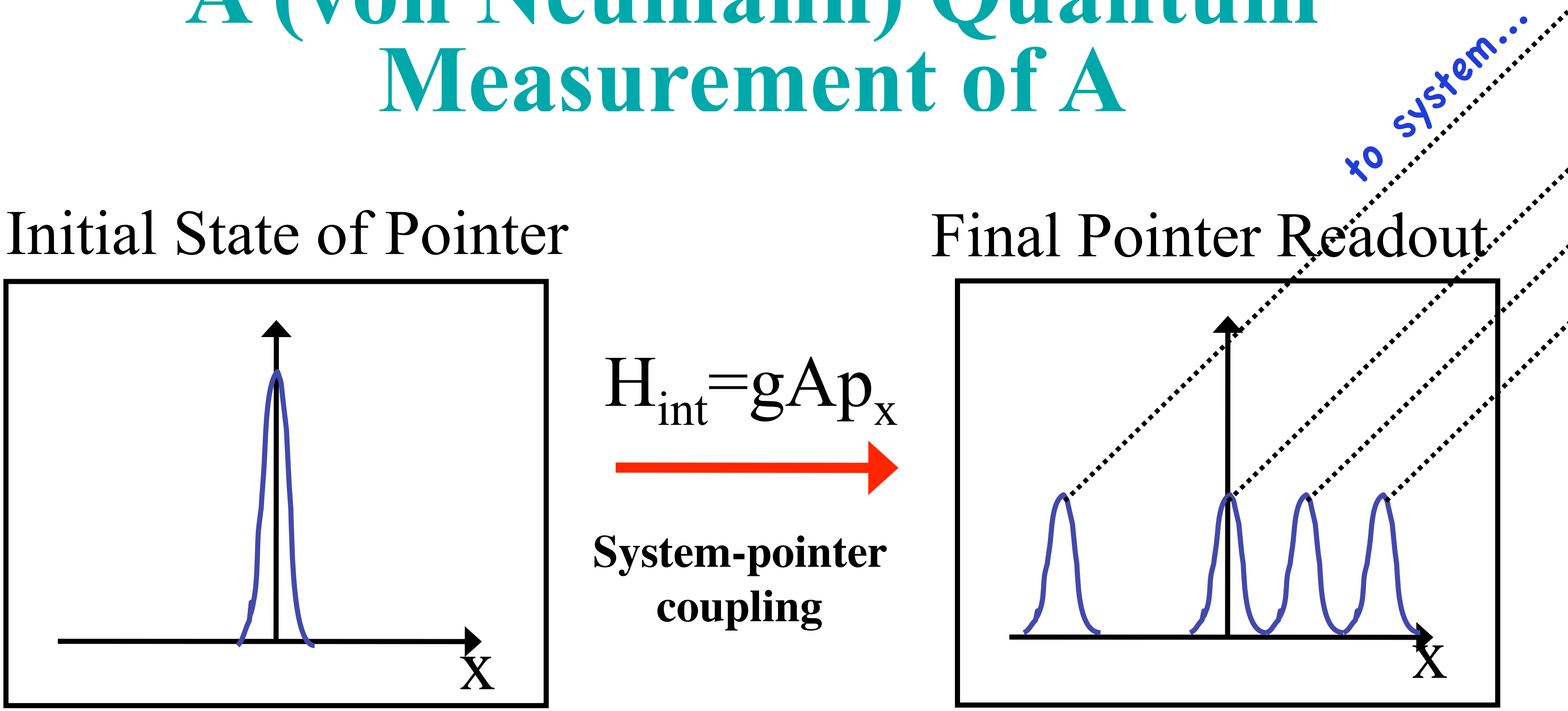


Does <A> depend more on i or f, or equally on both?
Clever answer: both, as Schrödinger time-reversible.
Conventional answer: i, because of collapse.

Reconciliation: measure A "weakly."
Poor resolution, but little disturbance.

the "weak value"
(but how to determine?)

A (von Neumann) Quantum Measurement of A



Well-resolved states

System and pointer become entangled

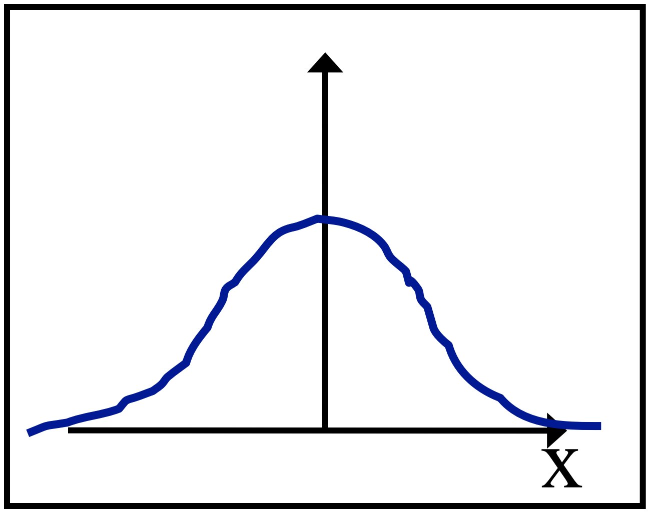
Decoherence / "collapse"

Large back-action

Aharonov, Albert, & Vaidman, PRL 60, 1351 (1988)

A Weak Measurement of A

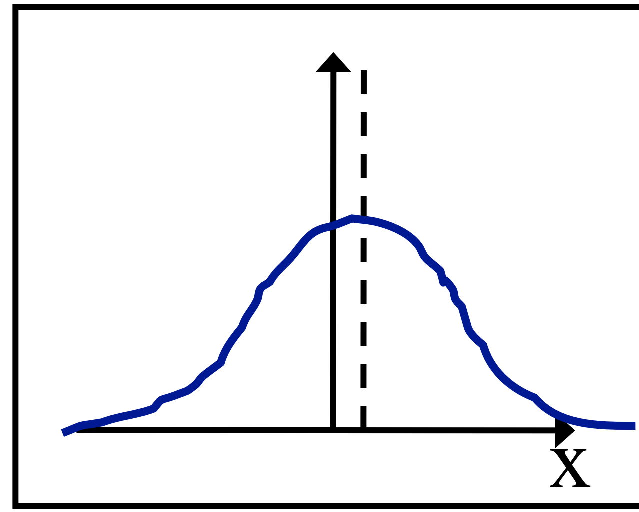
Initial State of Pointer



System-pointer coupling

$H_{int} = gAp_x$

Final Pointer Readout



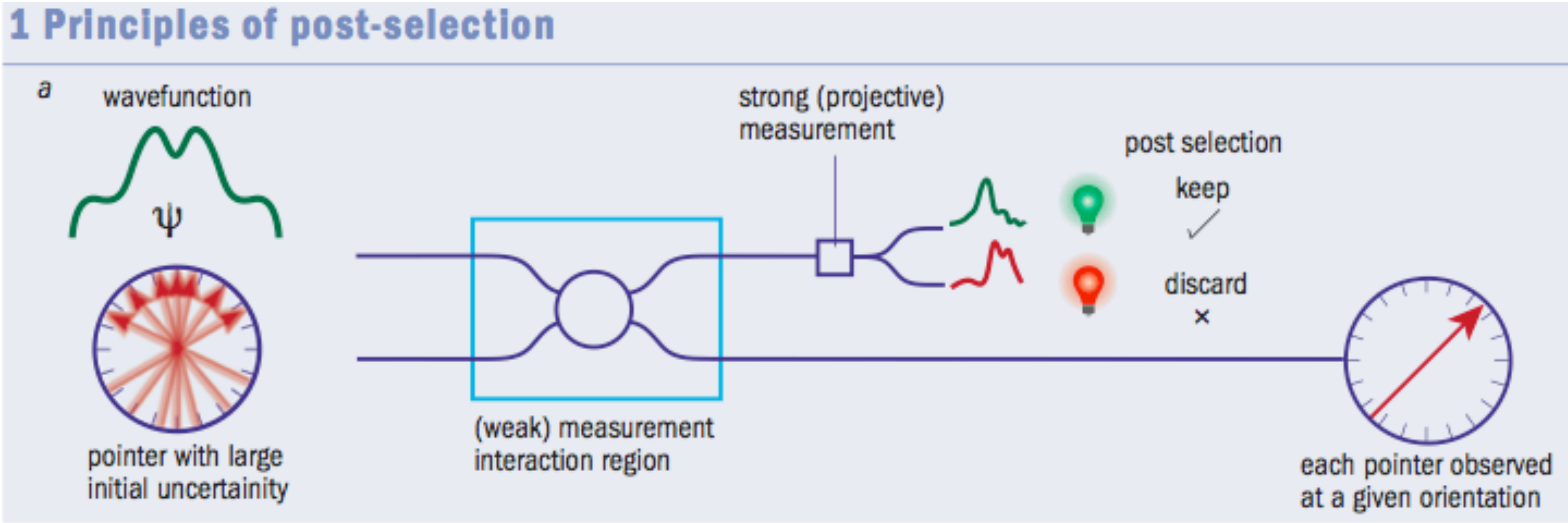
Poor resolution on each shot.

On the other hand, essentially no disturbance to the system

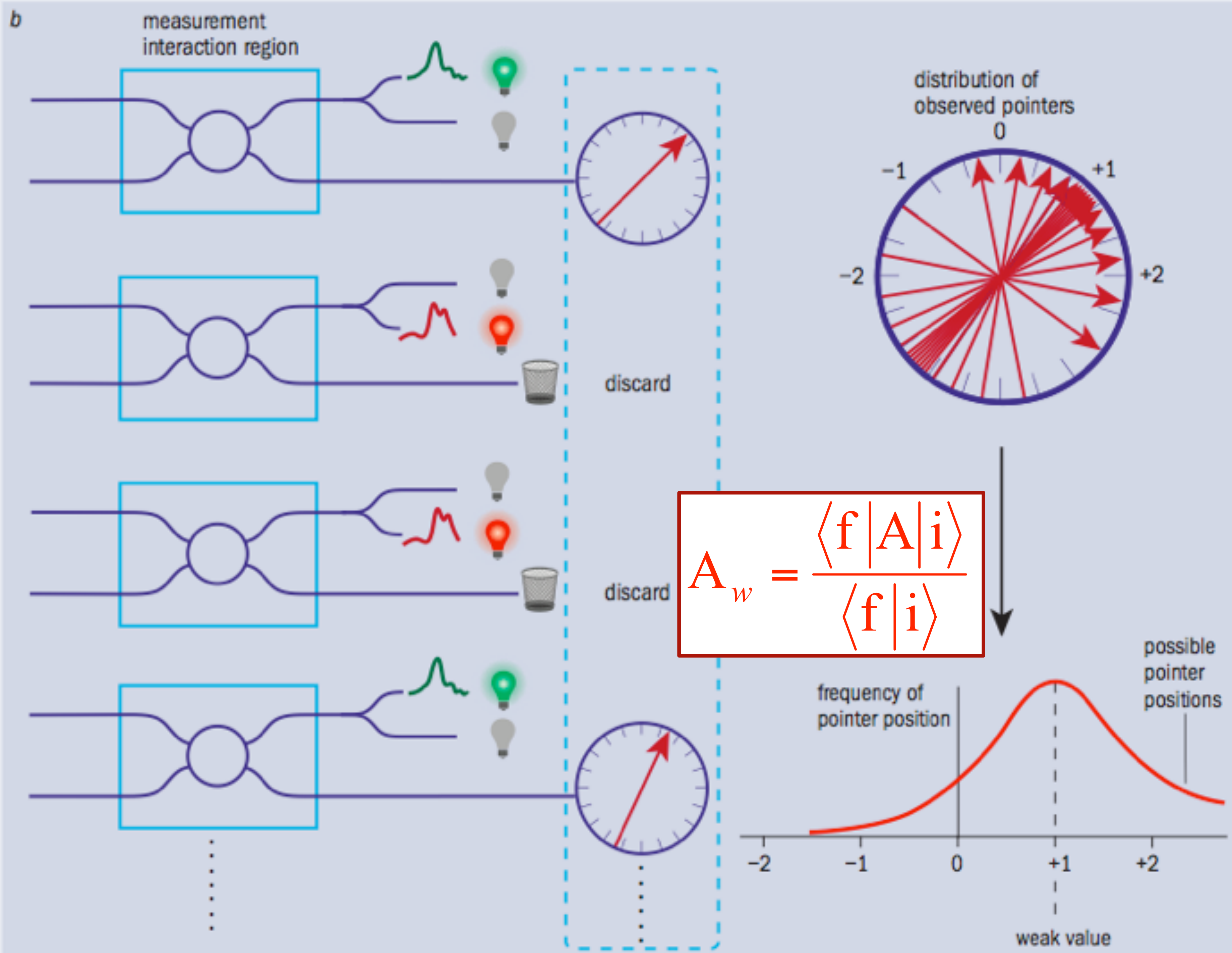
Strong: $|\Psi\rangle_s \phi_p(x) \rightarrow \sum_i c_i |\psi_i\rangle_s \phi_p(x - g a_i)$

Weak: $|\Psi\rangle_s \phi_p(x) \rightarrow |\Psi\rangle_s \phi_p(x - g\langle A_s\rangle)$

“Post-selecting” on the desired final state



And now, even though each pointer position seems to be pretty random, if you make millions of measurements and build up statistics, you can figure out the average shift --

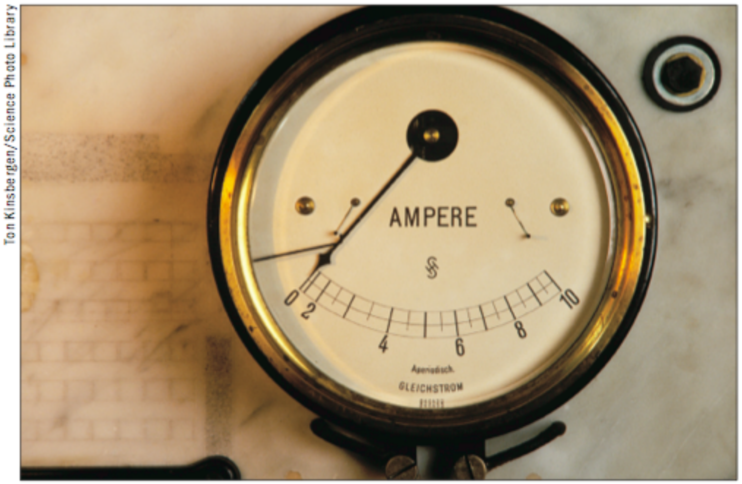


$$A_w = \frac{\langle f|A|i\rangle}{\langle f|i\rangle}$$

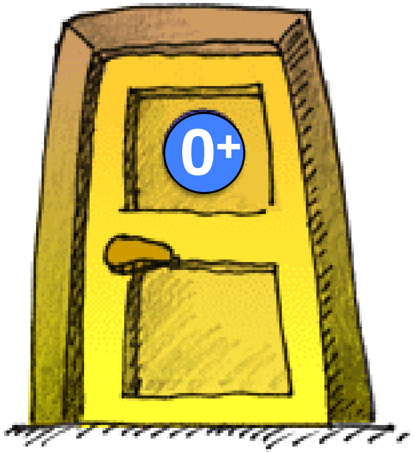
(This remains controversial)

Some would argue that whatever this Byzantine strategy yields, it is not really a "measurement" of anything (it's not on page 36 of the QM textbooks yet)...

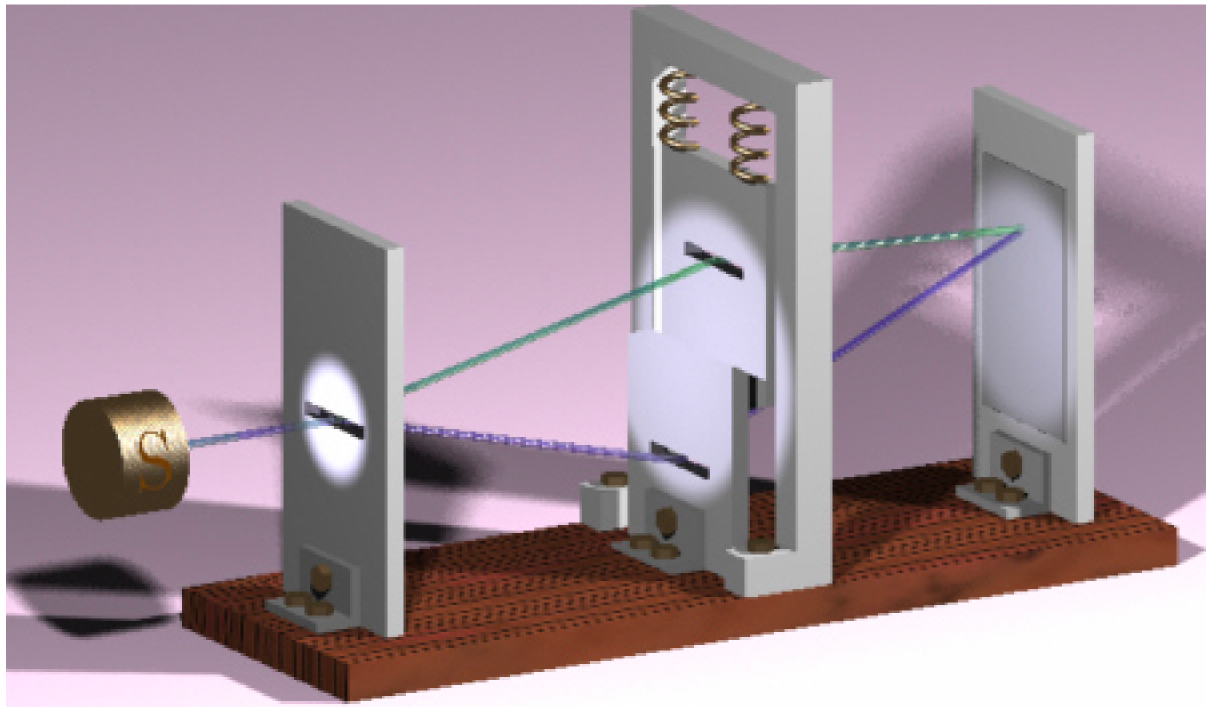
Ton Kinsbergen/Science Photo Library



Some of us instead maintain that the QM definition of measurement has only ever aimed to model what happens when we really interact with measuring devices, and if interacting with them strongly changes the results, it's only natural to investigate what interacting with them weakly does.



“Breaking” Heisenberg’s Uncertainty Principle ?



Any precise measurement of X is guaranteed to disturb P, by an amount $\Delta P \geq \hbar / 2\Delta X$

“Any precise measurement of X is guaranteed to disturb P, by an amount $\Delta P \geq h/2\Delta X$”

What I’ve always taught my students:

- **This is true, but it puts a limit on measurement only.**
- **A much deeper statement puts a limit on *reality*:**

“Any *state* in which X is *determined* precisely is guaranteed to have an *intrinsic* uncertainty in P, such that $\Delta P \geq h/2\Delta X$”

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“Any *state* in which X is *determined* precisely is guaranteed to have an *intrinsic* uncertainty in P, such that $\Delta P \geq h/2\Delta X$”

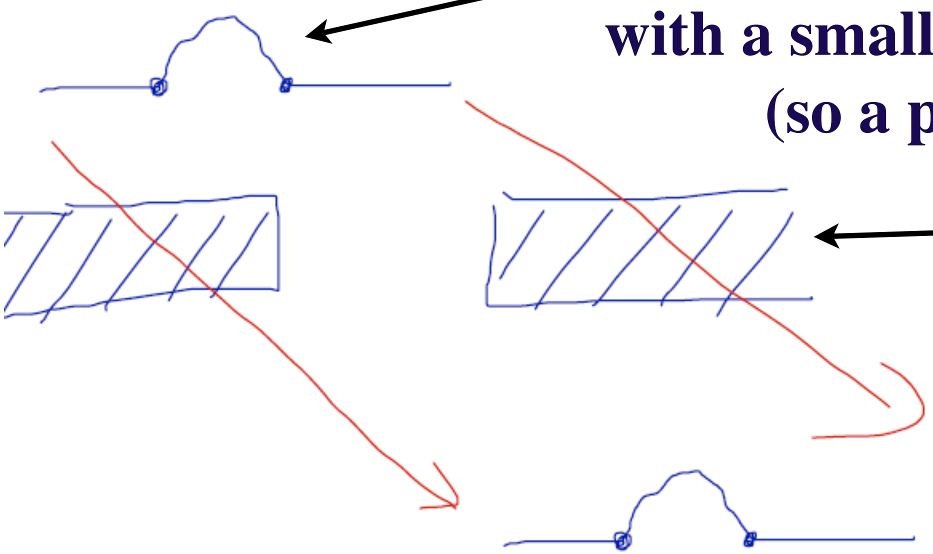
What I tell my students now:

Not only does the first version put a limit on measurement only, but it’s also *wrong*!

Rotating-arm approximation for x & p...

The wave function of some particle, with a small uncertainty in position (so a pretty big one in momentum)

Rotating-arm approximation for x & p...

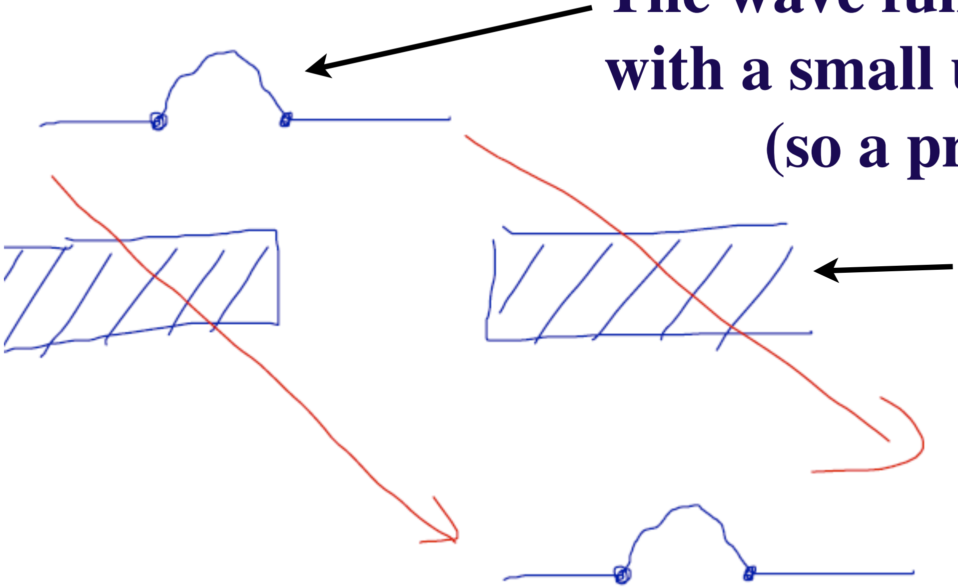


The wave function of some particle, with a small uncertainty in position
(so a pretty big one in momentum)

A slit – if the particle is transmitted, this constitutes a measurement that it was in the slit...

But if the slit is wider than the original wave function, the particle never even sees the walls;
how could the particle be disturbed at all?

Rotating-arm approximation for x & p...



The wave function of some particle, with a small uncertainty in position (so a pretty big one in momentum)

A slit – if the particle is transmitted, this constitutes a measurement that it was in the slit...

So we have *confirmed* that the particle is near x=0, with some finite precision – and we have done this without disturbing the momentum at all.

(Of course, the final momentum is uncertain – there was enough uncertainty in the state all along, and I didn't need to add any more with my measurement!)

Ozawa's relation

Heisenberg's uncertainty principle
for *variances* is proved in every textbook,
and we take no issue with it:

$$\Delta(A)\Delta(B) \geq \langle[A,B]\rangle / 2$$

A similar relation for measurement precision
$\varepsilon(A)$ of the probe vs. disturbance to the
system $\eta(B)$ is, however, false:

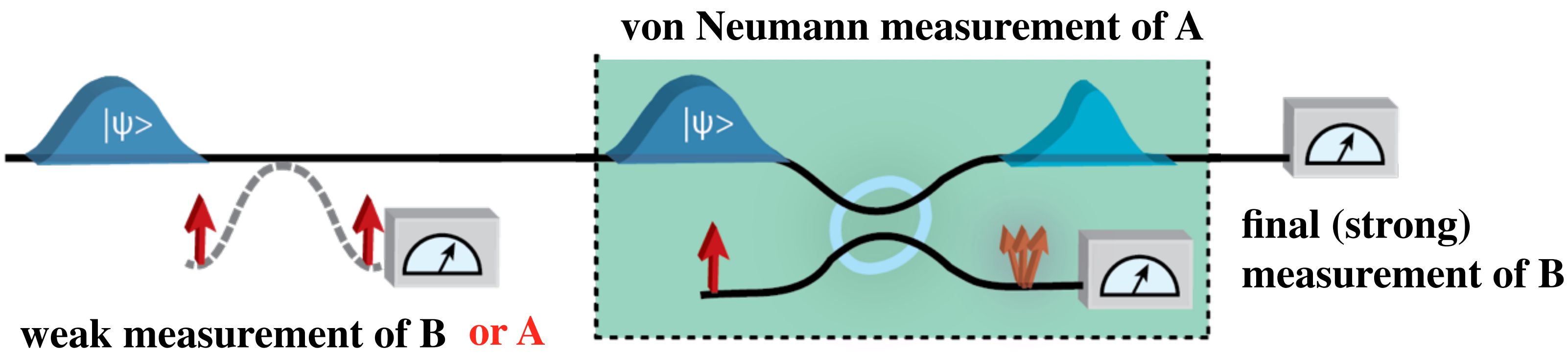
$$\varepsilon(A)\eta(B) \geq \langle[A,B]\rangle / 2$$

Ozawa, PRA 67, 042105 (2003):

$$\epsilon(A)\eta(B) + \epsilon(A)\Delta B + \eta(B)\Delta A \geq \frac{1}{2}\langle[A,B]\rangle$$

But how can you measure the disturbance due to a measurement?
You would need to know B before and after the measurement –
but unless you're already in an eigenstate of B, this would change
the state (and the RHS of the inequality).

Proposal Using Weak Measurements



Consider a von Neumann measurement of A

- The system becomes entangled with probe, disturbing the system

- **Define disturbance to B as the RMS difference between the value of B before and after the measurement**

- **Define precision of A as the RMS difference between the value of A of the system before the measurement and the value of A on the probe**

Lund & Wiseman, NJP 12, 093011 (2010)

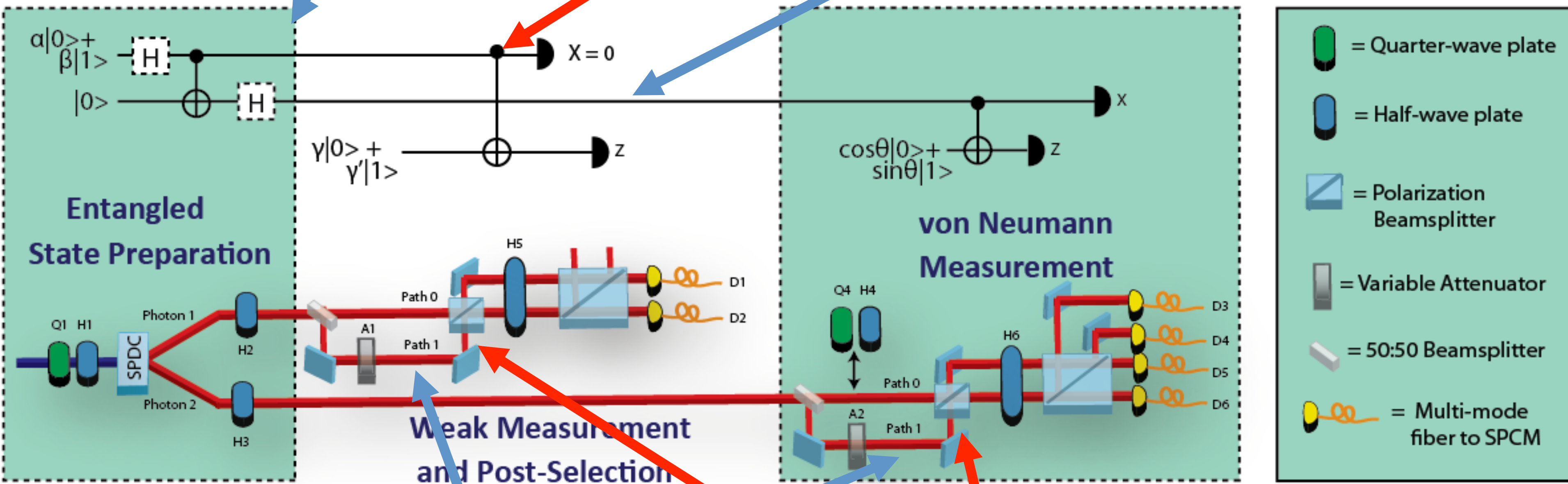
ALTERNATE APPROACH:
theory: **Ozawa, Ann. Phys. 311, 350 (2004)**
expt: **Erhart *et al*., Nature Physics 8, 185 (2012)**

Putting it all together

To implement consecutive C-NOT gates start with an entangled state

Do first C-NOT with qubit 1,

teleport state to qubit 2, leaving it free to control a second C-NOT



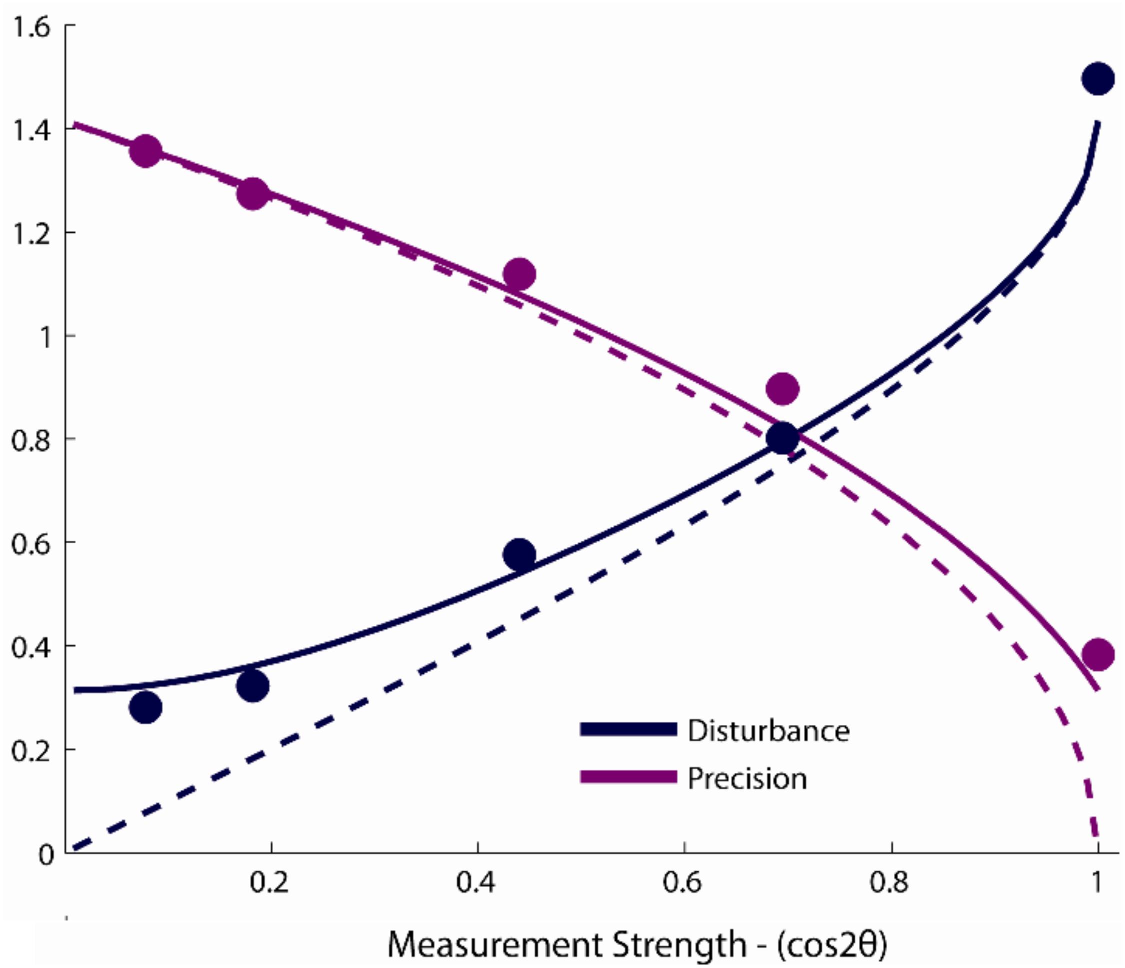
Probes are both path qubits
State set with variable attenuator

PBS's implement CNOTs
Polarization qubit controls the path qubit

Results – Disturbance & Precision

Fix the strength of the weak probe, vary the strength of the von Neumann measurement and observe the precision and disturbance

Dashed lines are theory, solid lines are simulations accounting only for imperfect entangled state preparation

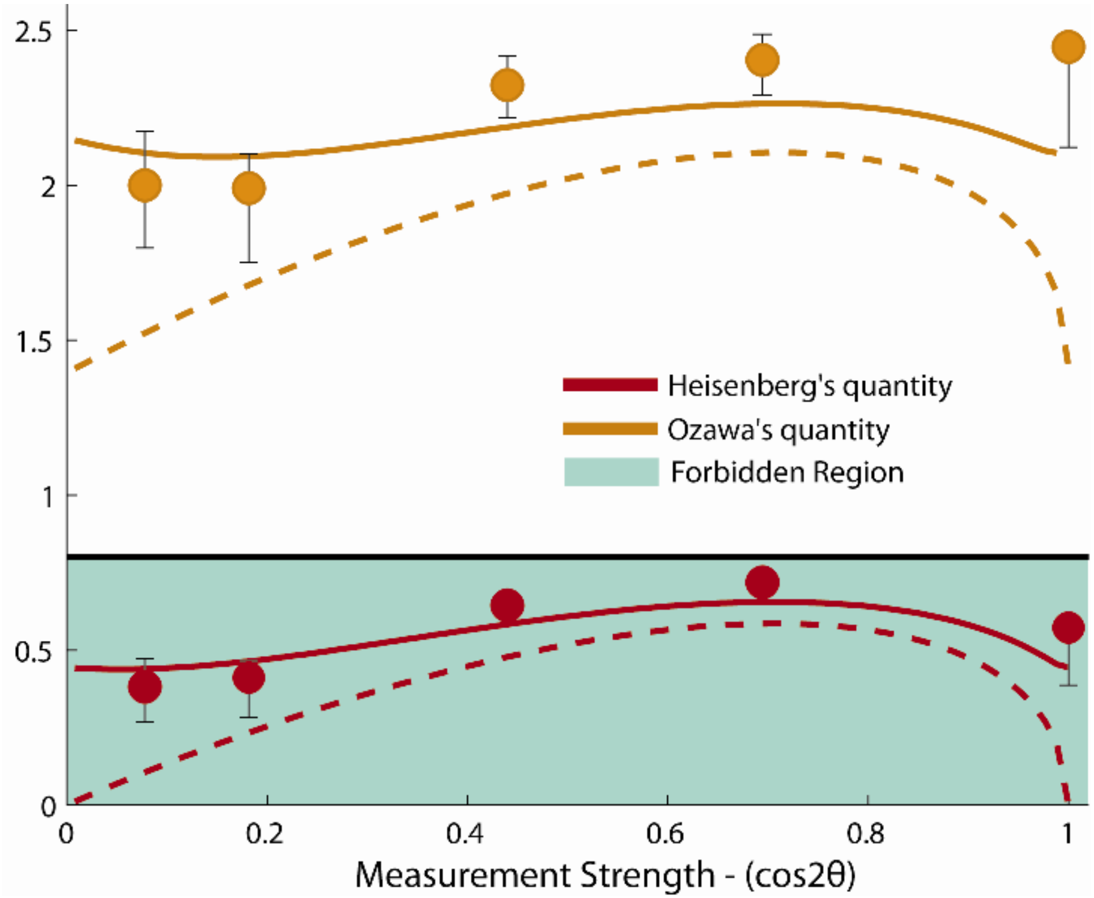


Results – Ozawa & Heisenberg's Quantities

Rozema et al., PRL 109, 100404 (2012)

Forbidden region set by measuring of <Y> on the qubit after the weak measurement and teleportation

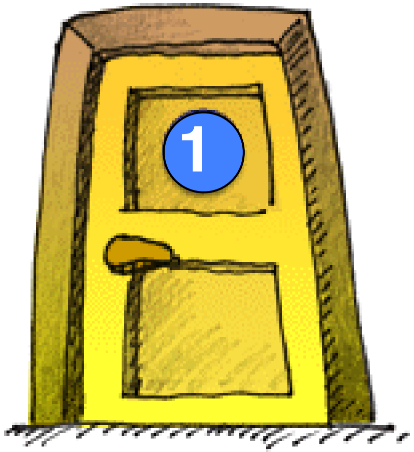
Dashed lines are theory, solid lines are simulations accounting only for imperfect entangled state preparation



Heisenberg's relation is clearly violated $\varepsilon(A)\eta(B) \geq 1/2\langle[A,B]\rangle$

Ozawa's remains valid $\epsilon(A)\eta(B) + \epsilon(A)\Delta B + \eta(B)\Delta A \geq \frac{1}{2}\langle[A,B]\rangle$

MAIN COURSE: COUNTING 1 PHOTON AND GETTING A RESULT OF 1000

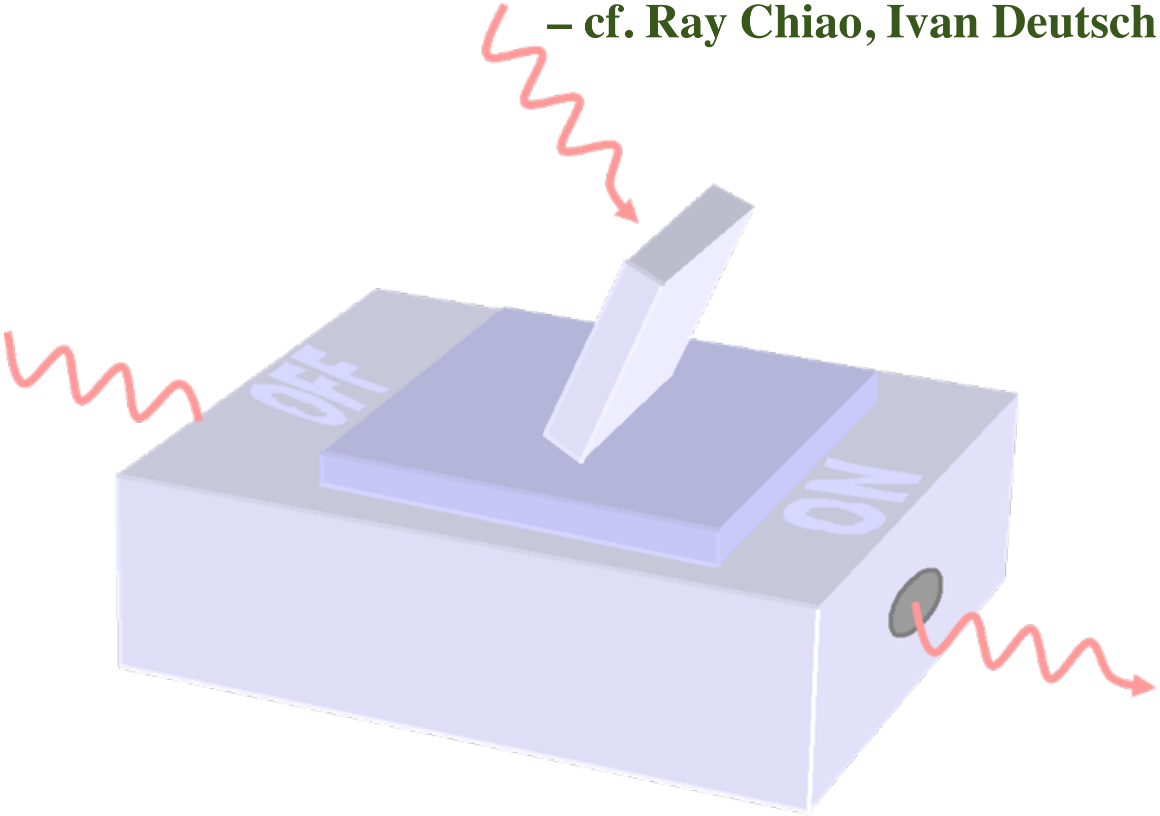


Background:

Quantum non-demolition measurements
via weak (/giant) optical nonlinearities

Practical motivation: quantum NLO (e.g., weak "giant nonlinearities")

"Giant" optical nonlinearities...
(a route to optical quantum computation;
and in general, to a new field of *quantum nonlinear optics*)
– cf. Ray Chiao, Ivan Deutsch, John Garrison)



Motivation: quantum NLO (e.g., weak "giant nonlinearities")

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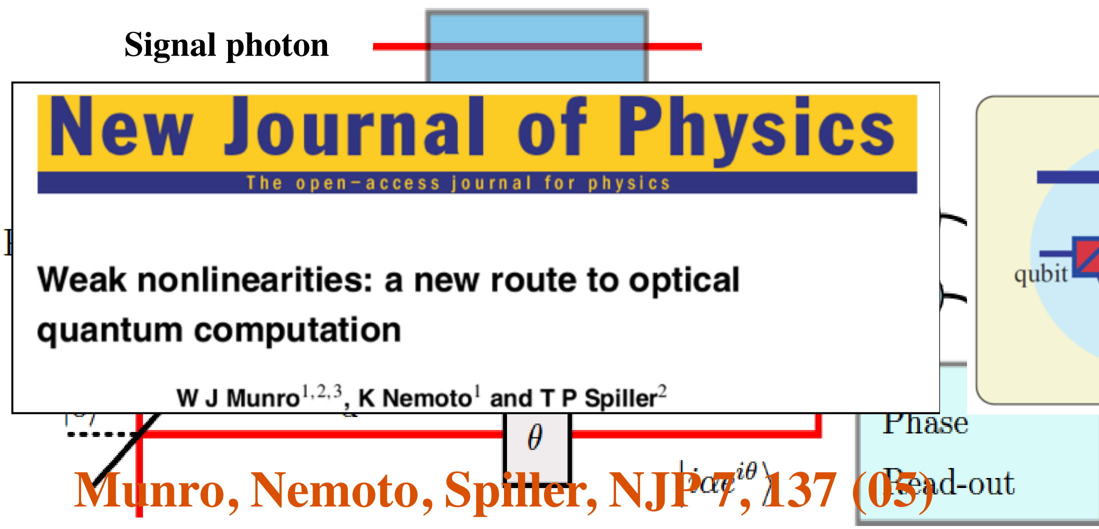
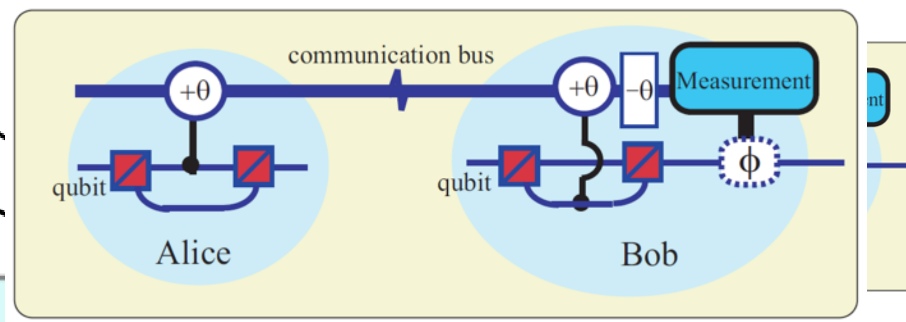
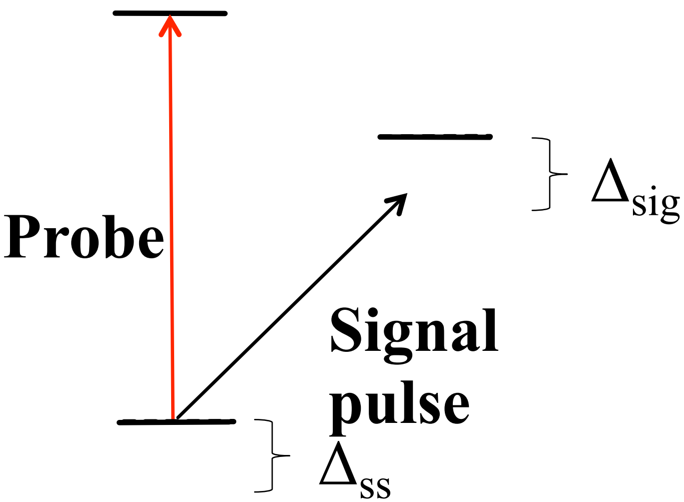


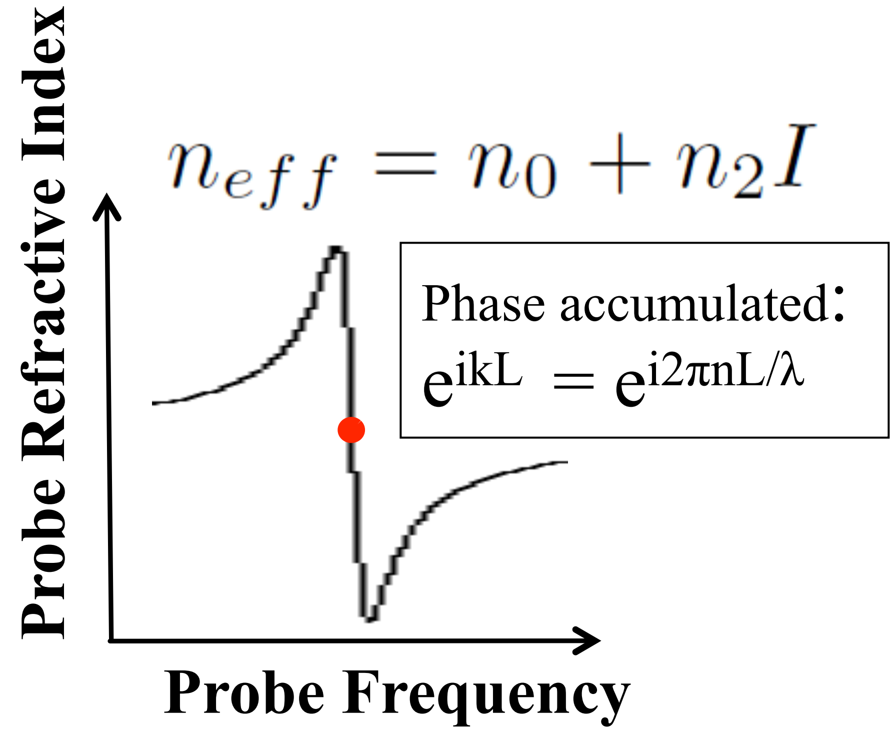
Munro, Nemoto, Spiller, NJP 7, 137 (05)



(Also of course, cf. "giant giant nonlinearities,"
e.g., Lukin & Vuletic and Rempe with Rydberg atoms;
Jeff Kimble *et al.* on nanophotonic approaches; Gaeta Rb in hollow-core fibres; et cetera)

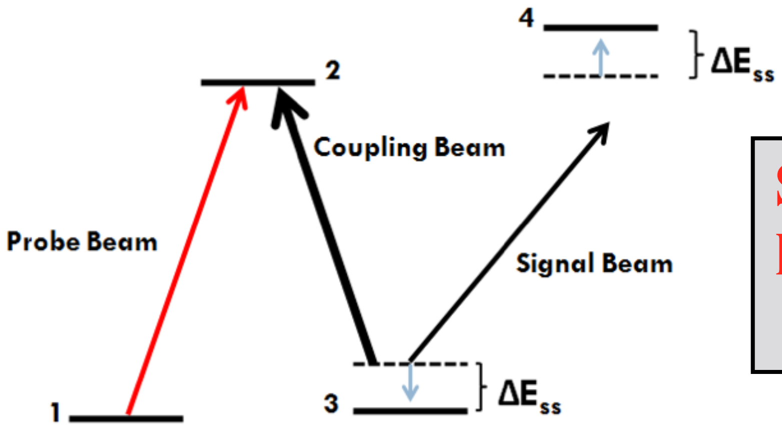
Cross-phase modulation (XPM)





AC Stark shift changes effective detuning, changing index of refraction experienced by probe

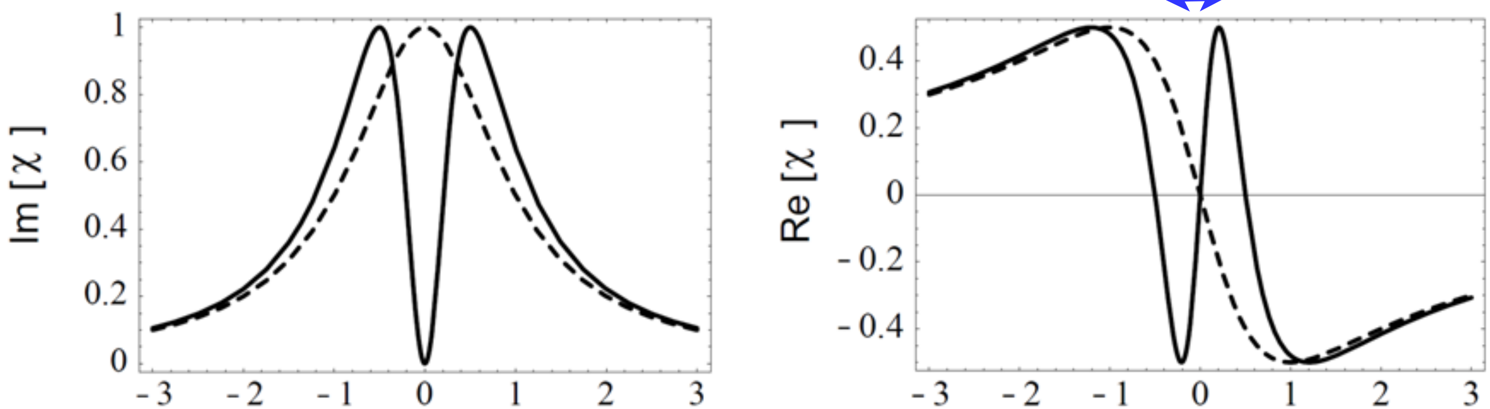
EIT-enhanced XPM



e.g., Schmidt & Imamoglu, Opt. Lett. 21, 1936 (96)

Steep slope of dispersion curve -> higher sensitivity to AC Stark shift (& transparency too)

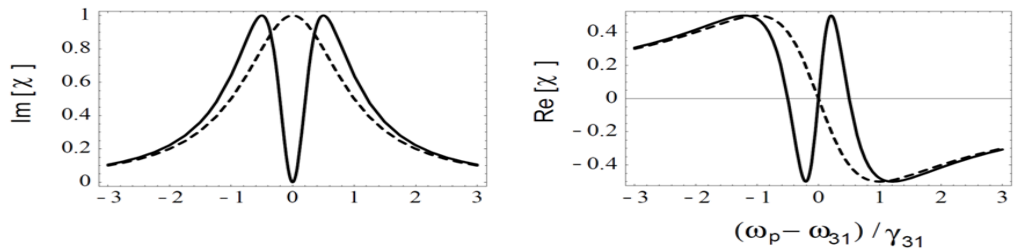
EIT width -> 0 as I_{coup} -> 0

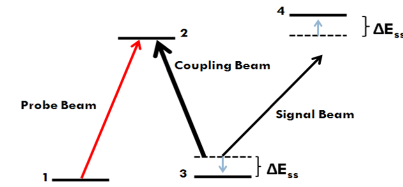


Narrower transparency windows yield larger cross-phase shifts

AC Stark shift is intensity-dependent – i.e. broadband signal pulses produce larger XPM

EIT-enhanced XPM ?





What is the use of a narrow transparency window if the signal pulse is broad? (E.g., 7 MHz single photons from our Rb-tuned OPO)

No problem: put a narrowband probe in the window, and the (broadband) signal on the other transition

But still: if the EIT bandwidth is 100 kHz, a 100 ns pulse is much shorter than the 10 us response time...

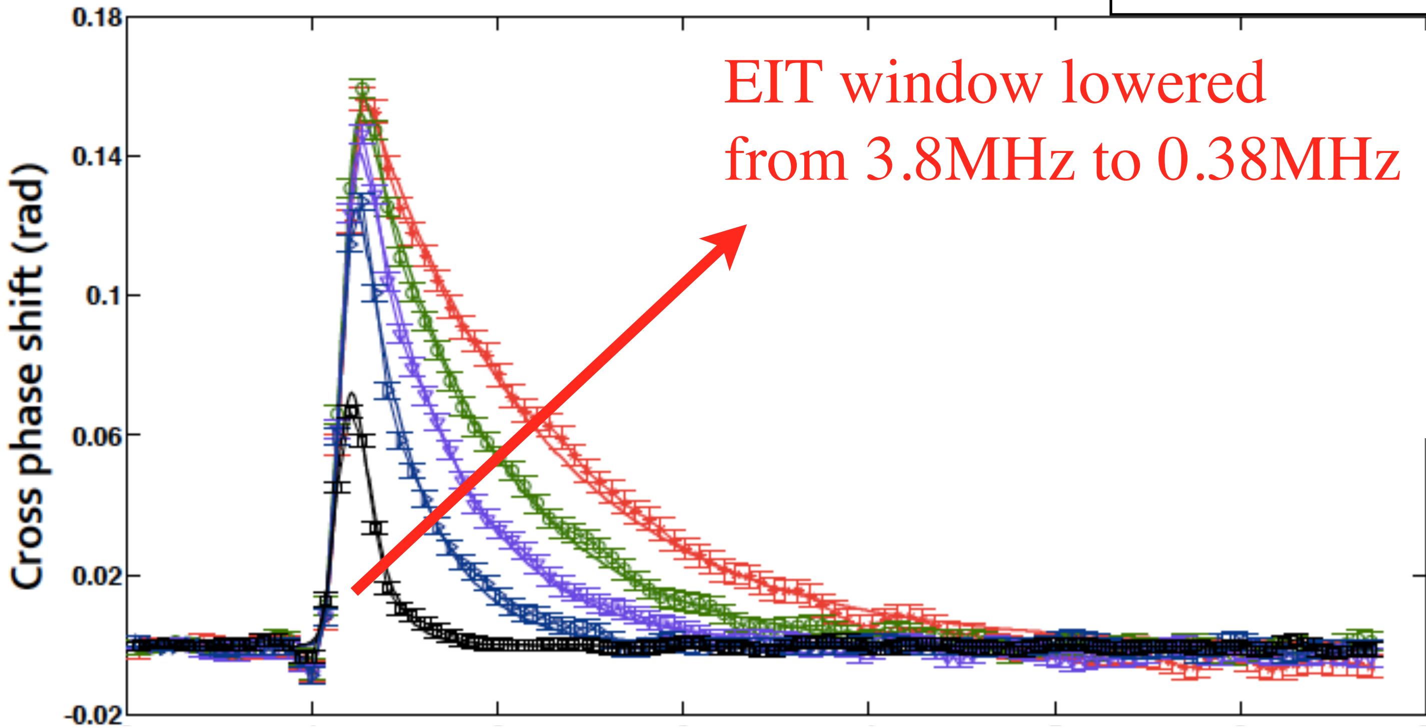
G. Sinclair, Physical Review A **79**, 023815 (2009)

M. Pack, R. Camacho, and J. Howell, Physical Review A **74**, 013812 (2006) **76**, 033835 (2007)

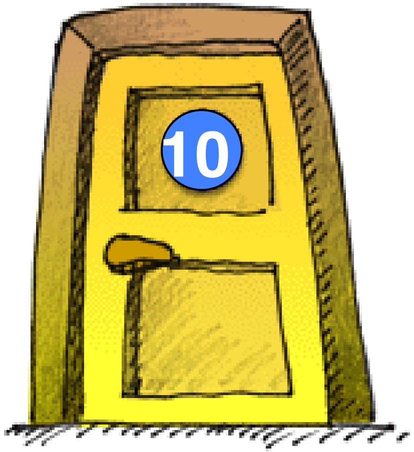
XPM for narrow EIT windows

40 ns (6 MHz) Gaussian pulse, 0.8 uW peak power, 40 MHz detuned from a sample of cold ^{85}Rb atoms (OD=3).

THEORY: PRA 93, 013843 (2016)
EXP'T: PRL 116, 173002 (2016)



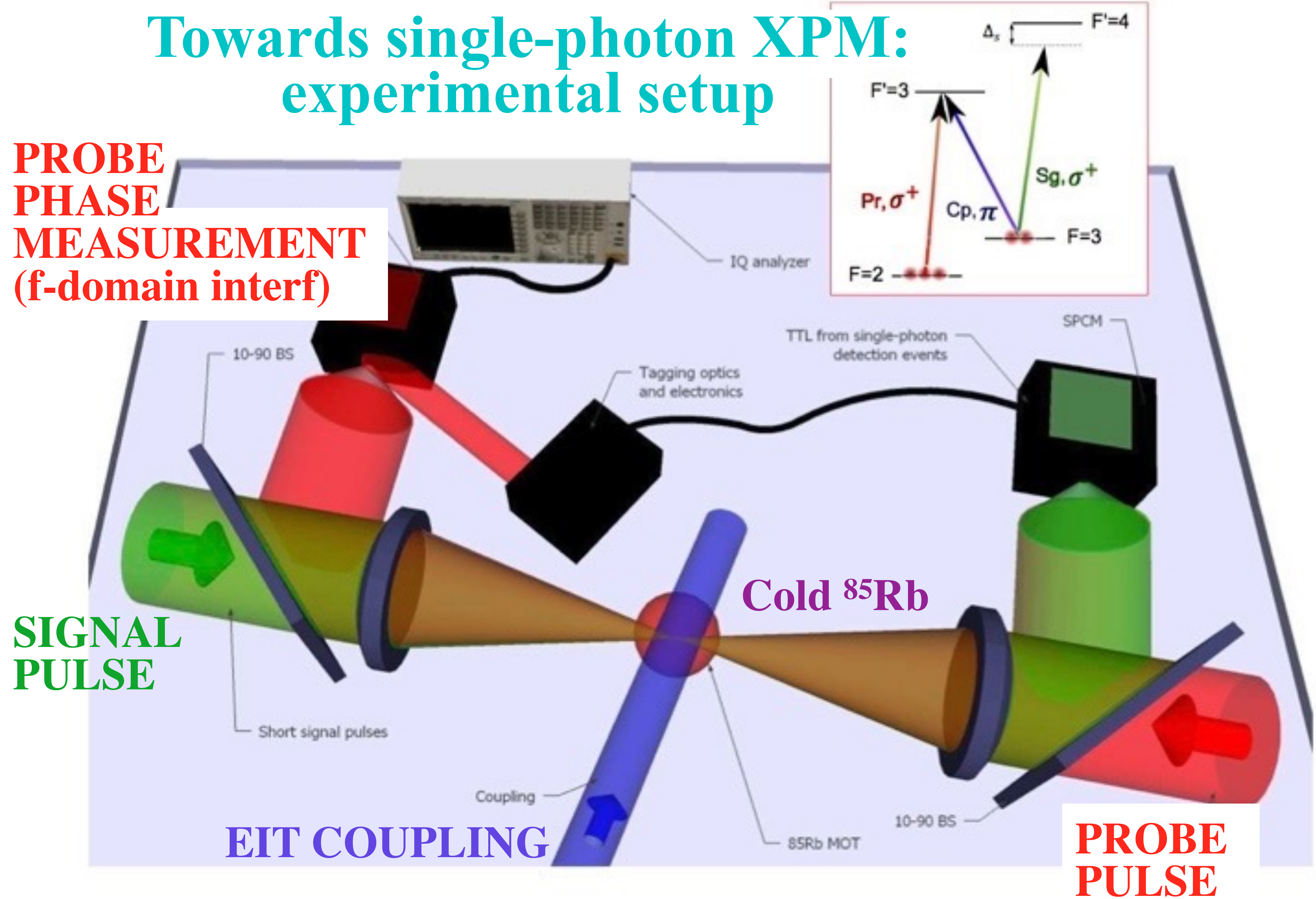
As EIT linewidth lowered below about half the pulse bandwidth, peak phase shift saturates – but it does not fall.
Moreover, the system memory time grows, so the narrow window continues to improve the *measurability* of the phase shift



Experiment:

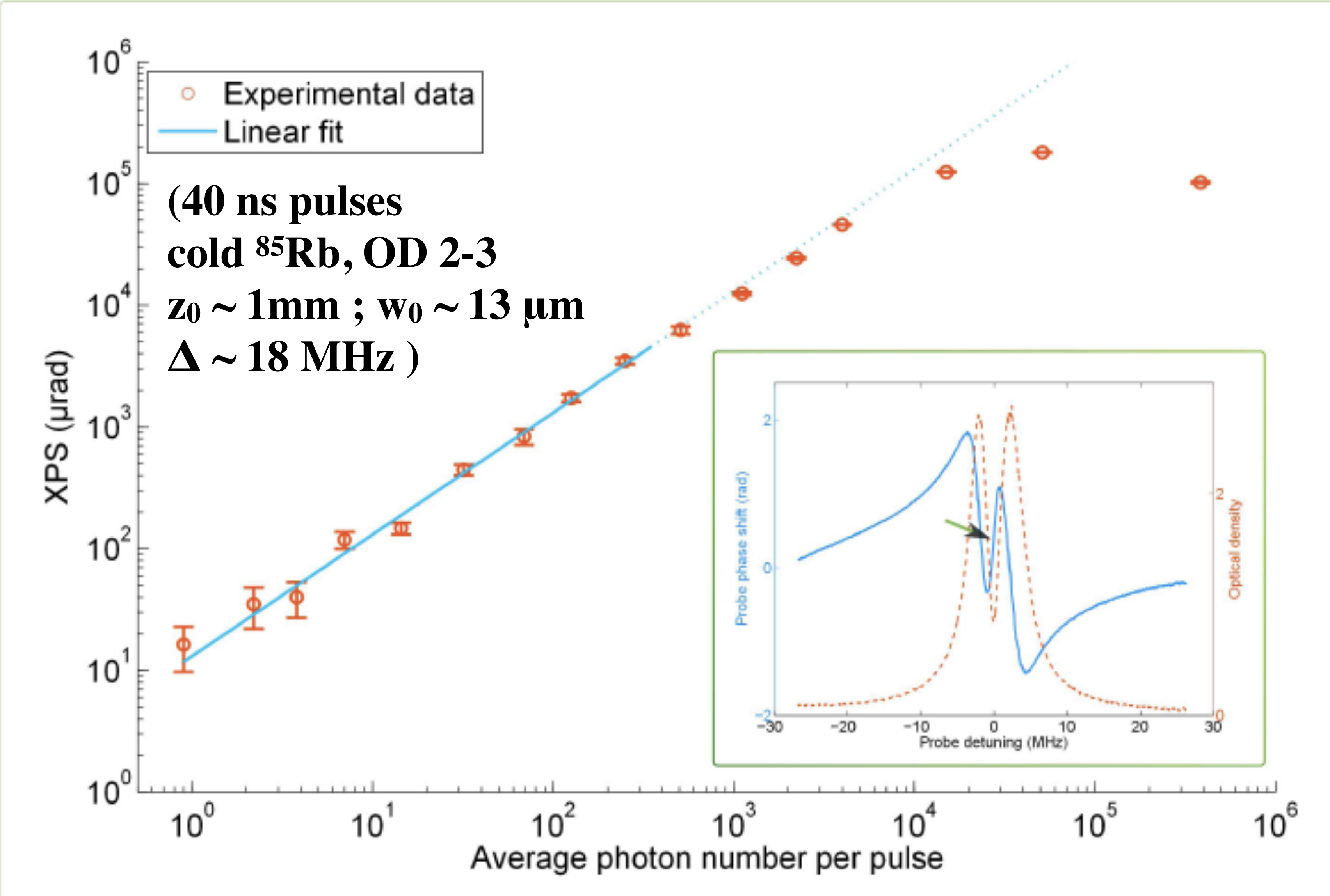
Observing the nonlinear effect of a single photon

Towards single-photon XPM: experimental setup

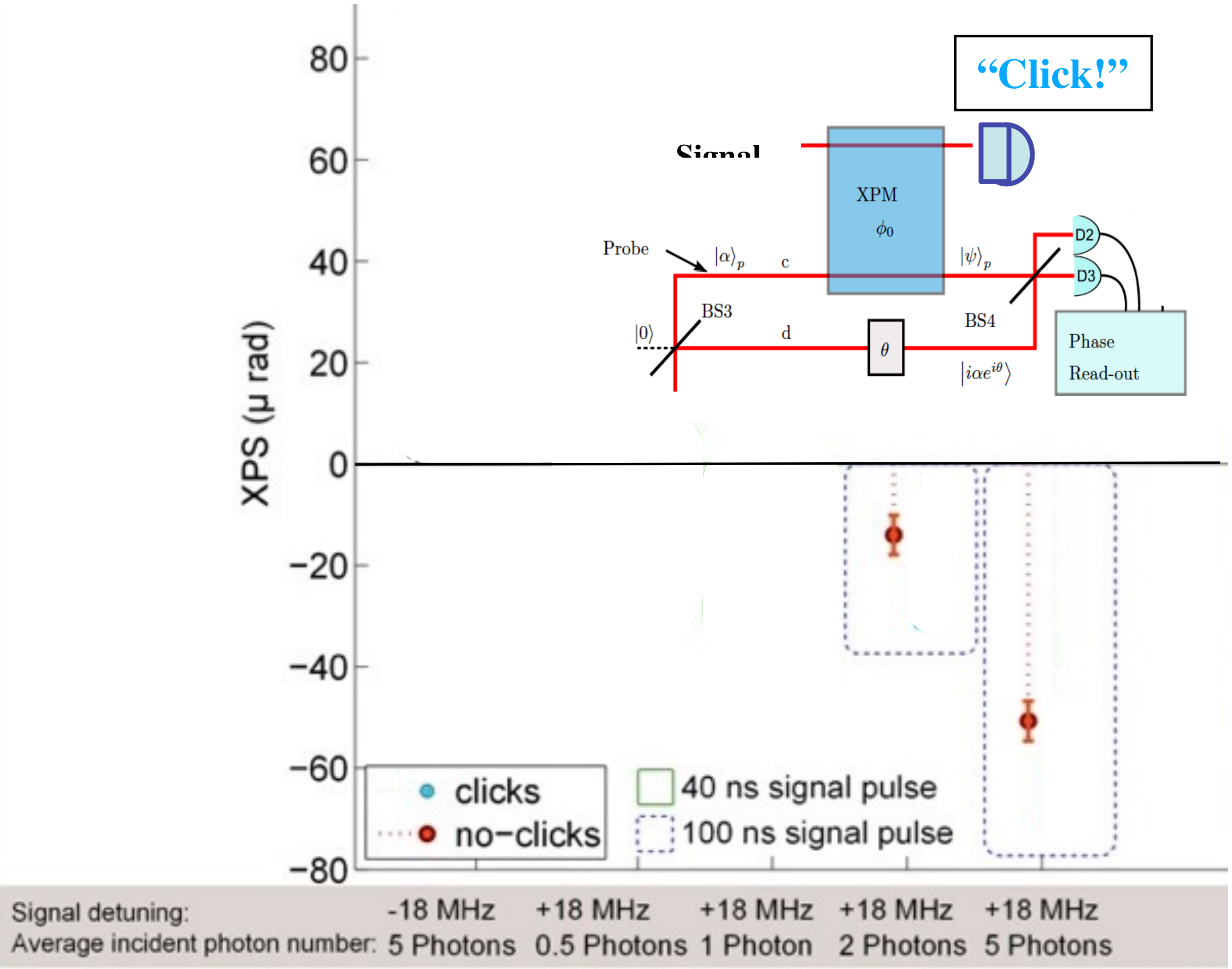


A. Feizpour et al., Nature Physics, DOI: 10.1038/nphys3433 (2015)

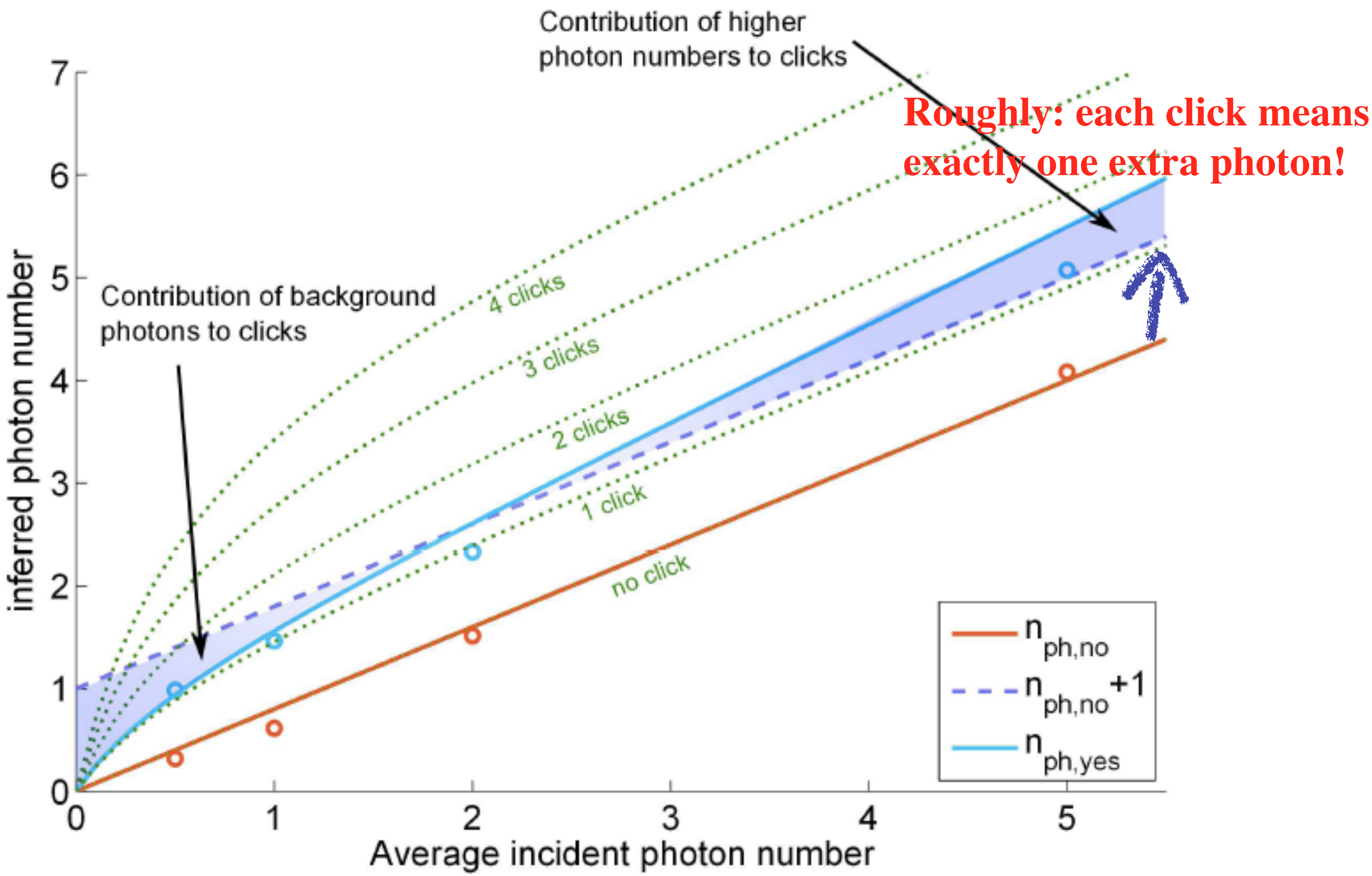
Measurement of cross phase shift, down to signal pulses with <n> = 1



Non-linear phase shift due to single photons



Post-selected single photons



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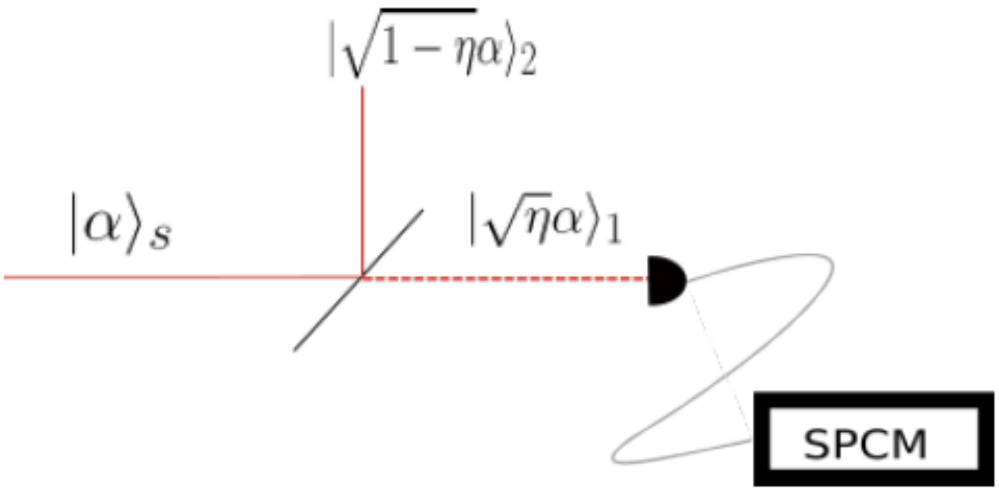
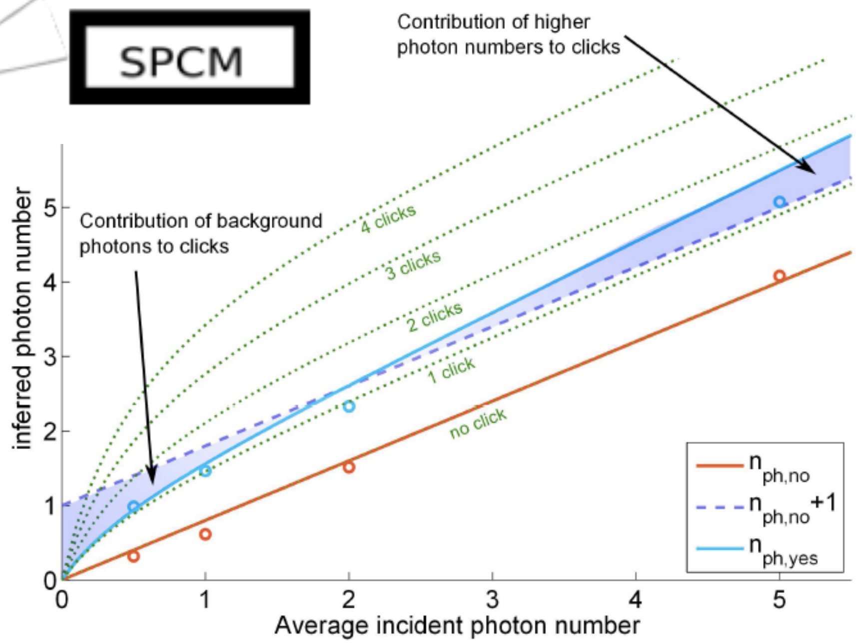
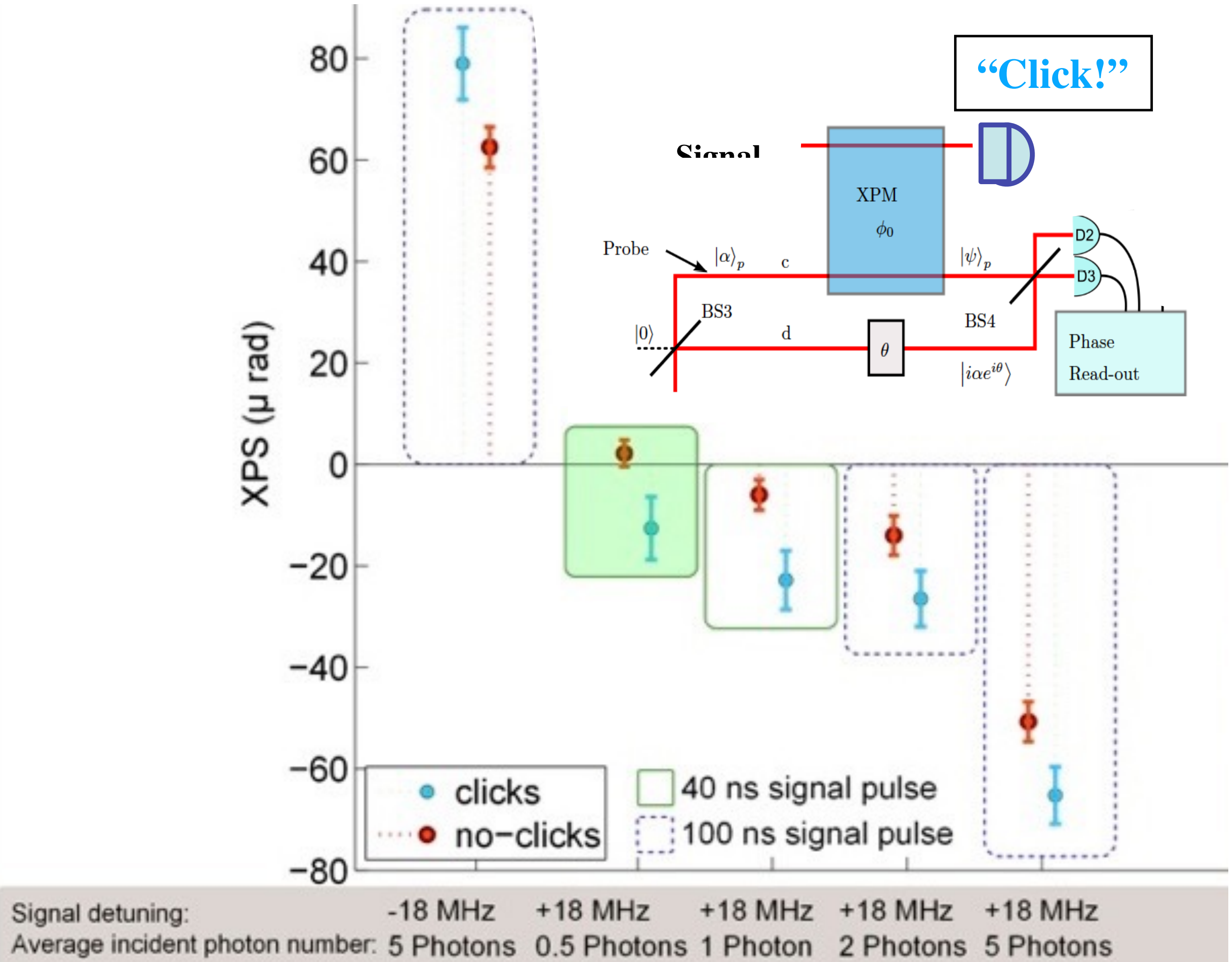




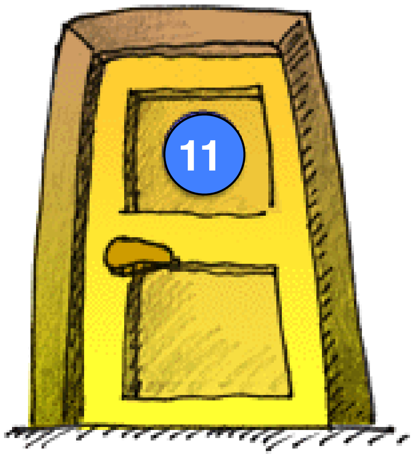
Non-linear phase shift due to single photons



A. Feizpour et al., Nature Physics, DOI: 10.1038/nphys3433 (2015)

Can we ask what "that" one photon was doing before we observed it?

(How should one describe post-selected states?)



OR:

Can a single photon have the effect of 1000 photons?

Aharonov, Albert, & Vaidman, PRL 60, 1351 (1988)

BER 14 PHYSICAL REVIEW LETTERS

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel

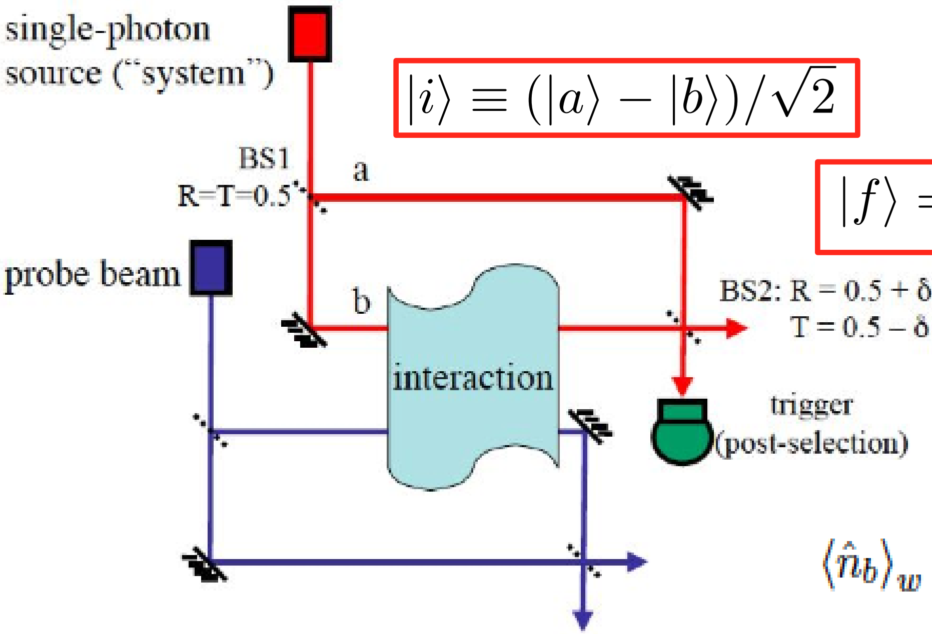
(Received 30 June 1987)

$$A_w = \frac{\langle f | A | i \rangle}{\langle f | i \rangle}$$

may be very big if the postselection is very unlikely (<f|i> very small)...

"Weak value amplification" – pioneering applications, e.g.,
Hosten & Kwiat, Science 319, 5864 (08);
Ben Dixon, Starling, Jordan, & Howell, PRL 102, 173601 (09); etc

How the result of the measurement of the number of 1 photon can be 100



$$|i\rangle \equiv (|a\rangle - |b\rangle)/\sqrt{2}$$

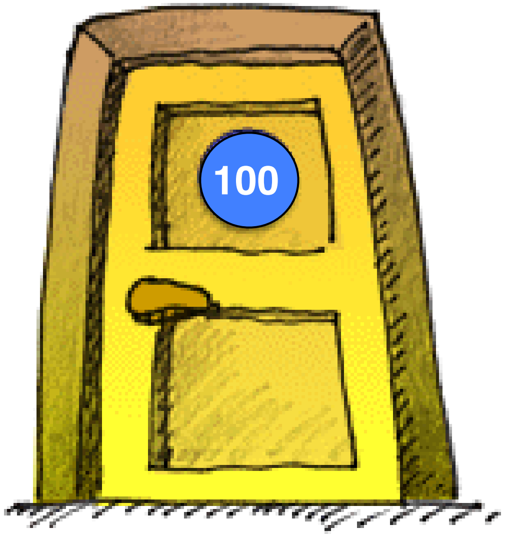
$$|f\rangle = r\,|a\rangle + t\,|b\rangle$$

$$\langle \hat{n}_b \rangle_w = \frac{\langle f|\,\hat{n}_b\,|i\rangle}{\langle f|\,i\rangle} = \frac{t/\sqrt{2}}{(t-r)/\sqrt{2}} = \frac{(1+\delta)/2}{\delta} \simeq \frac{1}{2\delta}\,.$$

When the post-selection succeeds, the phase shift on the probe may be much larger than the phase shift due to a single photon -- *even though there only ever* is *at most one signal photon*!

$\langle n \rangle_w$ may be >> 1.

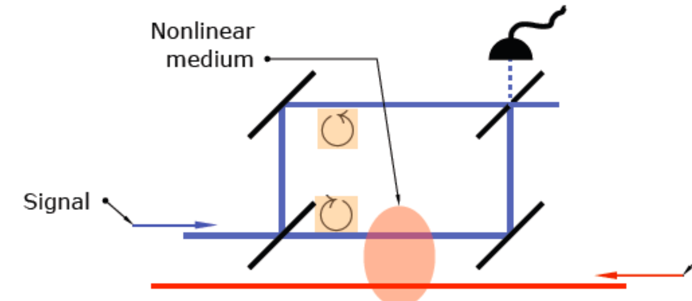
<u>Weak Measurement Amplification of Single-Photon Nonlinearity</u>, Amir Feizpour, Xingxing Xing, and Aephraim M. Steinberg Phys Rev Lett 107, 133603 (2011)

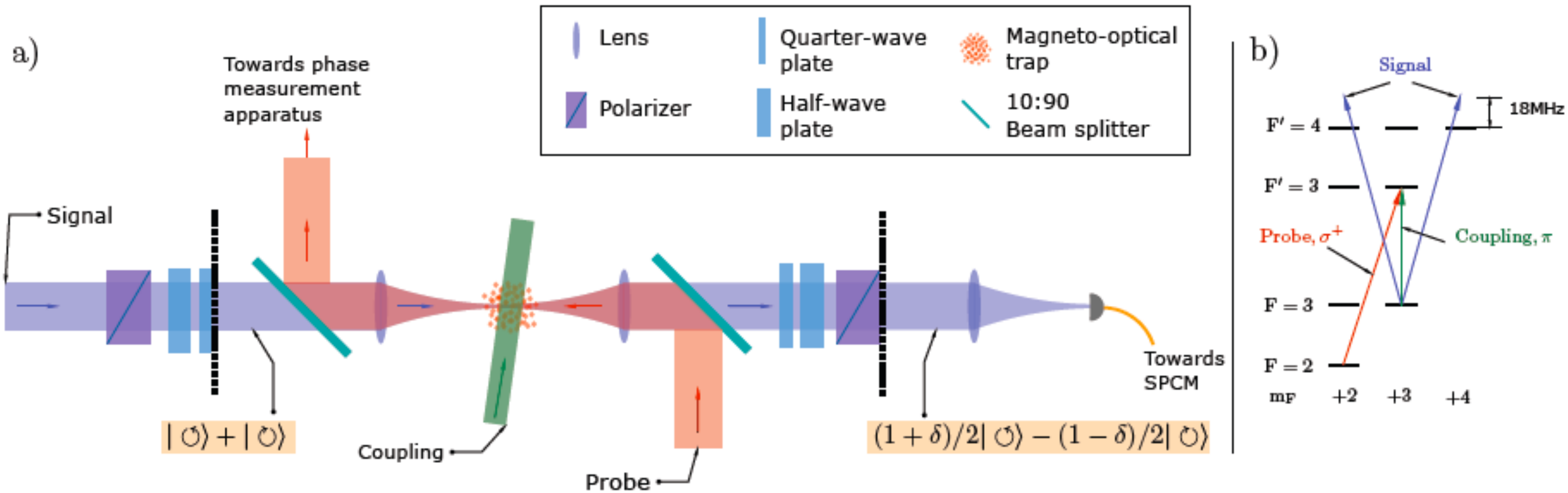


A photon in the hand
is worth 1000* in the vacuum chamber

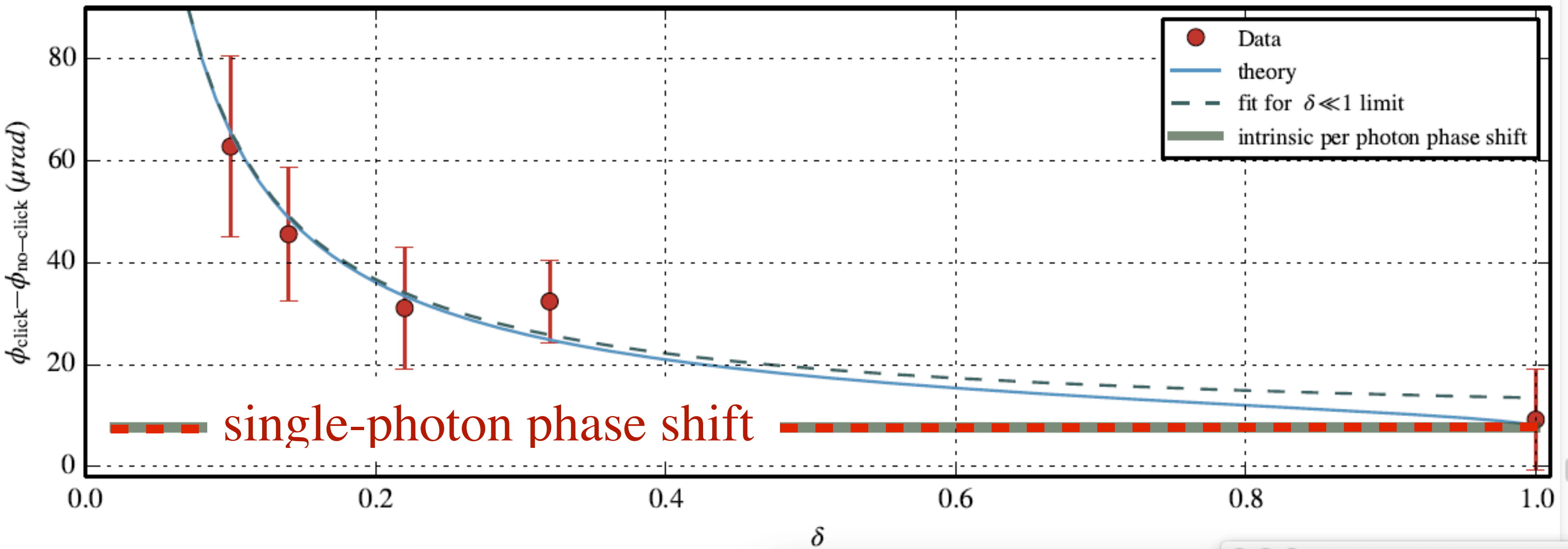
* – (base 2)

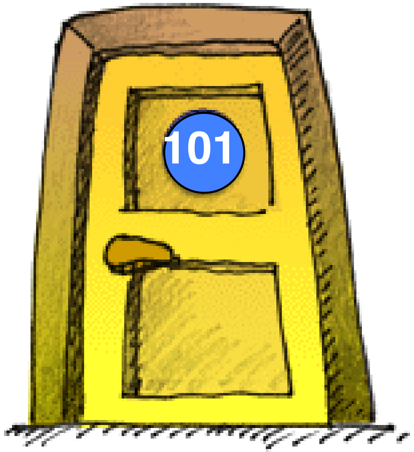
Polarisation interferometer





The phase shift due to an appropriately post-selected photon





Is it any *practical use* for 1 photon to act like 100?

Is weak measurement good for anything *practical*?

"Weak value amplification" has been proposed as a way to enhance the signals of small effects (like our nonlinearity...?):

Hosten & Kwiat, Science 319, 5864 (08); and, more quantitatively --

Selected for a Viewpoint in *Physics*

RL **102,** 173601 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification

P. Ben Dixon, David J. Starling, Andrew N. Jordan, and John C. Howell

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA

(Received 12 January 2009; published 27 April 2009)

We report on the use of an interferometric weak value technique to amplify very small transverse deflections of an optical beam. By entangling the beam's transverse degrees of freedom with the which-path states of a Sagnac interferometer, it is possible to realize an optical amplifier for polarization independent deflections. The theory for the interferometric weak value amplification method is presented along with the experimental results, which are in good agreement. Of particular interest, we measured the angular deflection of a mirror down to 400 ± 200 frad and the linear travel of a piezo actuator down to 14 ± 7 fm.

DOI: 10.1103/PhysRevLett.102.173601 PACS numbers: 42.50.Xa, 03.65.Ta, 06.30.Bp, 07.60.Ly

PRL 112, 040406 (2014) PHYSICAL REVIEW LETTERS week ending 31 JANUARY 2014

Weak Value Amplification is Suboptimal for Estimation and Detection

Christopher Ferrie and Joshua Combes
Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA
(Received 25 July 2013; revised manuscript received 21 November 2013; published 31 January 2014)

Experimentally quantifying the advantages of weak-value-based metrology

Gerardo I. Viza, Julián Martínez-Rincón, Gabriel B. Alves, Andrew N. Jordan, and John C. Howell
Phys. Rev. A **92**, 032127 – Published 22 September 2015

Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA
(Received 16 March 2014; revised manuscript received 18 July 2014; published 18 September 2014)

We show that the phenomenon of anomalous weak values is not limited to quantum theory. In particular,

Anomalous weak values are proofs of contextuality

Matthe
Perimeter Institute for Theoretical Physics, 31
(Dated: S

The average result of a weak measurement
measured quantum system, exceed the largest
well as the presence of post-selection and hen
has led to a long-running debate about wheth
"anomalous weak values" are non-classical in

Lev Vaidman's riposte

Comment on "How the result of a single coin toss can turn out to be 100 heads"

In a recent Letter, Ferrie and Combes [1] claimed to show "that weak values are not inherently quantum, but rather a purely statistical feature of pre- and post-selection with disturbance." In this Comment I will show that this claim is not valid. It follows from Ferrie and Combes misunderstanding of the concept of weak value.

SNR controversy: the short version

Weak value ~ 1 / <f|i>
Success probability ~ |<f|i>|^2

Pointer shift gets 10 times bigger,
as data rate gets 100 times smaller; noise 10 times bigger too.

TRUE IF --- **the noise is "statistical," as opposed to "technical."**

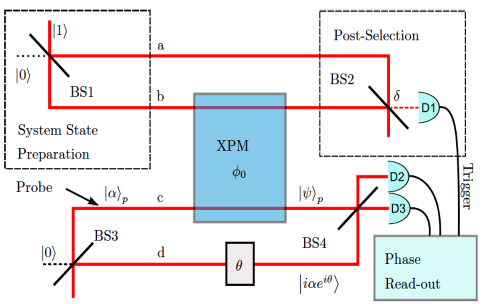
Early conjectures: something like pixel size in a detector array is insurmountable. Use WVA to make shift > pixel size ("technical")
Truth: you can still fit the center of a distribution to better than the pixel size, and 1/N$^{1/2}$ still applies in principle.

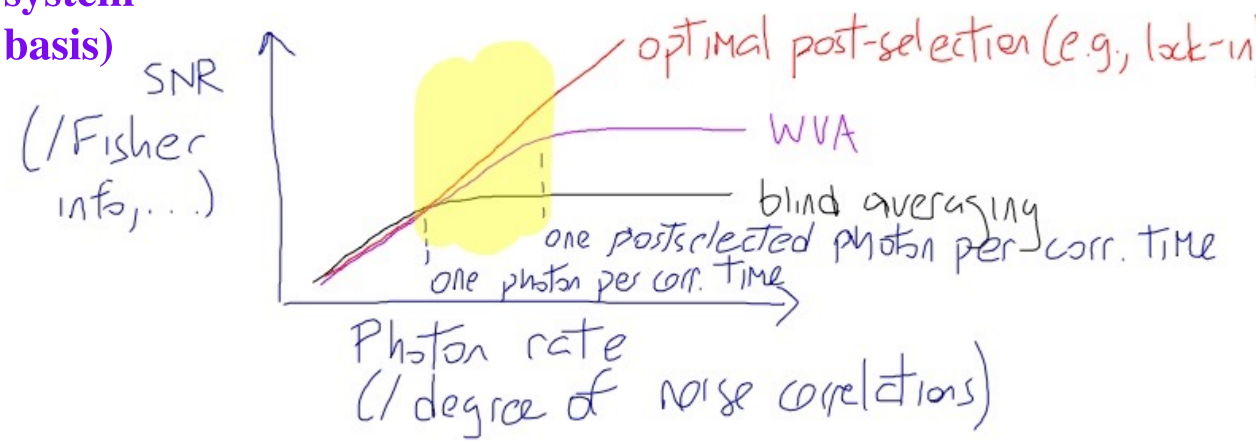
BUT: noise only drops as 1/N$^{1/2}$ because of the random walk, i.e., the fact that the noise on different data points is uncorrelated. Adding more data points within a noise correlation time *does not* let you keep averaging the noise away; better to post-select, and get a bigger signal.

One (of many) perspective(s) on the signal-to-noise issues… "technical noise"

NOTE: some language issues?
To most theorists, "postselection" means "throwing something out"; to some experimentalists, it means "doing a measurement on the system at all" (and perhaps choice of basis)

A. Feizpour et al., Phys. Rev. Lett. 107, 133603 (2011) + experiment & theory to appear…





WE CONTEND WVA IS USEFUL IN THE FOLLOWING SITUATIONS:

(1) limited by detector saturation

(2) most bins "empty" anyway

(3) noise correlation time > time between photons

(IN THIS REGIME, IT IS BETTER THAN STRAIGHT AVERAGING, YET STRICTLY SUB-OPTIMAL. IT IS RELATED TO THE BETTER – AND BETTER-KNOWN – "LOCK-IN" TECHNIQUE, BUT POTENTIALLY MORE "ECONOMICAL")

One unexpected advantage

Given the extensive discussion in recent years over the possible merits of WVA for making sensitive measurements of small parameters, it is interesting to contrast the present experiment with an earlier one, in which we measured the nonlinear phase shift due to post-selected single-photons, but without any weak-value amplification (*31*). In our previous experiment, a total of approximately 1 billion trials (300 million events with post-selected photons, and 700 million without) were used to measure the XPS due to σ^+-polarized photons. By looking at the difference between the XPS measured for "click" and "no-click" events, we measured peak XPS ϕ_+ of $18 \pm 4\mu$rad. In this experiment, where we use the WVA technique, we used a total of around 830 million trials (200 million successful post-selections) to extract an average XPS ϕ_+ of $10.0 \pm 0.6\mu$rad (for more information regarding the reported average XPS see the Probe phase measurement section in the supplementary material). Note that this number it agrees well with our classical calibration of the peak XPS of $13.0 \pm 1.5\mu$rad (*31*). It is evident that the WVA

Recap Main Course

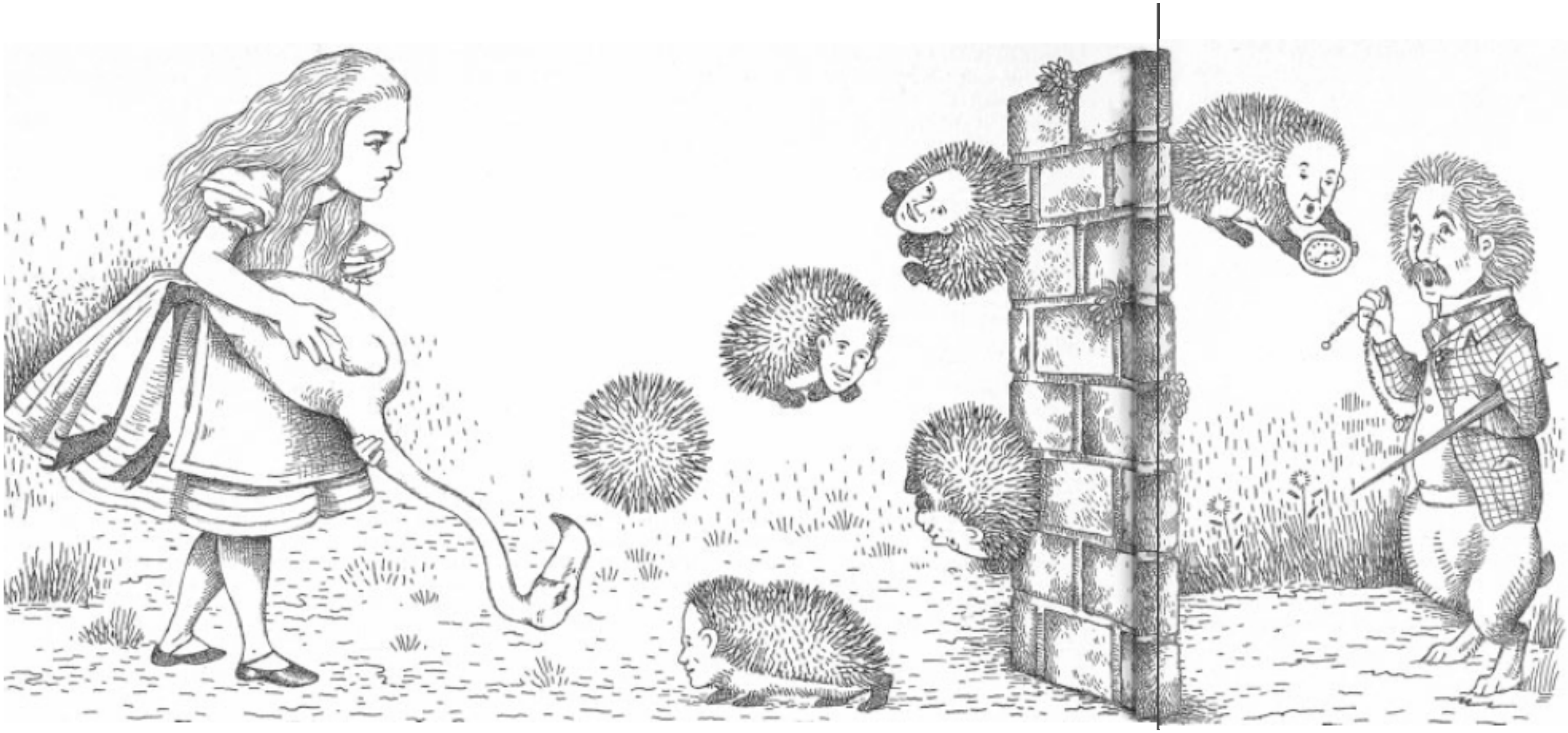
• We were able to generate a "big" (10^{-5} rad) per-photon nonlinear phase shift, and measure it – and confirm that properly post-selected photons may have an amplified effect on the probe, as per the weak value.

• We believe WVA is potentially useful in (at least) the following circumstances: when
(1) you are limited by detector saturation
(2) most bins "empty" anyway
(3) noise correlation time > time between photons

NB: in the third case, this is closely related to background subtraction & lock-in amplification, and in fact cannot outperform such techniques.

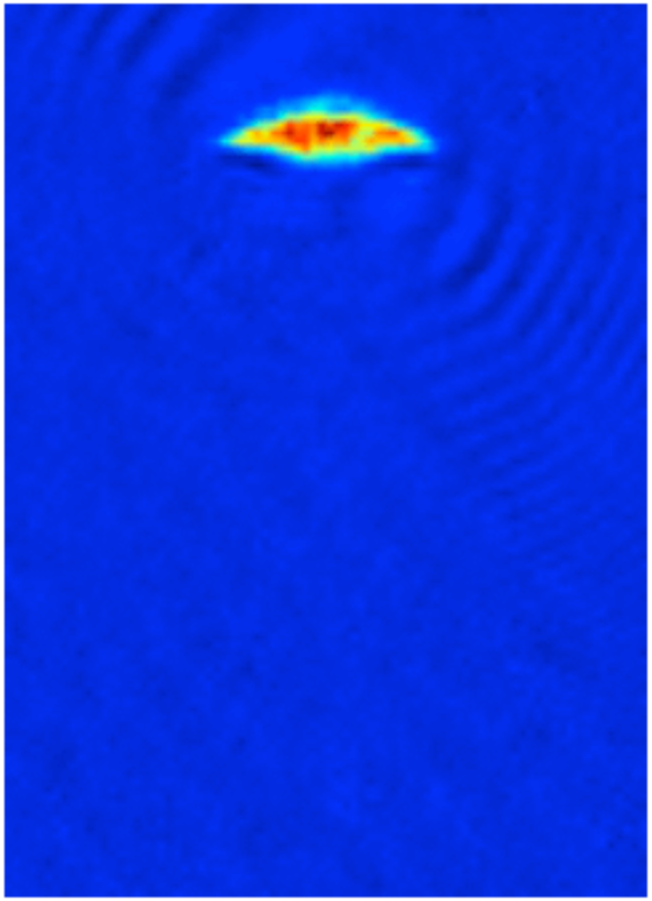
Dessert: some progress with ultracold atoms

Watching a particle in a region it's "forbidden" to be in

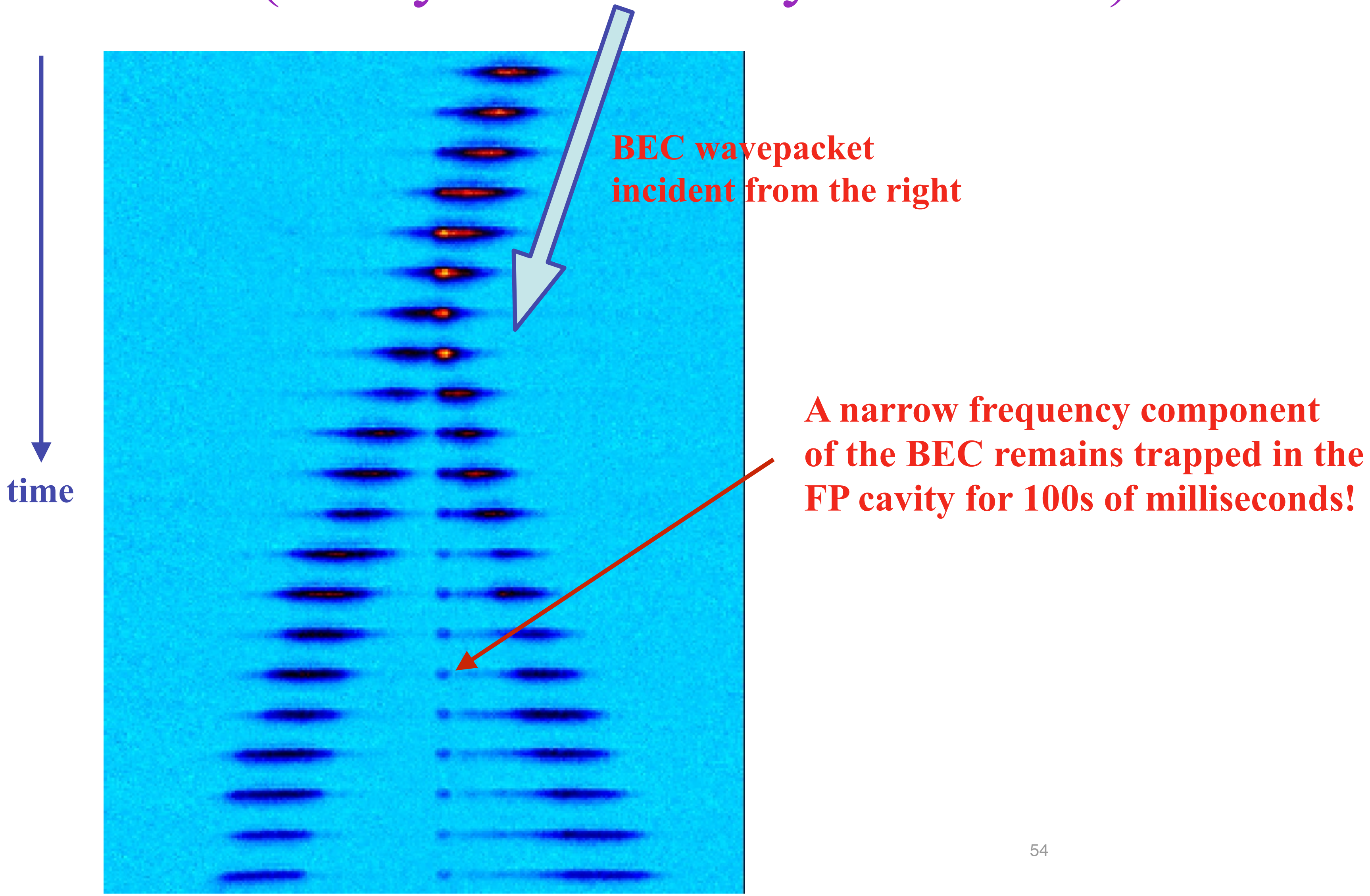


How long has the transmitted particle spent in the region?

Atoms spilling *around* an optical "ReST" trap

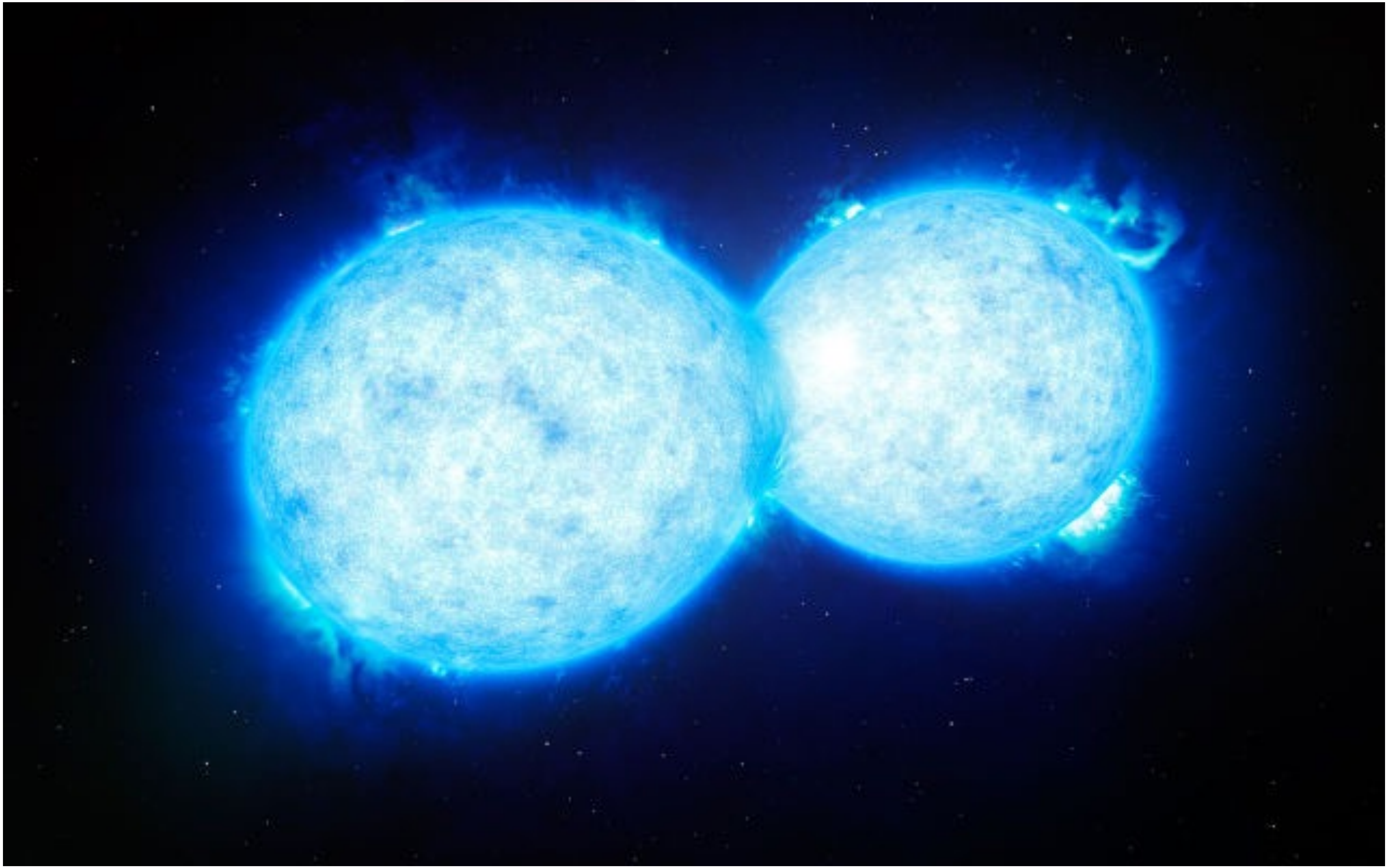


Preliminary evidence of tunneling through a *double* barrier (Fabry-Perot cavity for atoms)

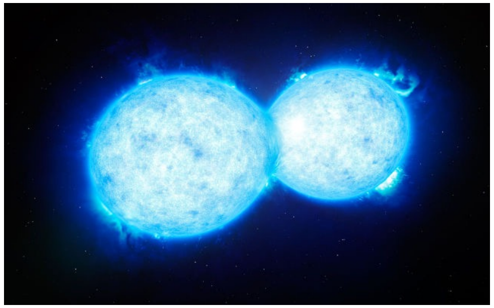


Digestif: Evading Rayleigh's Curse

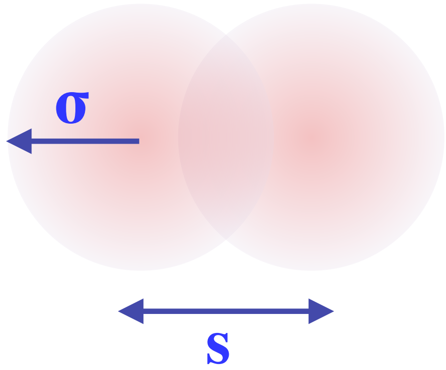
Toy problem: imaging a binary star



Toy problem: imaging a binary star



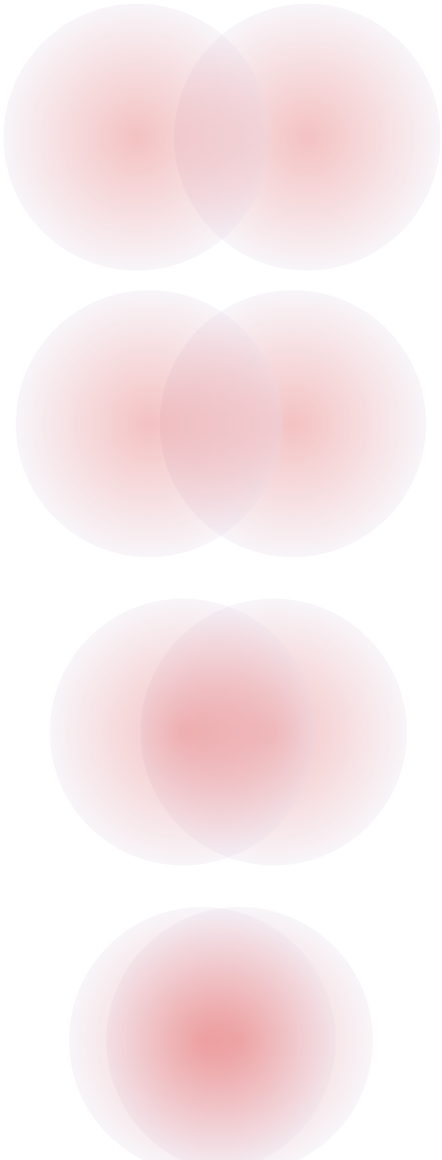
Toy problem: imaging a binary star



As we all know, if objects separated by less than ~width σ of the PSR (diffraction limit), we can't "resolve" them

… of course, that's not to say that with enough data, we can't tell there are two objects there, and where they are…

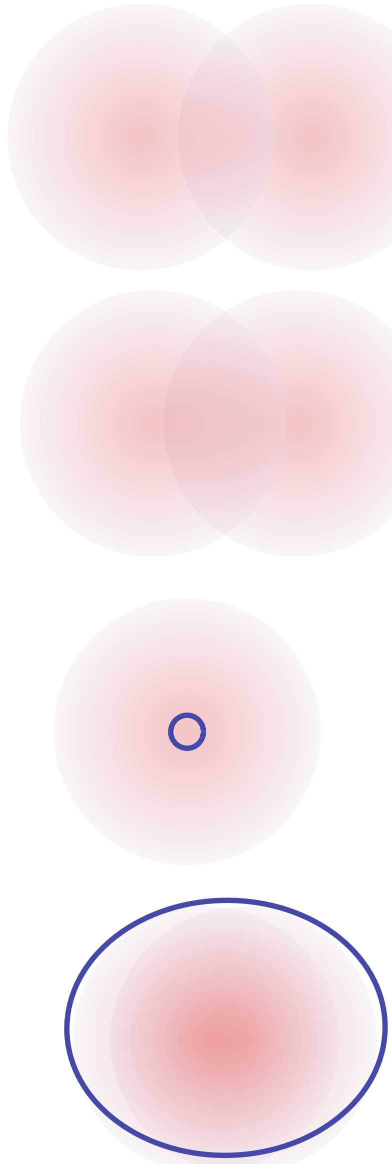
Toy problem: imaging a binary star



How well can we estimate the separation s of two objects, for s < width σ of PSR, given N photons?

σ / sqrt{N} for N photons would seem reasonable?

No such luck!

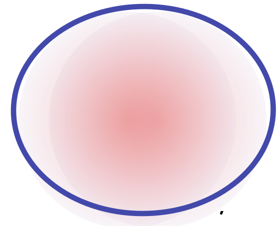


σ / sqrt{N} is indeed how well you can find the centre of *one* object.

But two closely separated gaussians just look like a slightly broader gaussian – the problem is to estimate the *width*, which proves much harder.

How well can you estimate a width?

$$V = \langle x^2 \rangle = \frac{1}{N} \Sigma_{(i=1)}^{N} x_i^2$$



Uncertainty ΔV can be calculated from $\Delta V^2 = \overline{V^2} - \overline{V}^2$.

$$V^2 = \frac{1}{N^2} \Sigma_i \Sigma_j x_i^2 x_j^2$$

For $i \neq j$, $\overline{x_i^2 x_j^2} = \sigma^4$

For $i = j$, $\overline{x_i^2 x_j^2} = 3\sigma^4$

$$\overline{V^2} = \frac{1}{N^2} \left\{ N^2 \sigma^4 + 2N\sigma^4 \right\} = \sigma^4 + \frac{2}{N} \sigma^4$$

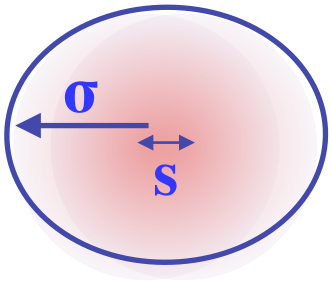
$$\overline{V}^2 = \sigma^4$$

$$\Delta V = \sqrt{\frac{2}{N} \sigma^4}$$

How well can you estimate a separation?

$$\Delta V = \sqrt{\frac{2}{N}\sigma^4}$$

$$V = \sigma^2 + s^2$$

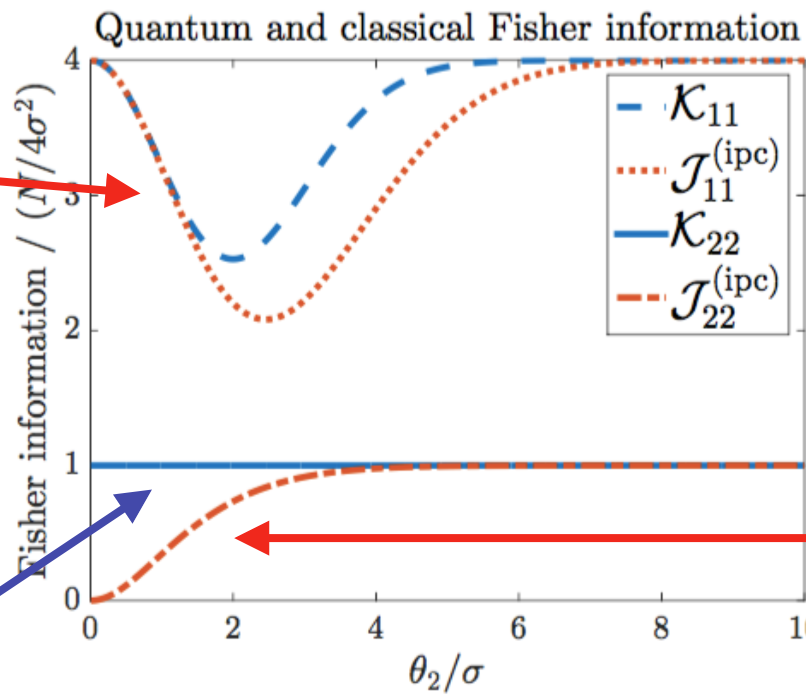


$$\Delta s = \Delta V / \frac{dV}{ds} = \frac{\sigma^2\sqrt{2/N}}{2s} = \frac{\sigma^2}{s}\sqrt{\frac{1}{2N}}$$

The uncertainty in s does not merely *remain* large (σ/sqrt{N}) as s -> 0 ; it actually *diverges* as 1/s!

M. Tsang, R. Nair, and X.-M. Lu, Phys. Rev. X 6, 031033 (2016).

Information about *centroid*



***Classical* Fisher Information about *separation* (vanishes at small sep.)**

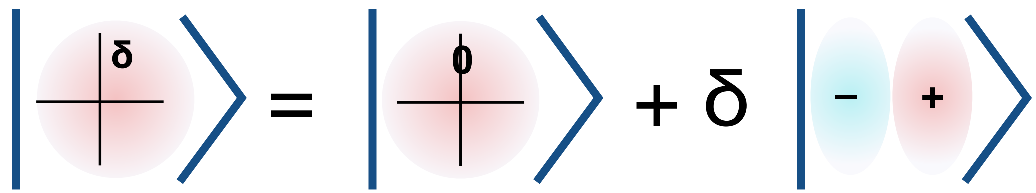
***Quantum* Fisher Information about separation —- constant!!**

FIG. 2. Plots of Fisher information quantities versus the separation for a Gaussian point-spread function. $\mathcal{K}_{11}$ and $\mathcal{K}_{22}$ are the quantum values for the estimation of the centroid $\theta_1 = (X_1 + X_2)/2$ and the separation $\theta_2 = X_2 - X_1$, respectively, while $\mathcal{J}_{11}^{(\mathrm{ipc})}$ and $\mathcal{J}_{22}^{(\mathrm{ipc})}$ are the corresponding classical values for image-plane photon counting. The horizontal axis is normalized with respect to the point-spread function width σ, while the vertical axis is normalized with respect to $N/(4\sigma^2)$, the value of $\mathcal{K}_{22}$.

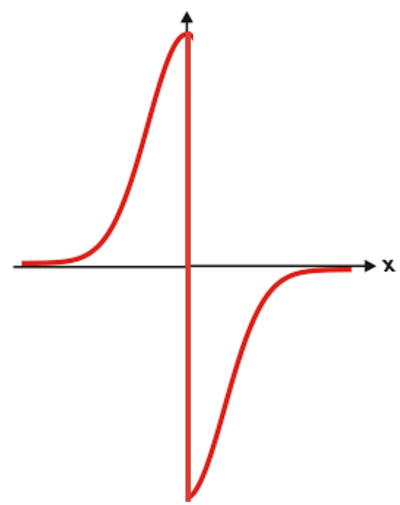
The Fisher information drops to 0 — the error of any unbiased estimator of s goes to infinity.

For two incoherent sources, the 2-spot distinguishability is essentially the same as the 1-spot distinguishability… how to optimally distinguish?

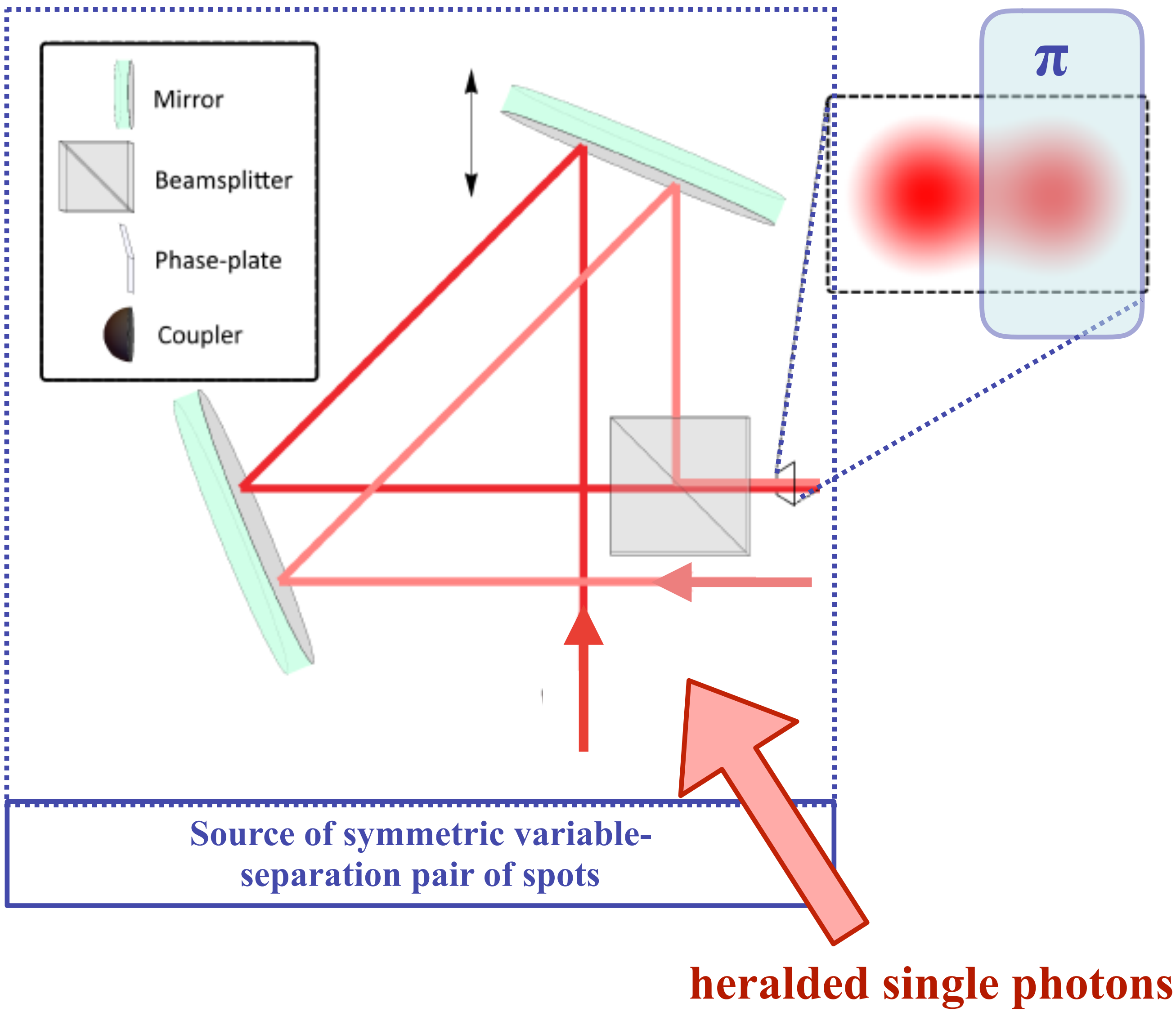
This becomes a quantum state discrimination problem



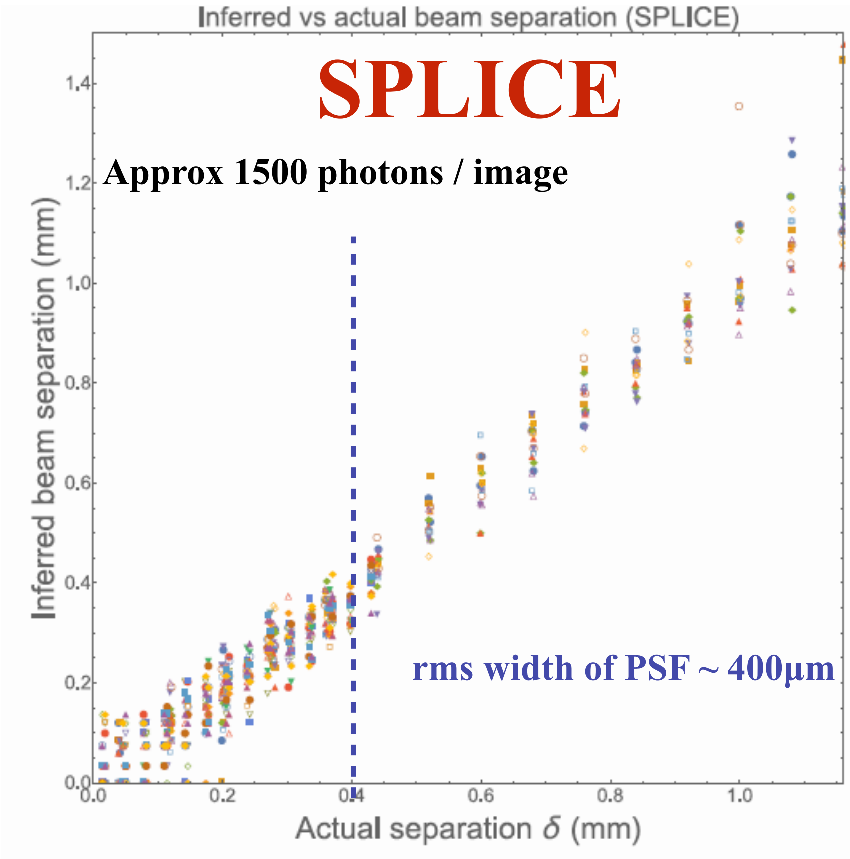
SPLICE:
Project onto any odd-parity mode, not necessarily TEM01 in particular —

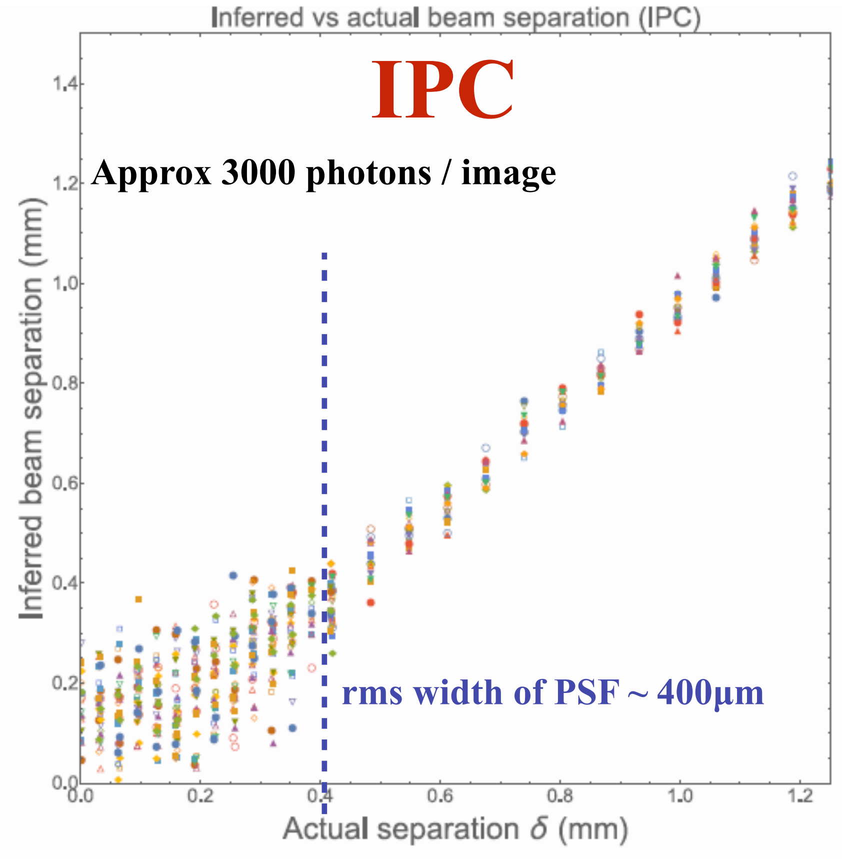


Projecting a double-spot onto an odd-parity mode



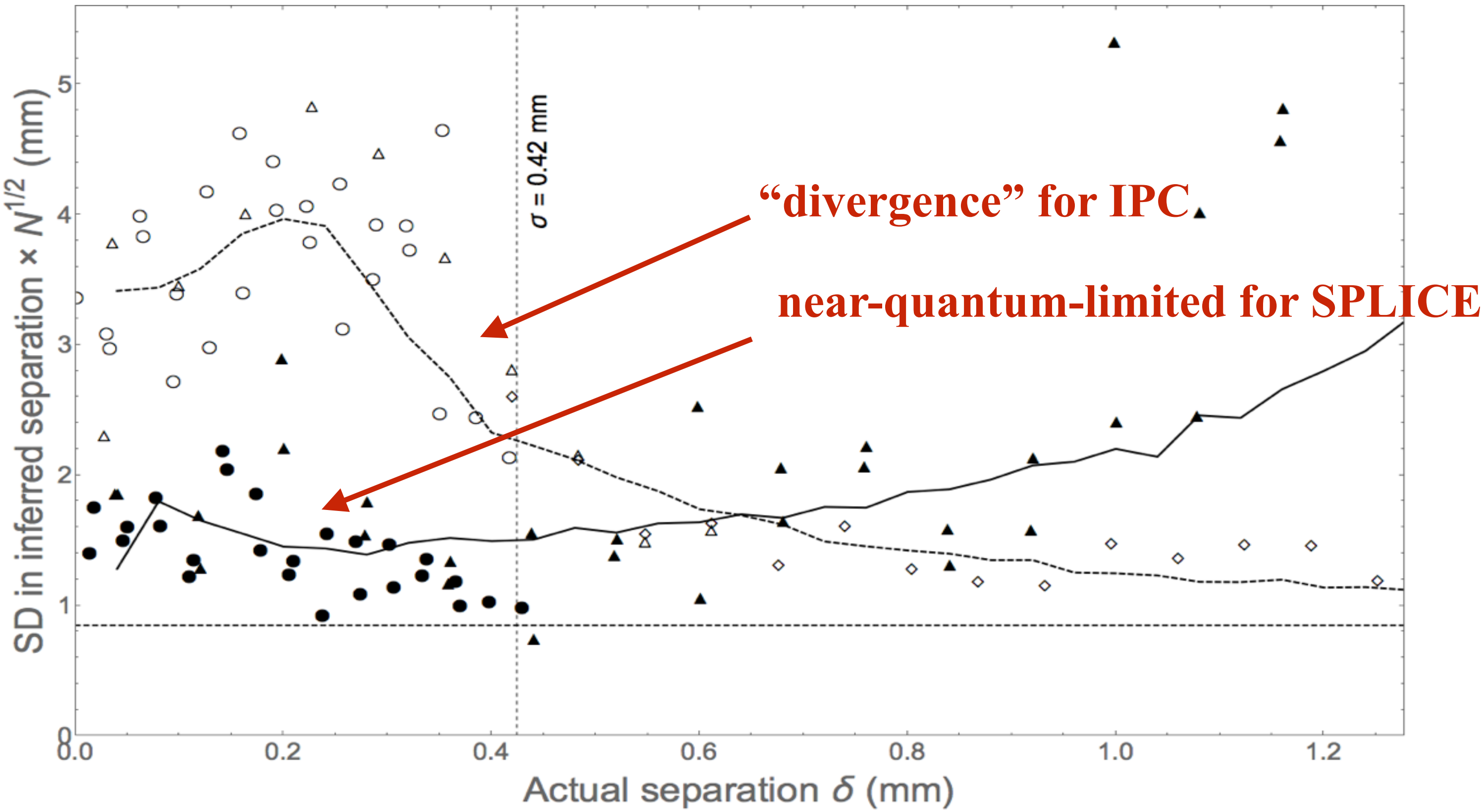
Observed vs. actual separation





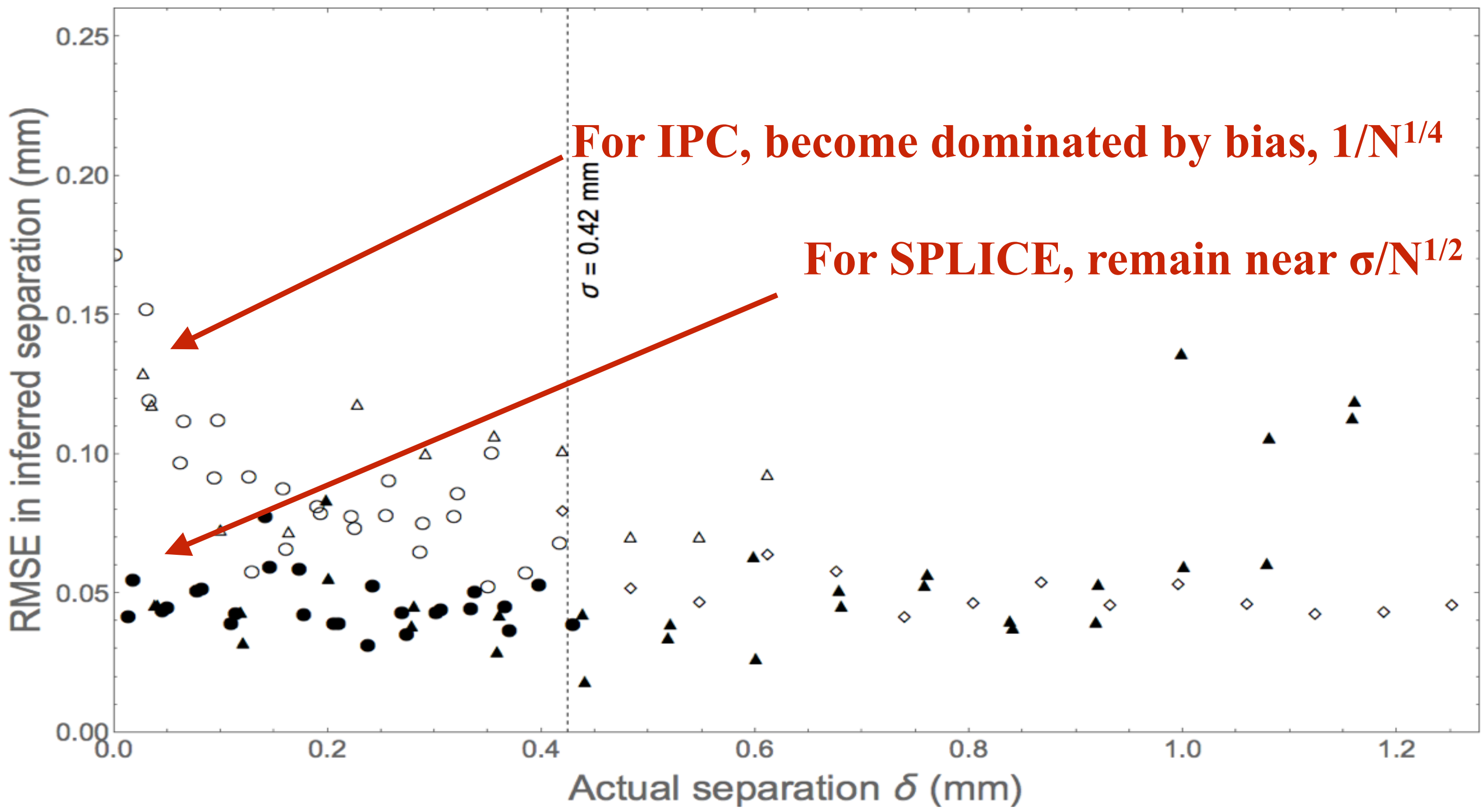
W.K. Tham, H. Ferretti, AMS arXiv:1606.02666 (2016)

SD in inferred separation, vs. s_{actual}



W.K. Tham, H. Ferretti, AMS arXiv:1606.02666 (2016)

Total RMS error, *including* bias



CONCLUSION: We have shown that a simple phase-mask technique removes the 1/s catastrophe, and permits us to achieve near-quantum-limited resolution, providing an unbiased estimator with $\sigma/N^{1/2}$ resolution, yielding a quadratic-in-N advantage over even the best *biased* estimator possible with image-plane counting.

With about 1500 photons, SPLICE determined the separation 3 times more accurately than IPC could with about 3000 photons

W.K. Tham, H. Ferretti, AMS arXiv:1606.02666 (2016)

See also: T. Z. Sheng, K. Durak, and A. Ling, arXiv preprint arXiv:1605.07297 (2016); M. Paur et al. arXiv: 1606.08332 (2016); F. Yang, A. I. Lvovsky et al arXiv:1606.02662 [physics.optics].

Summary

• We were able to generate a "big" (10^{-5} rad) per-photon nonlinear phase shift, and measure it – and confirm that properly post-selected photons may have an amplified effect on the probe, as per the weak value.

A. Feizpour et al., Nature Physics, DOI: 10.1038/nphys3433 (2015)
Weak-msmt theory: Phys Rev Lett 107, 133603 (2011)
Weak-msmt exp't: under review

• After talking about it for 20 years, we are getting close to being able to probe atoms while they tunnel through an optical barrier, using weak measurement to ask "where they were" before being transmitted!

We have preliminary evidence that our Fabry-Perot cavity for ultracold Rubidium atoms is working.

In progress – for previous work, see e.g. S. Potnis, R. Ramos, K. Maeda, L.D. Carr, AMS, 1604.06388; R. Chang, S. Potnis, R. Ramos, C. Zhuang, M. Hallaji, A. Hayat, F. Duque-Gomez, J. Sipe, AMS, PRL 112, 170404 (2014)

• Even in the image plane, much (even most) of the information may be in the optical phase and not the intensity – a new route to super-resolution, requiring no structured illumination!

W.K. Tham, H. Ferretti, AMS arXiv:1606.02666 (2016)