Consider the Icicle

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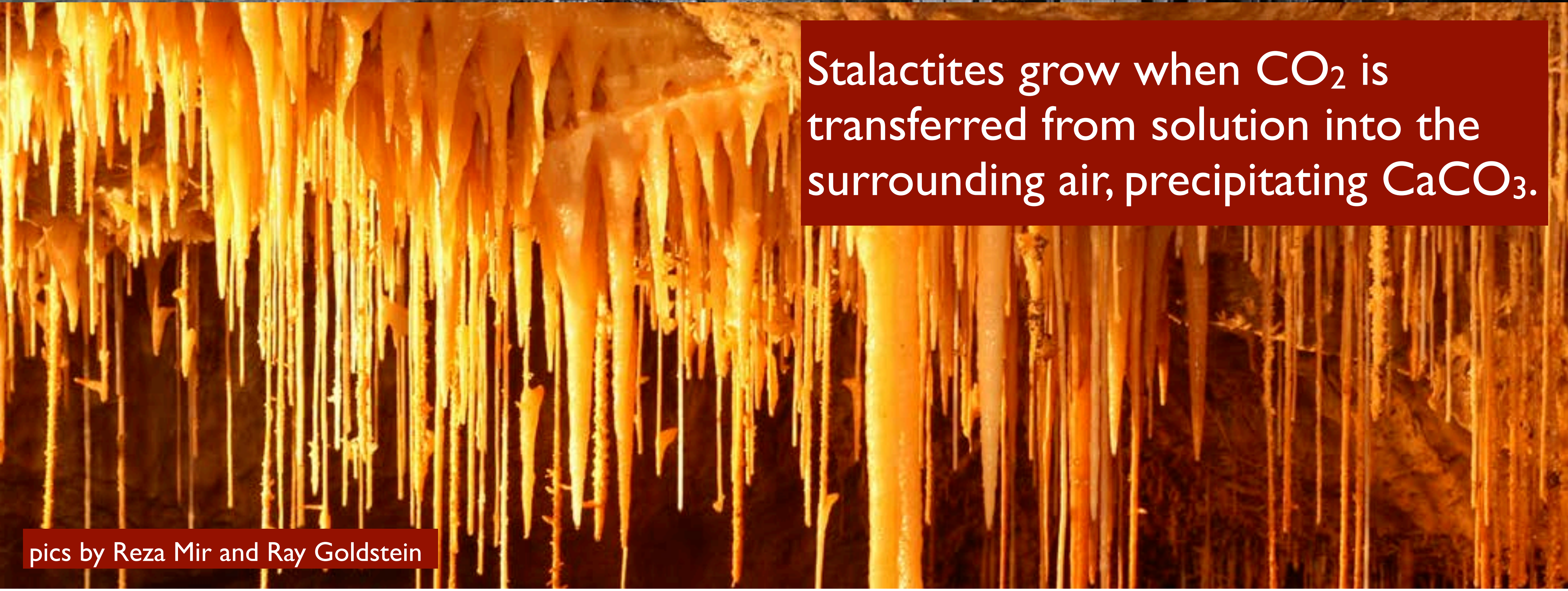
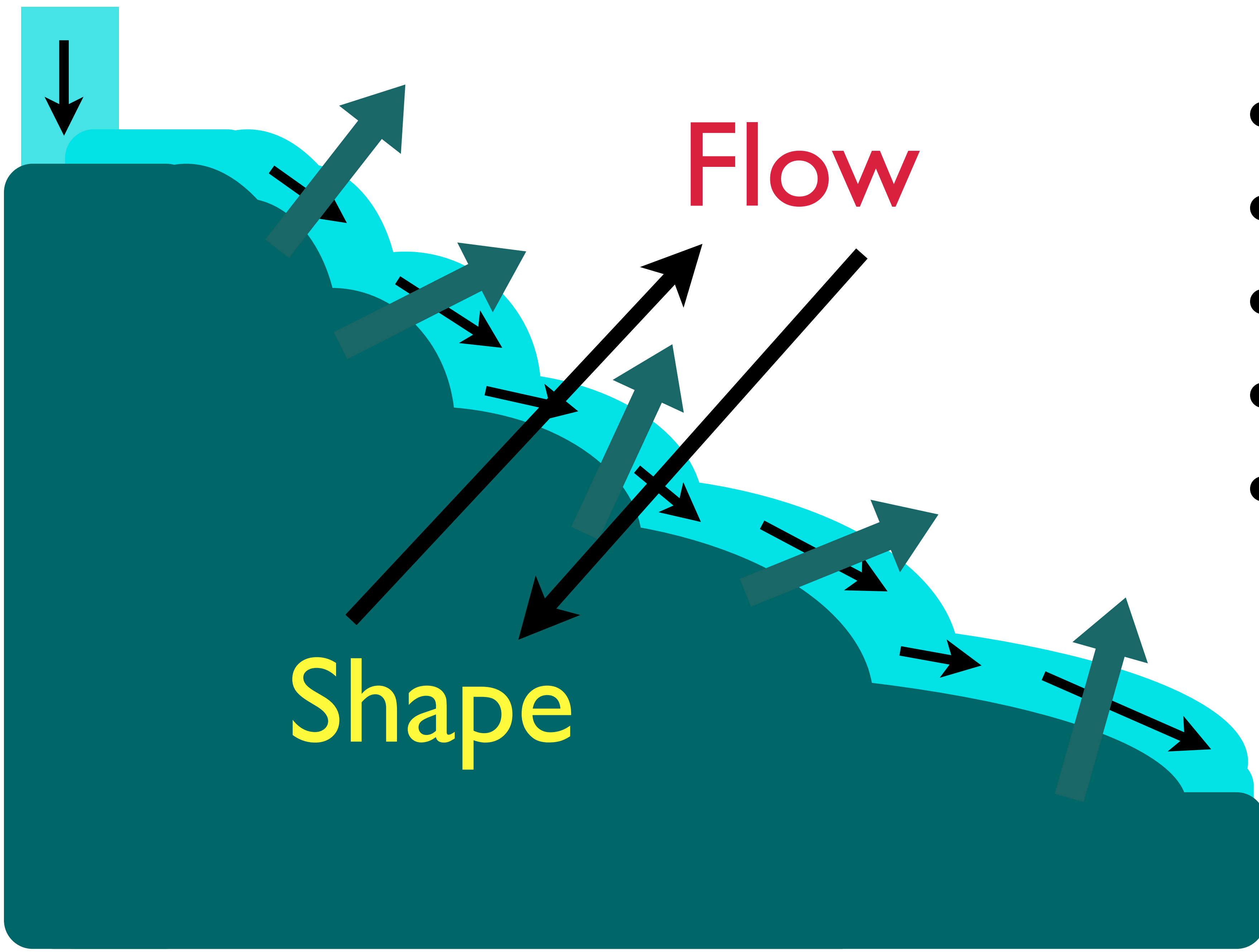
Icicles grow when latent heat is transferred into the surrounding air *via* a thin film of water flowing down their surface.


Stalactites grow when CO_2 is transferred from solution into the surrounding air, precipitating $CaCO_3$.
pics by Reza Mir and Ray Goldstein

Precipitative pattern formation

"… ubiquitous in Nature"

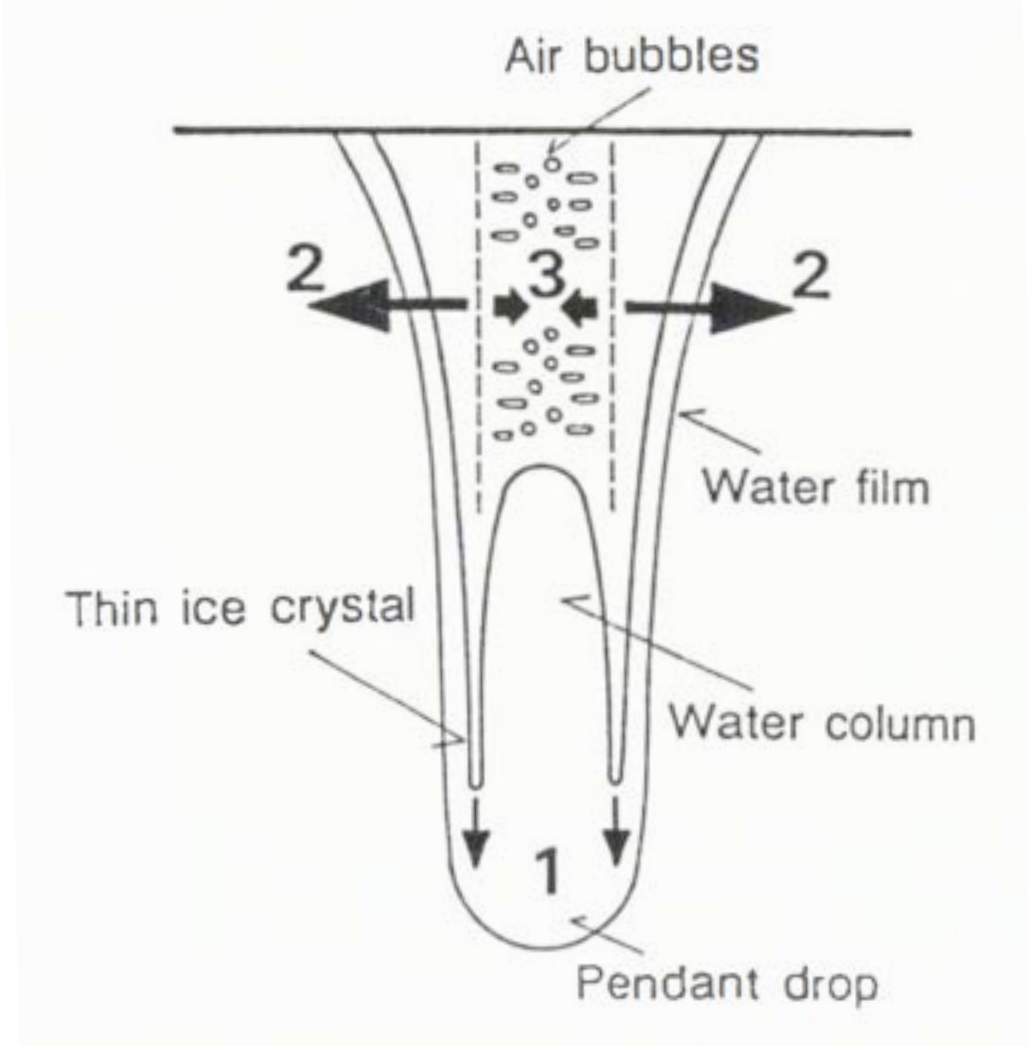


- erosion
- transport
- deposition
- dissolution
- growth

Complex feedback

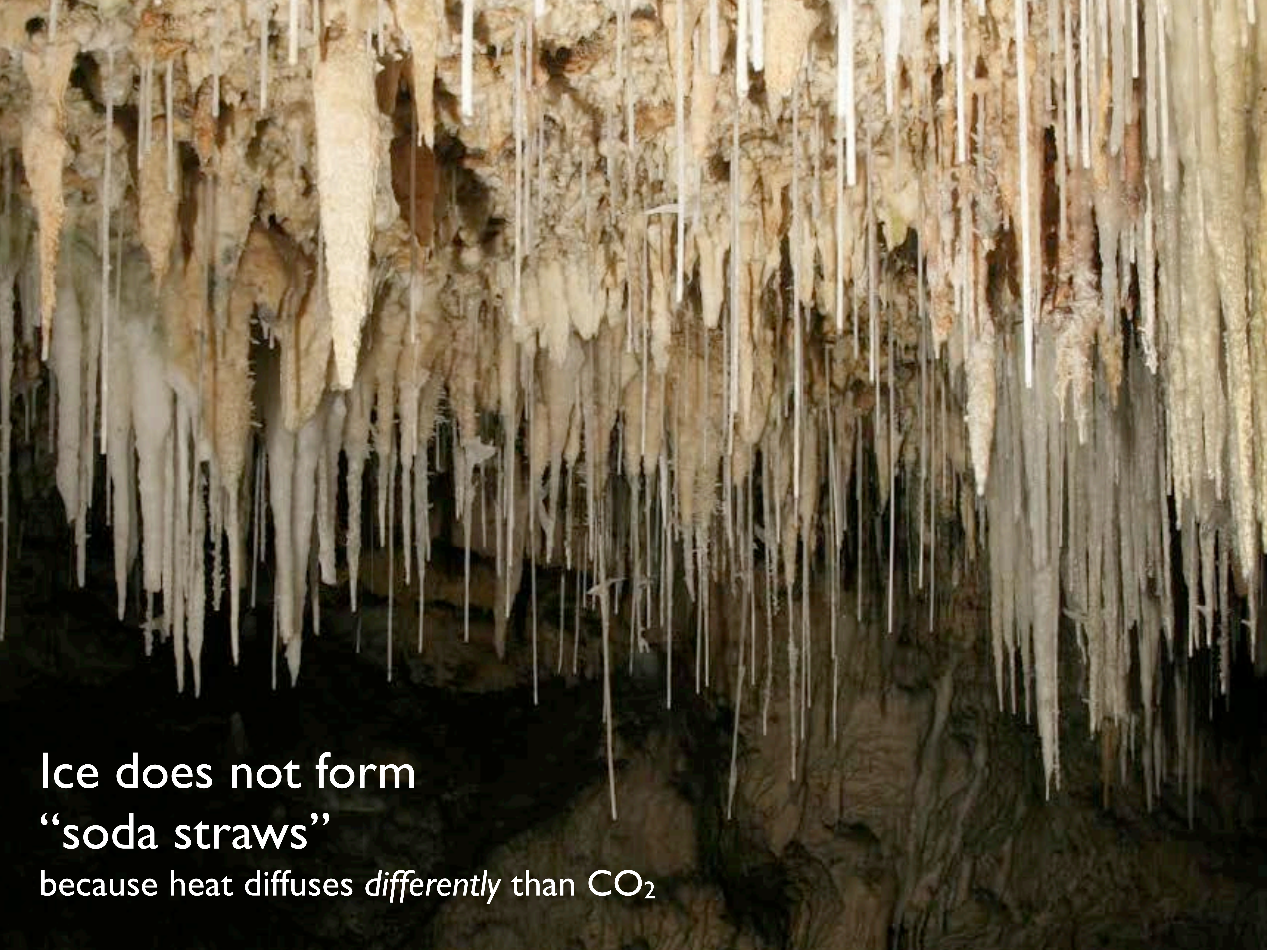

"... ubiquitous in Nature"

Icicle growth modes



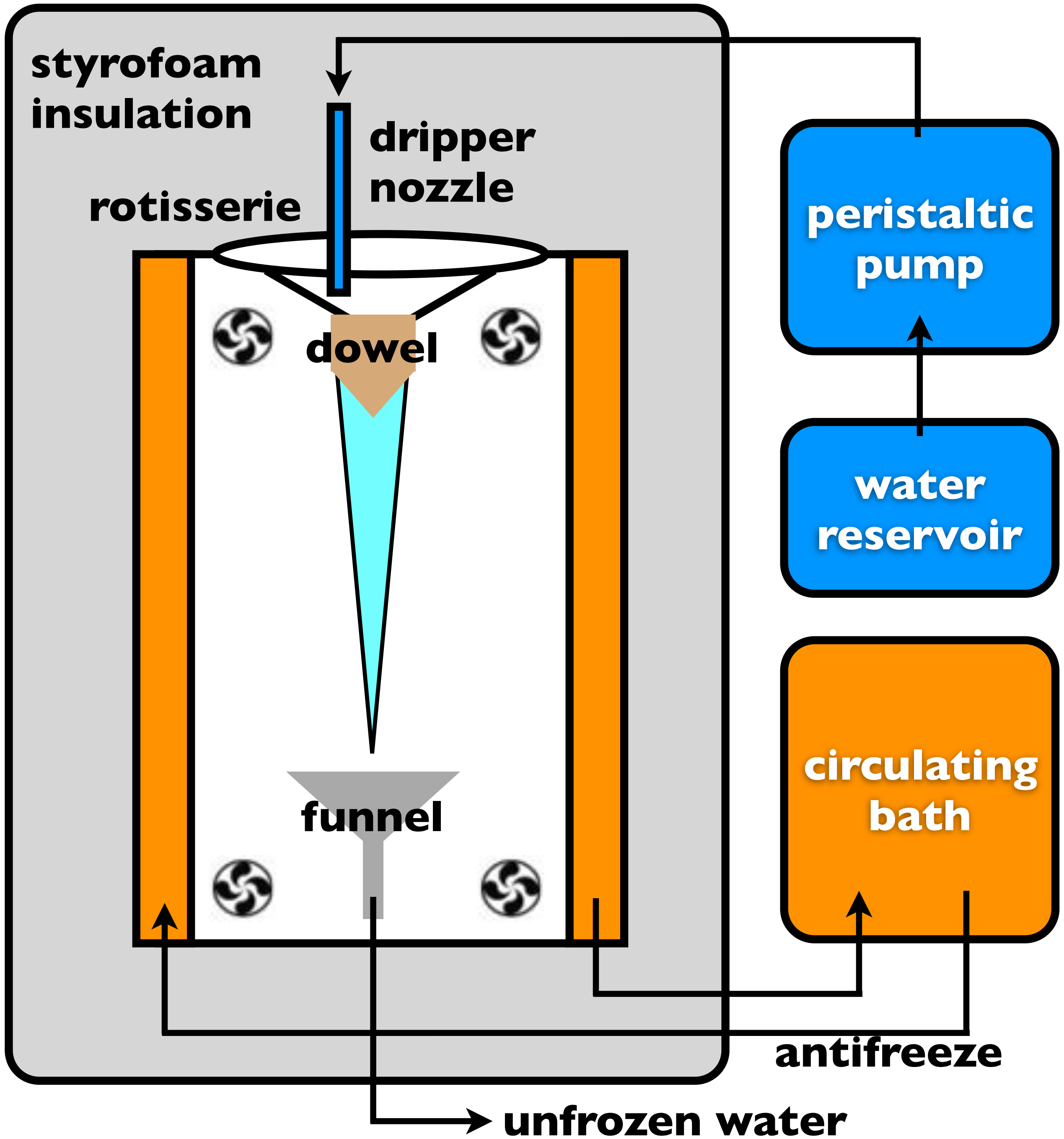
1. Tip growth from bulk water drop
2. Radial growth from thin film
3. Inward "pipe filling" growth

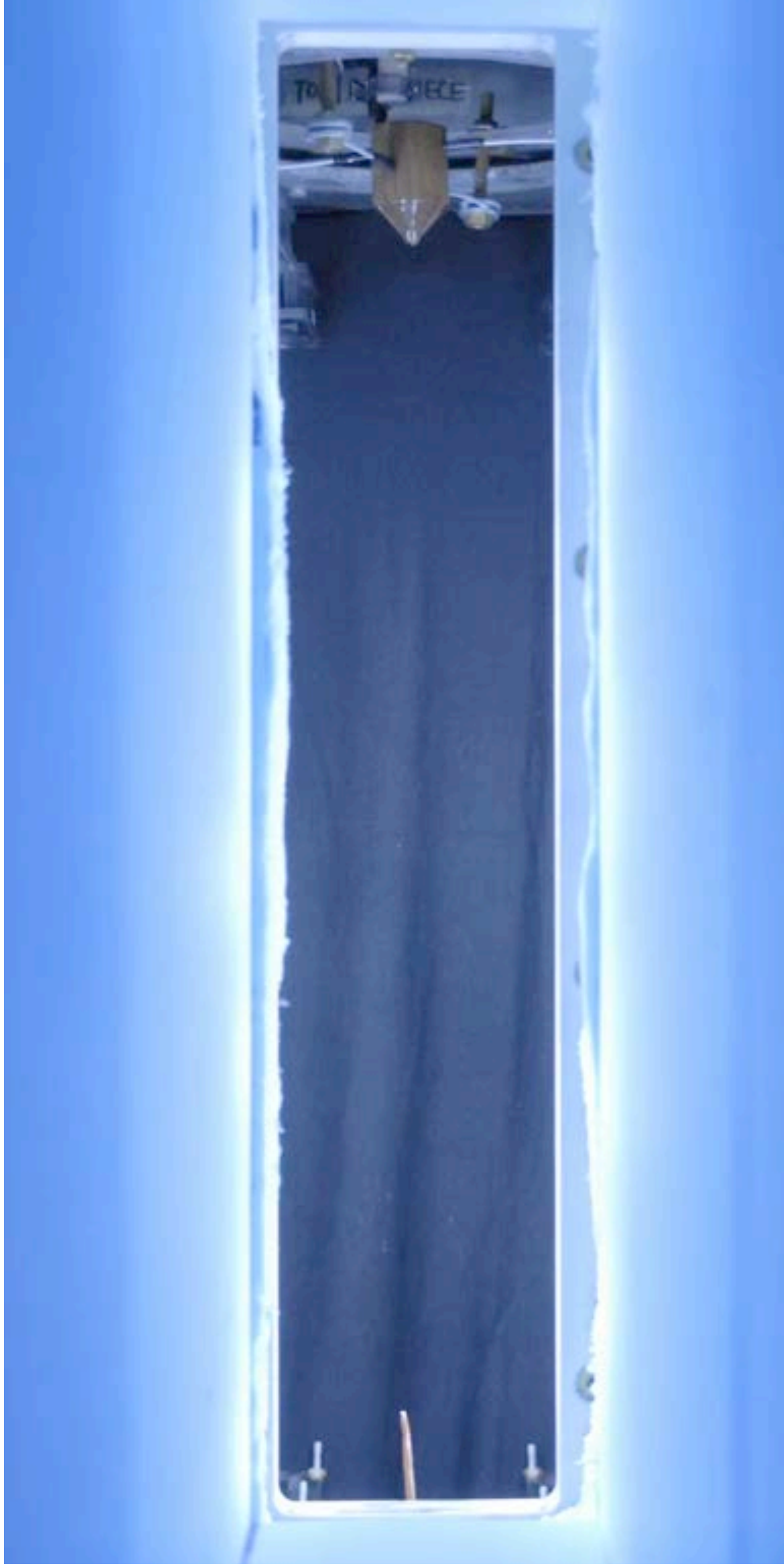
N. Maeno, L. Makkonen, K. Nishimura, K. Kosugi and T. Takahashi, *J. Glaciol.*, **40**, 319 (1994).



Ice does not form
“soda straws”
because heat diffuses *differently* than CO_2

The icicle machine



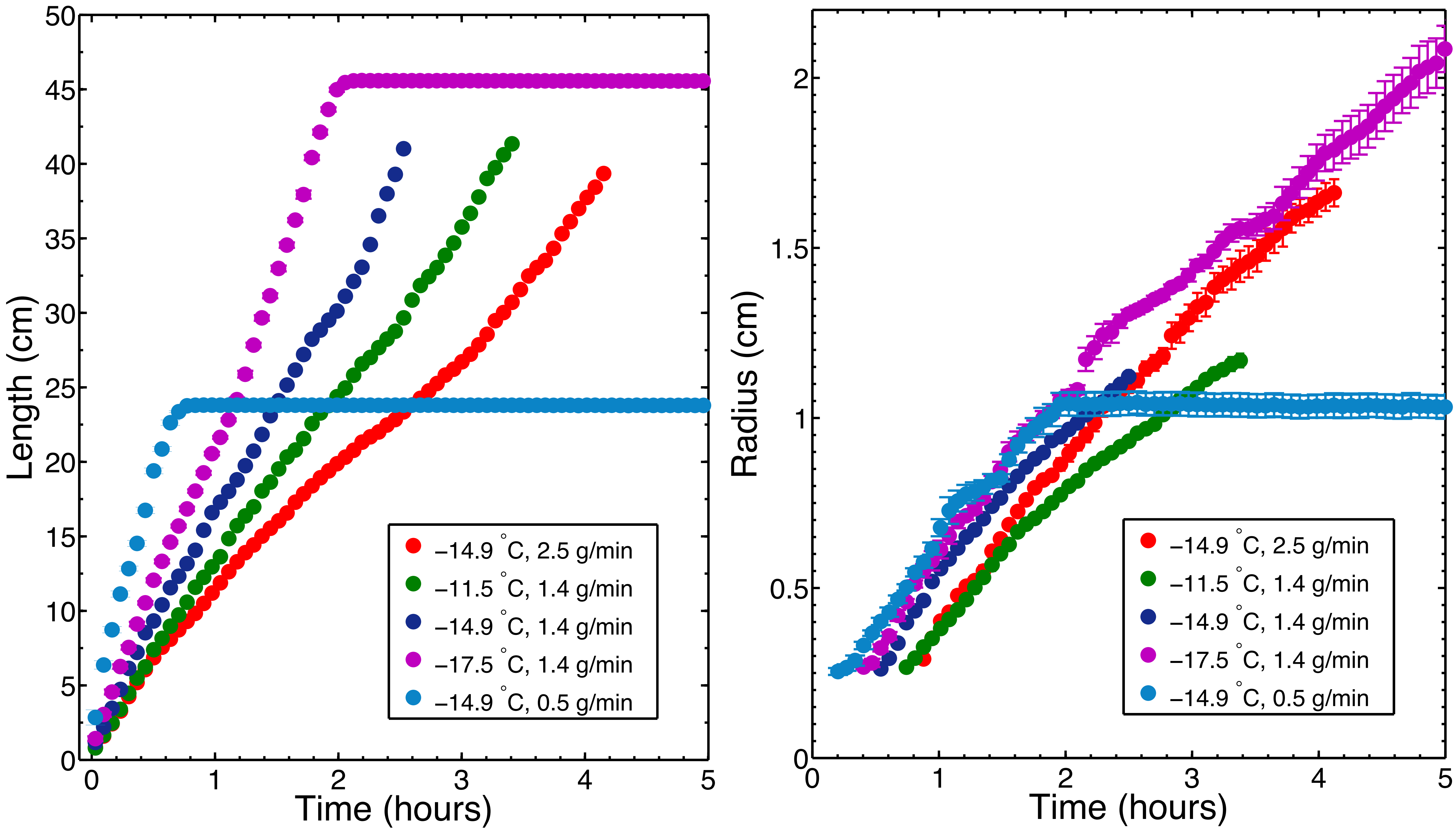


Using time-lapse photography, we can make movies of the icicle's evolving morphology.

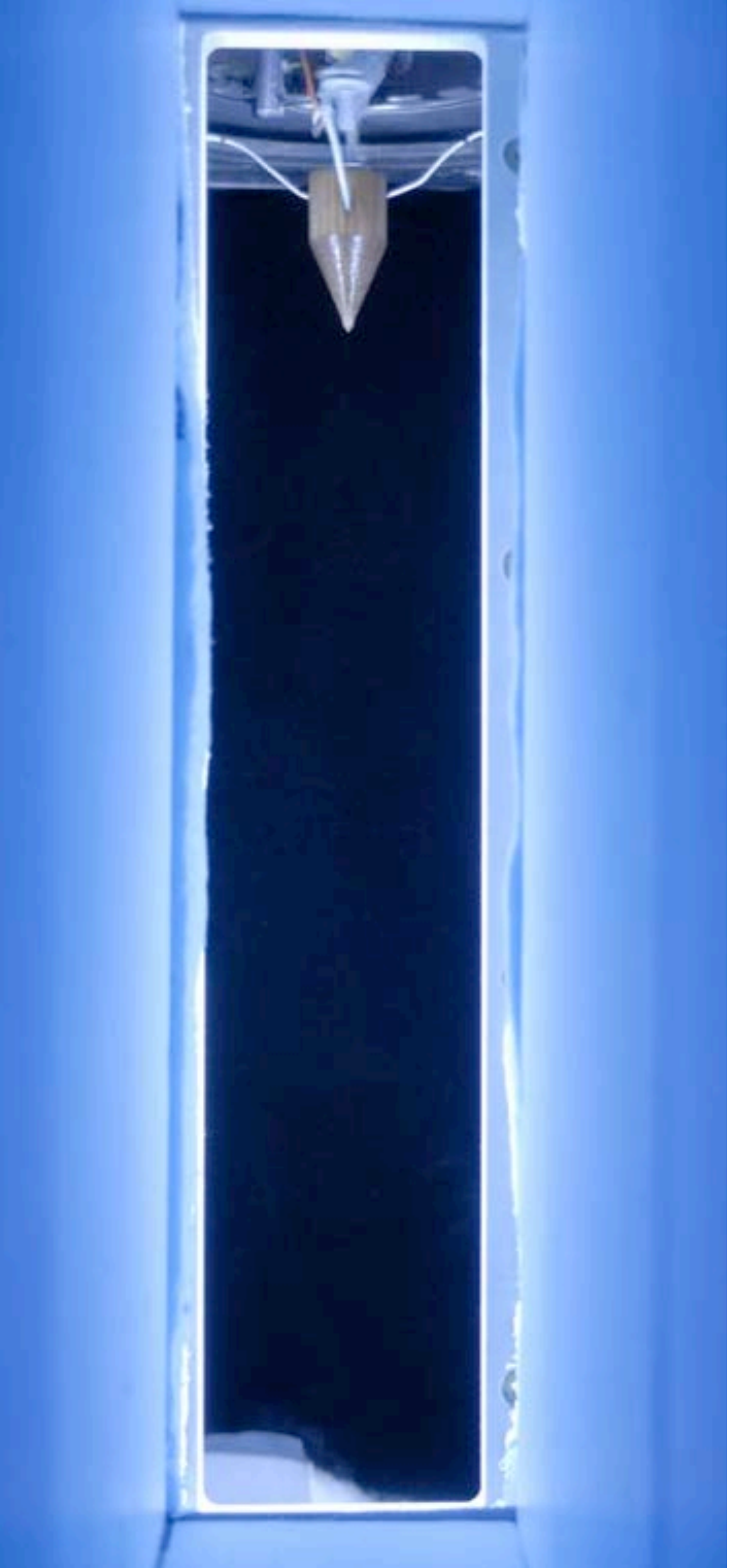
1 sec of movie = 10 min (1 rotation / 4 min)

Tip growth *vs.* radial growth

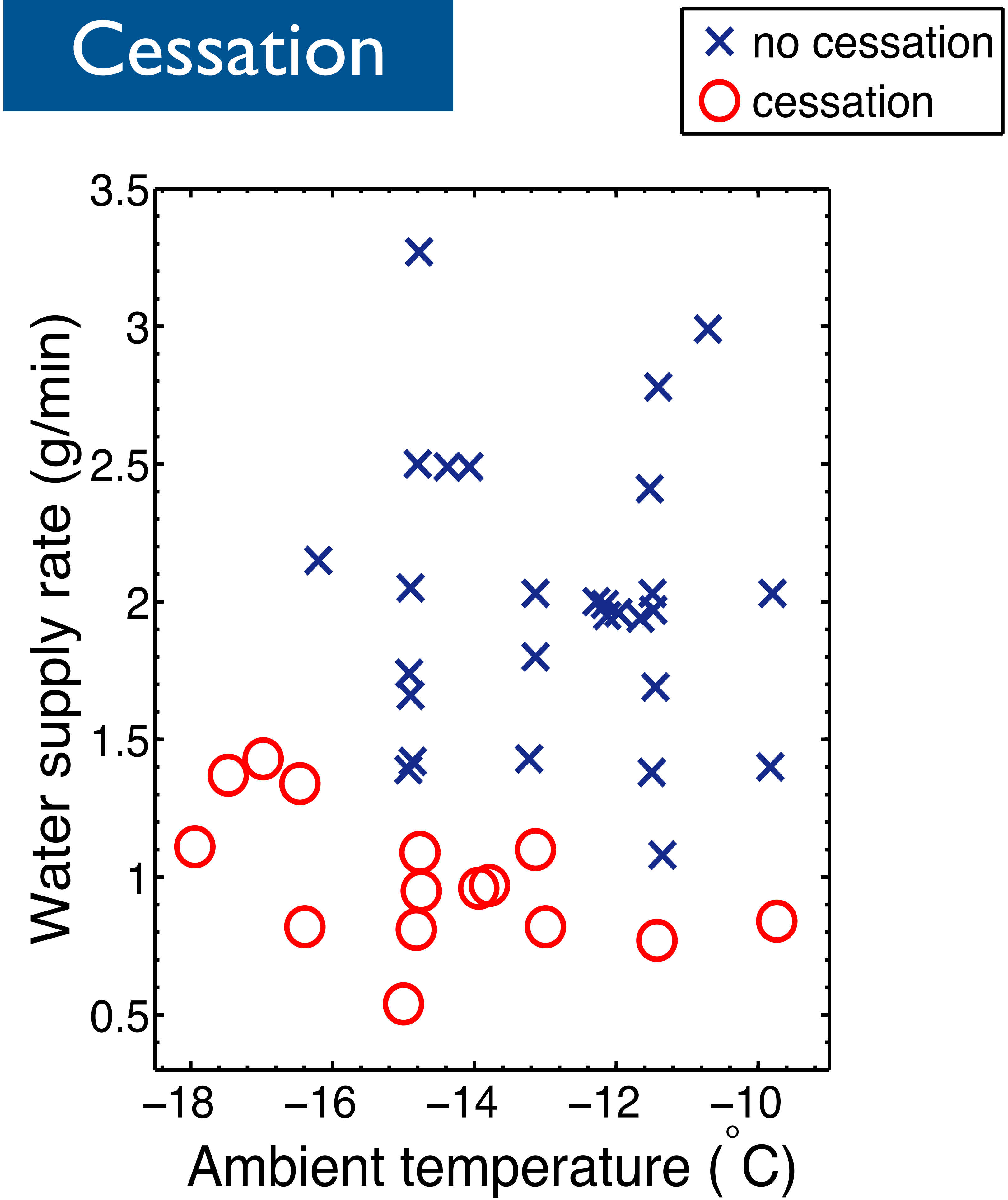
Tip and radial growth scale differently with conditions



Some icicle just ***stop*** growing under constant input flow rate



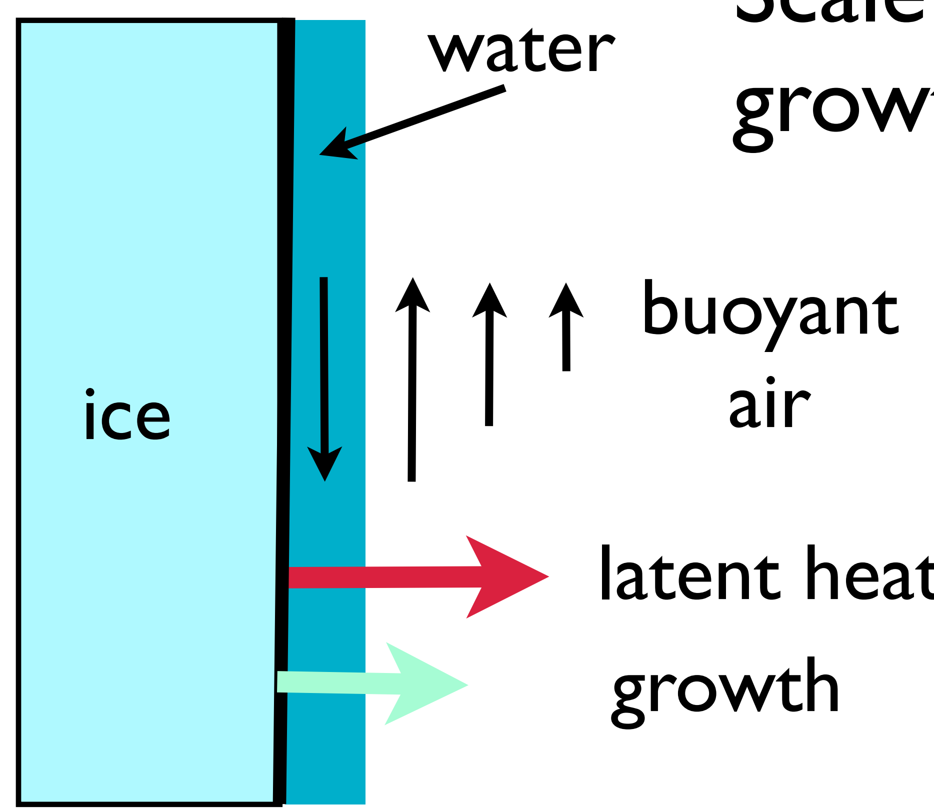
Cessation



no cessation
cessation
Water supply rate (g/min)
3.5
3
2.5
2
1.5
1
0.5
−18
−16
−14
−12
−10
Ambient temperature (°C)

The shape of an icicle

Scale out tip growth



Predicting the emergent shape of an icicle is a tricky free-boundary growth problem.

Short, *et al.* derived an ODE for the platonic icicle shape:

$$\rho = \frac{r}{a} \text{ and } \zeta = \frac{z}{a}$$

$$\rho'(\zeta) = \frac{1}{\sqrt{\zeta^{\frac{1}{2}} - 1}}$$

$$\rho(\zeta) = \frac{4}{3}(\zeta^{\frac{1}{2}} + 2)\sqrt{\zeta^{\frac{1}{2}} - 1}$$

Away from the tip: $\rho \sim \zeta^{\frac{3}{4}}$

Surface tension is neglected.
Axisymmetry assumed.

"All *platonic* icicles have the same shape."

M. B. Short, J. C. Baygents, R. E. Goldstein, Phys. Fluids, ***18****, 083101 (2006).*

Fitting the shape

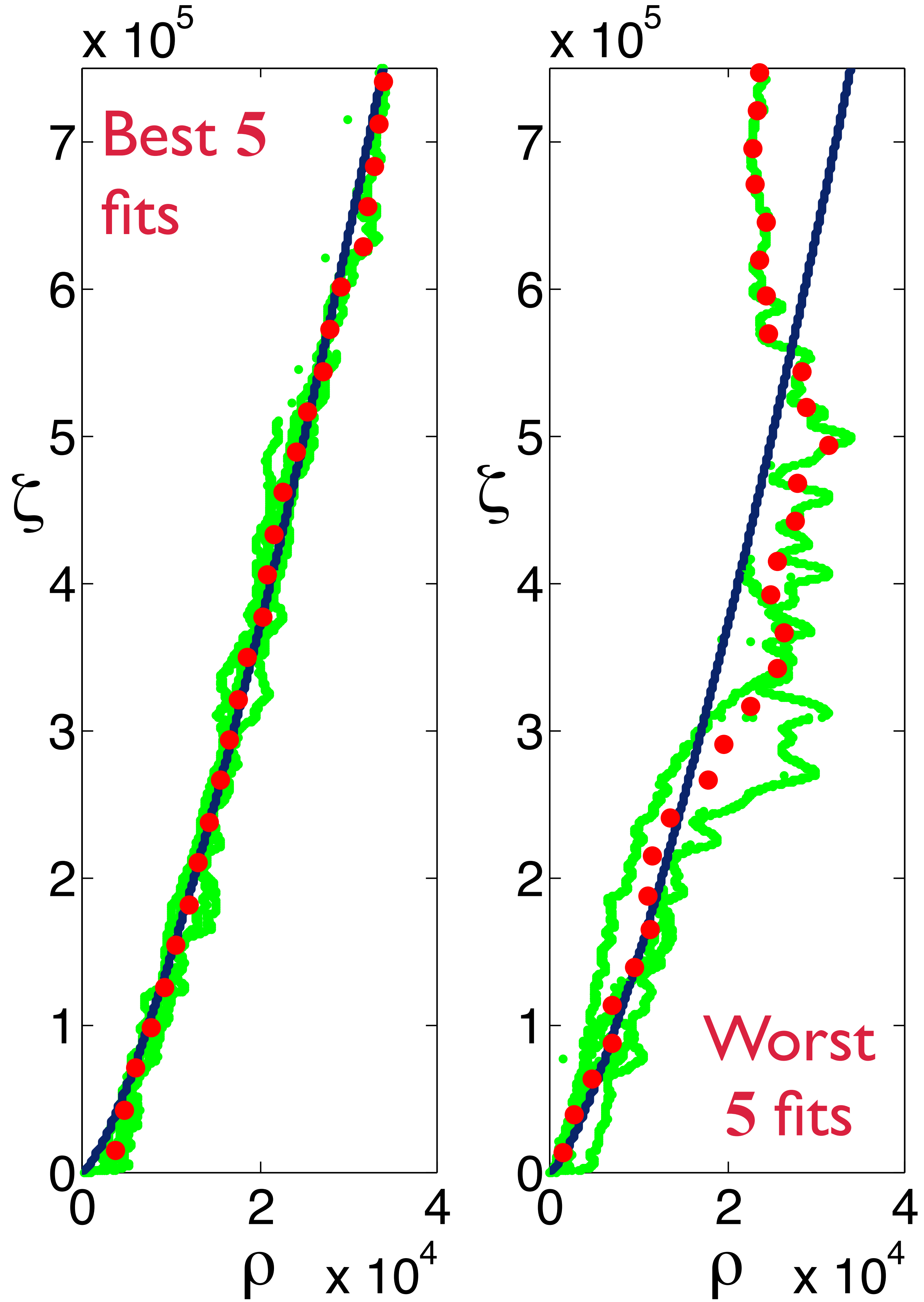
For each icicle, the profile $r(z)$ was fit to the theoretical shape:

$$r = a\left(\frac{4}{3}\left[\left(\frac{z}{a}\right)^{\frac{1}{2}} + 2\right]\sqrt{\left(\frac{z}{a}\right)^{\frac{1}{2}} - 1}\right)$$

via a least-squares analysis to get the best-fit scaling factor a.

For over 200 icicles, we find that *some* icicles fit the theory to a very remarkable degree, while others *do not*.

A. S. Chen and S. W. Morris, *Phys. Rev. E* 83, 026307 (2011).



The scale factor a

$$a = \ell(v_c/v_t)^4$$

$$= \frac{g\beta_a \Delta T^5}{\nu_a^2}\left[\frac{\Lambda_a}{v_t L C}\right]^4$$

$\ell =$ air boundary layer scale

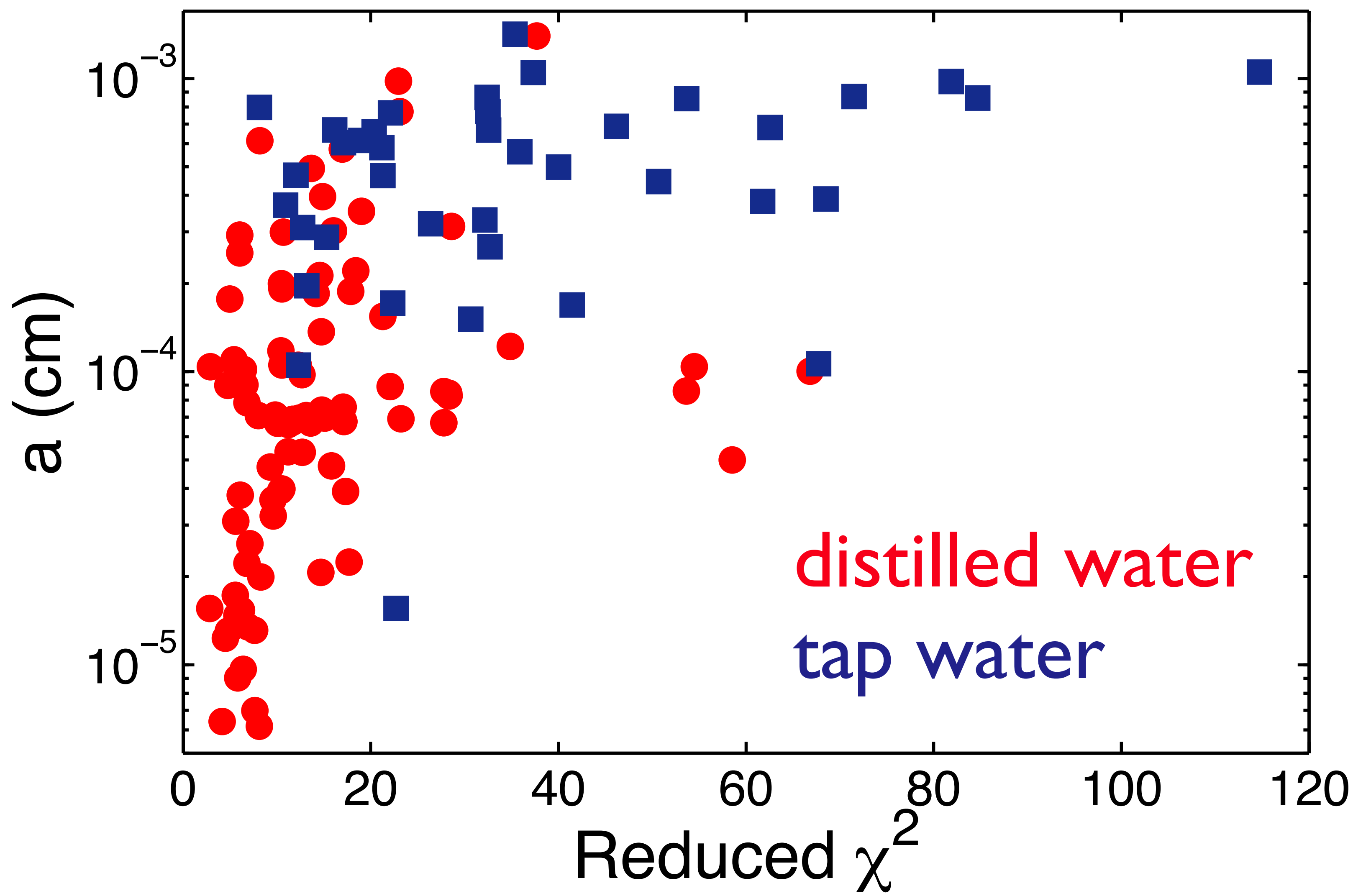
$v_c =$ surface growth speed

$v_t =$ tip growth speed

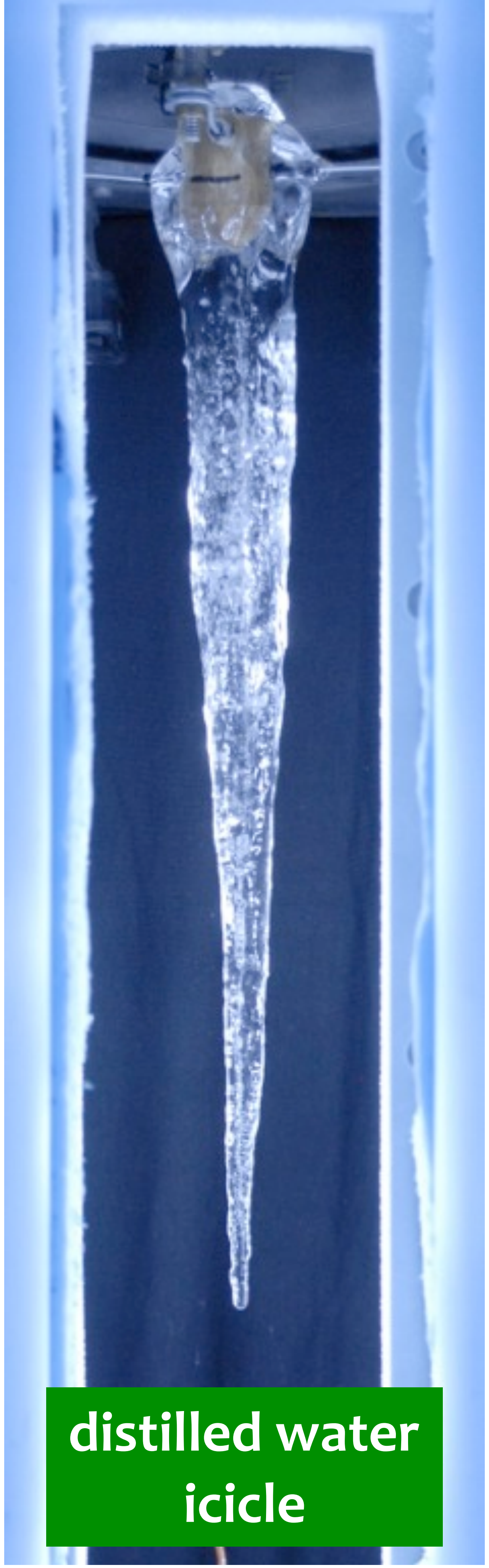
Theory predicts:

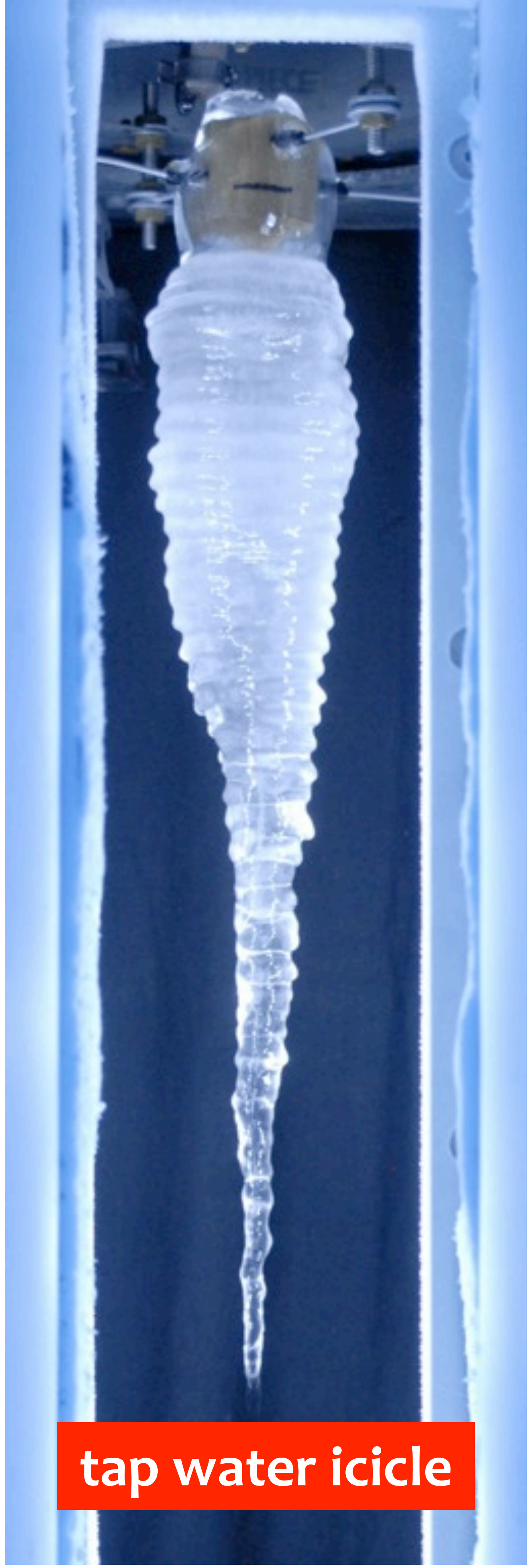
$$a_{\rm th} \sim 2 \times 10^{-5}\ {\rm cm}$$

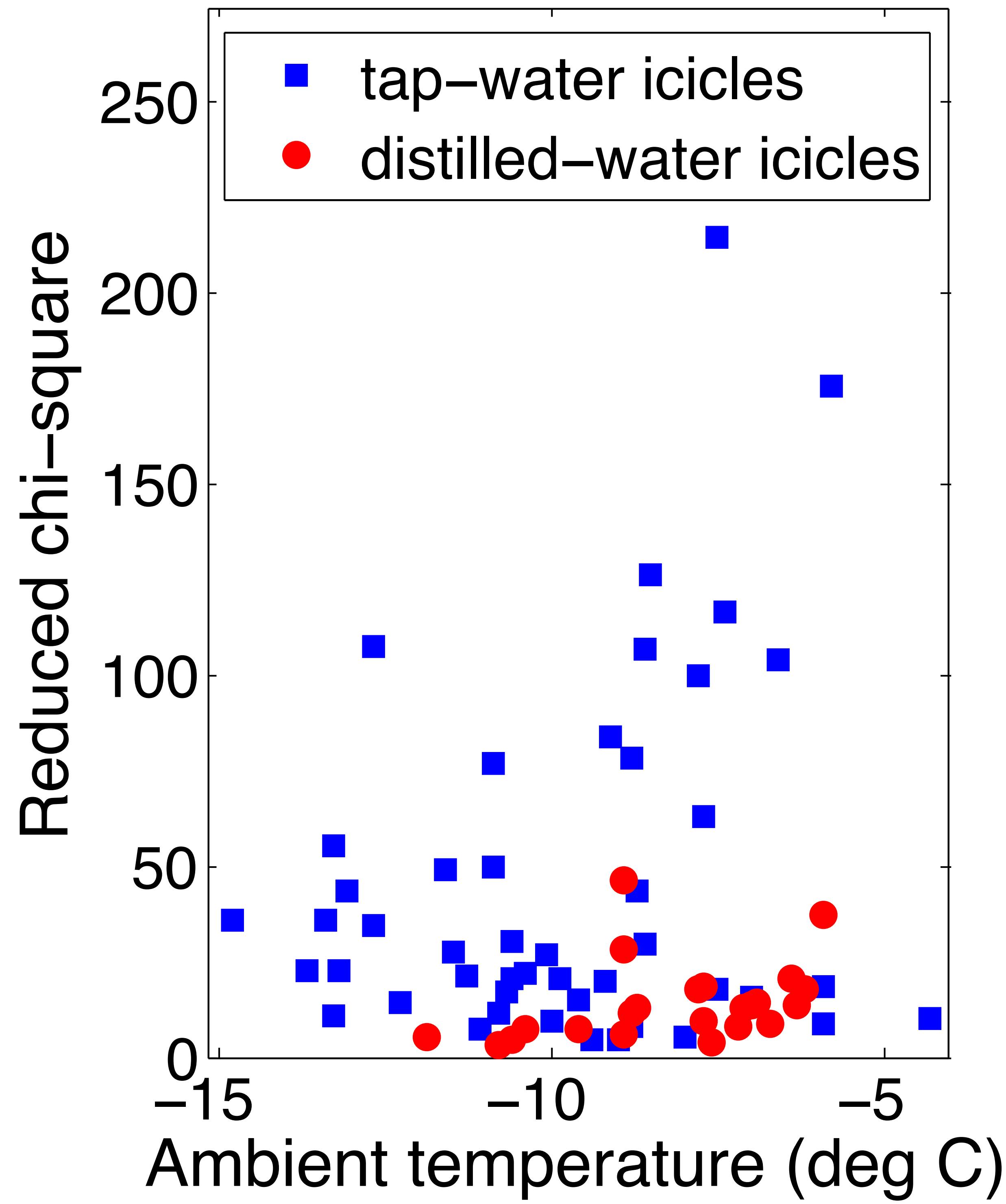
When the theory fits, the scale factor is about the right size



Not-so-platonic icicles

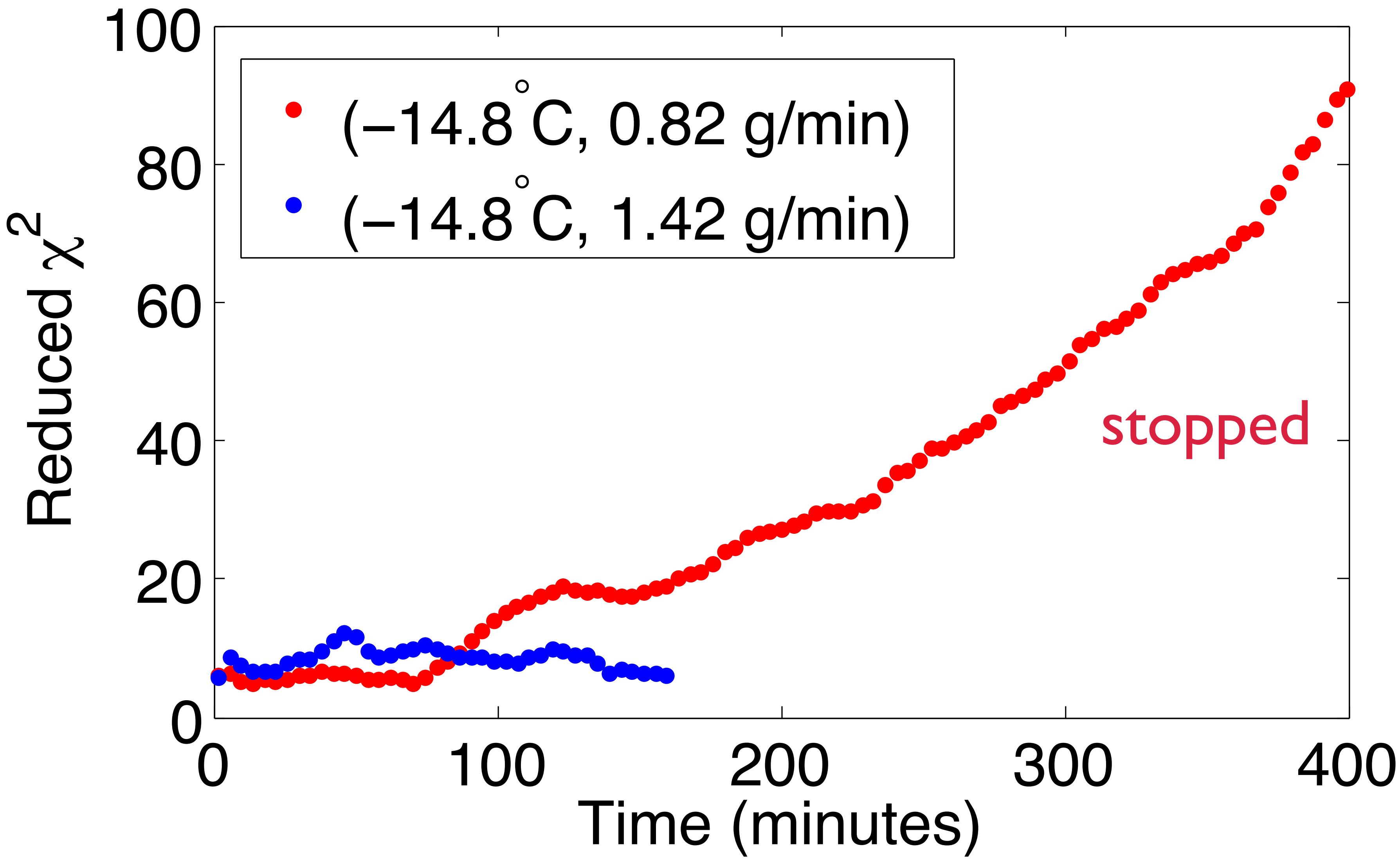






Water purity matters

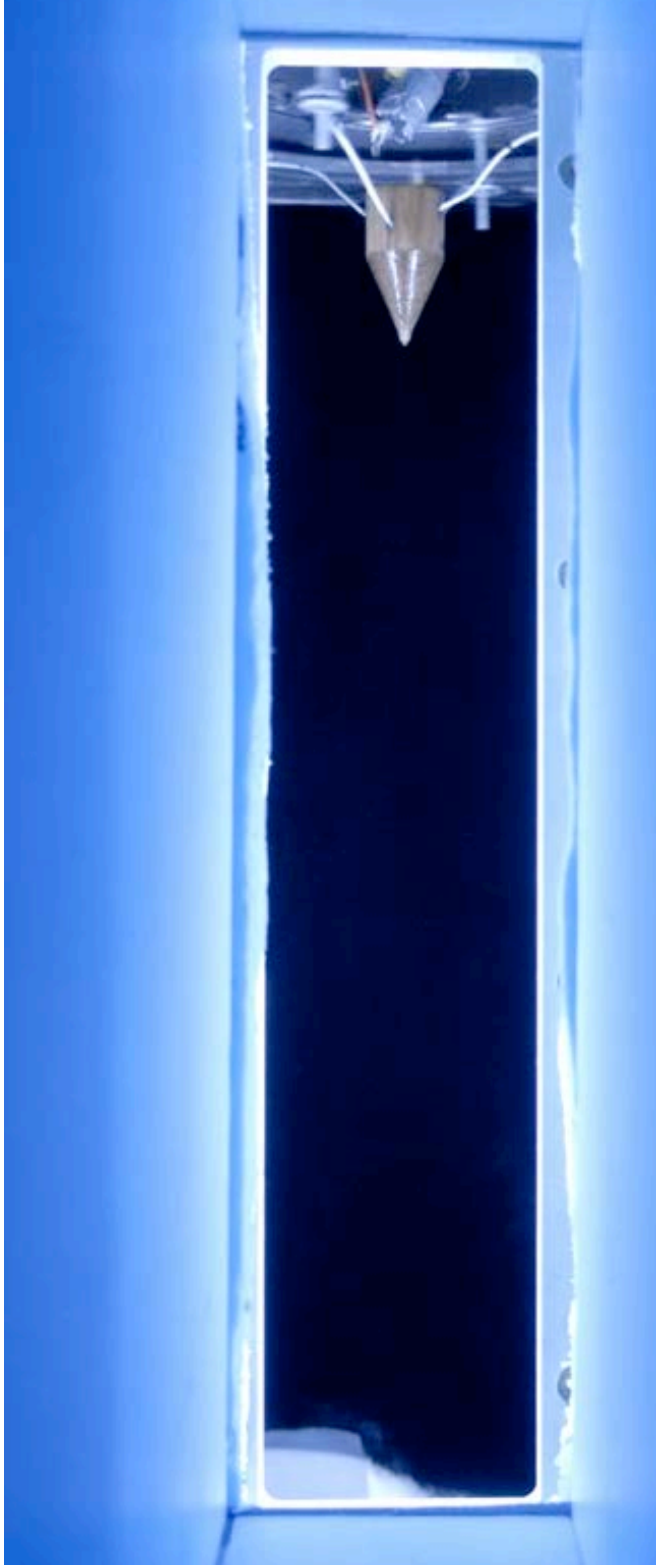
Cessation of growth and shape



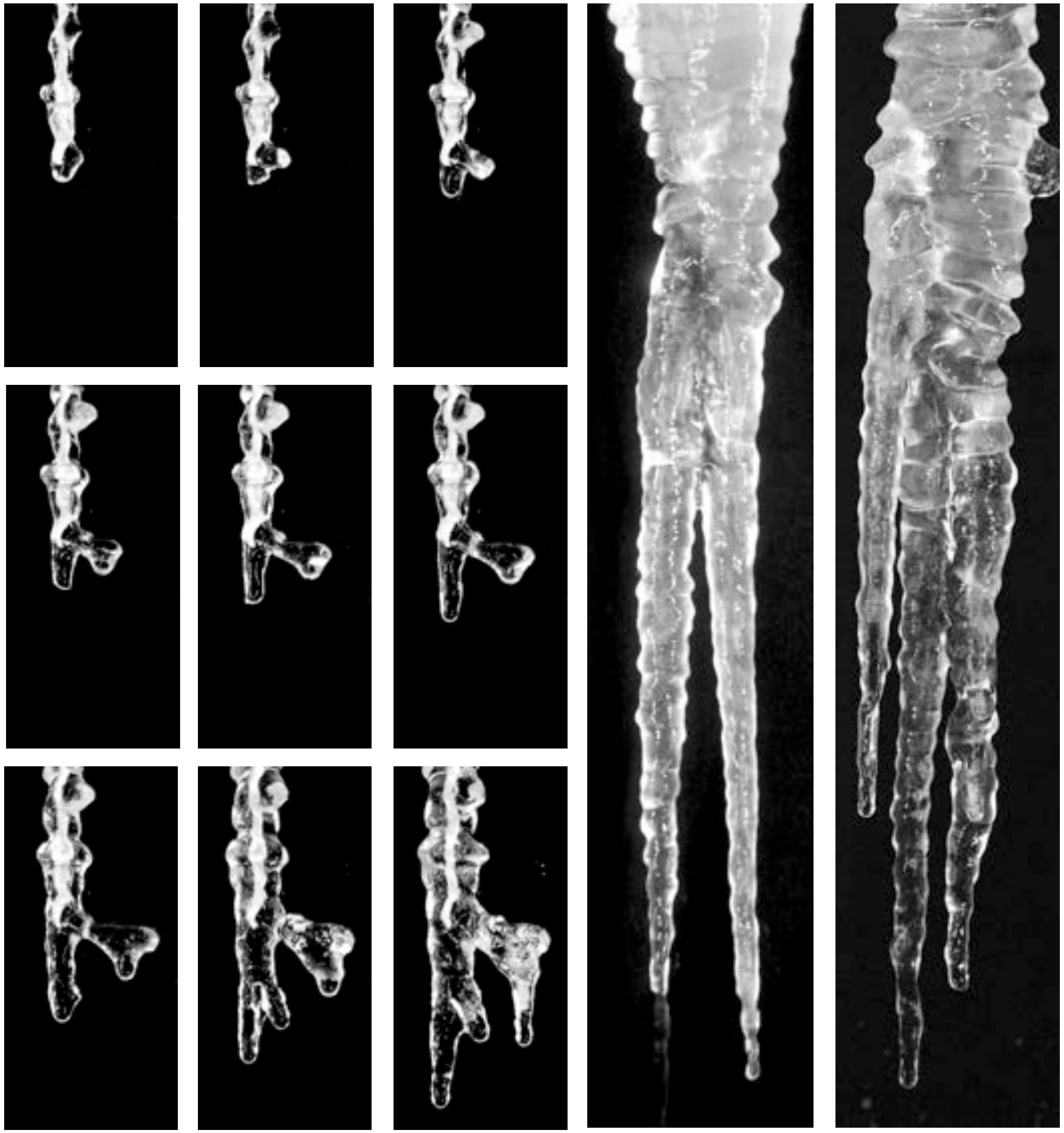
Low flow rate, very cold icicles stop growing, and then become much less "platonic".

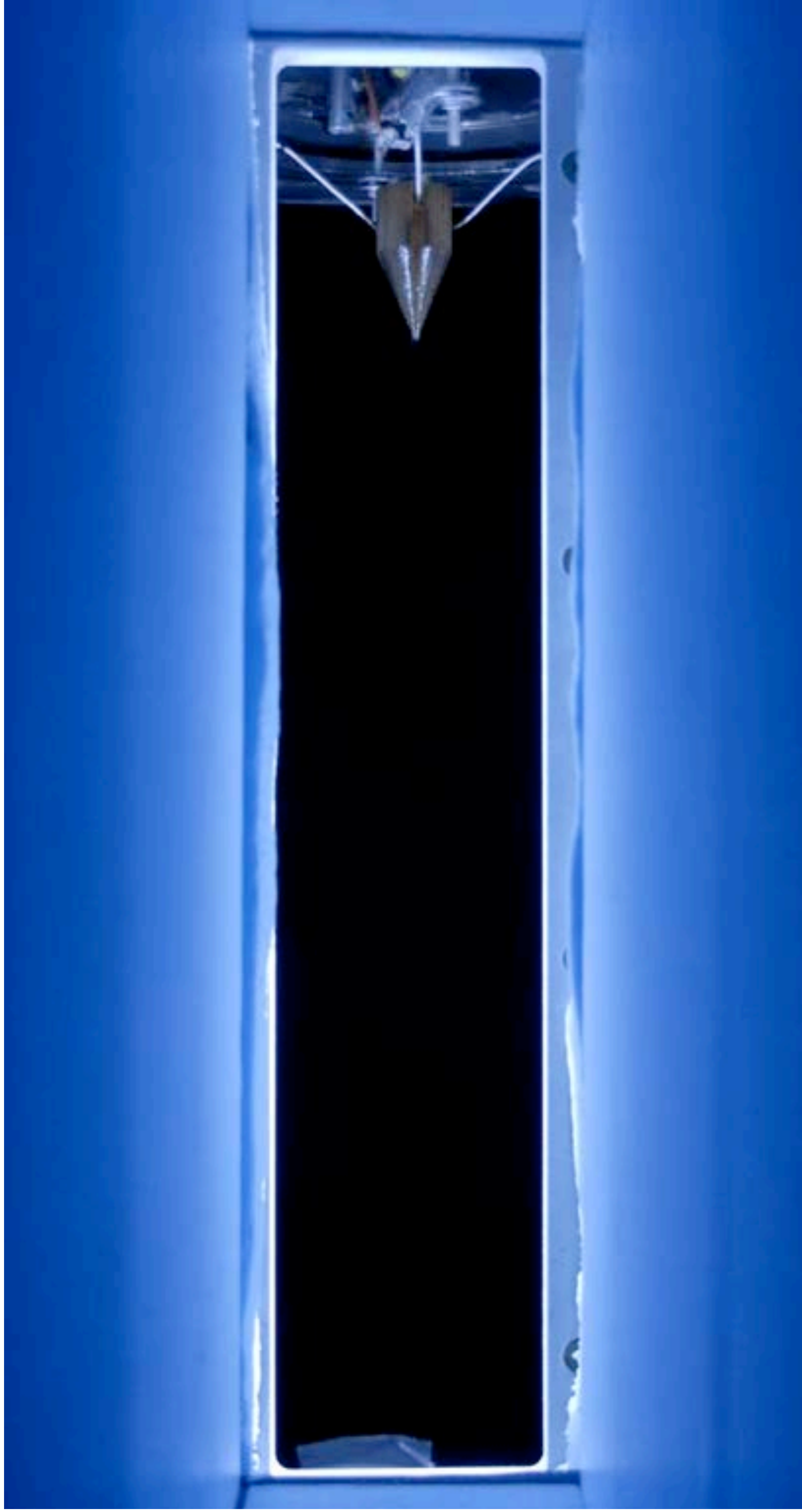
Stalactites do not do this.

-18° C
1.14 g/min



Air motion matters





Icicles tend to form multiple tips
if air is *not* stirred.

Advection / diffusion
Evaporation / condensation
Radiation

All heat transport mechanisms matter

$$\rho_{\mathrm{i}} L v = \Lambda_a \frac{\partial T}{\partial x}\bigg|_{x=0} + F_{\mathrm{ec}} + F_{\mathrm{rad}}$$

The Stefan condition: the water film is almost isothermal, ΔT less than 0.01°C

$F_{\mathrm{ec}} =$ Heat flux due to evaporated water advected by the buoyancy driven flow as a passive tracer. Depends on relative humidity.

$$F_{\mathrm{rad}} = \sigma (T_\infty^4 - T_0^4)$$

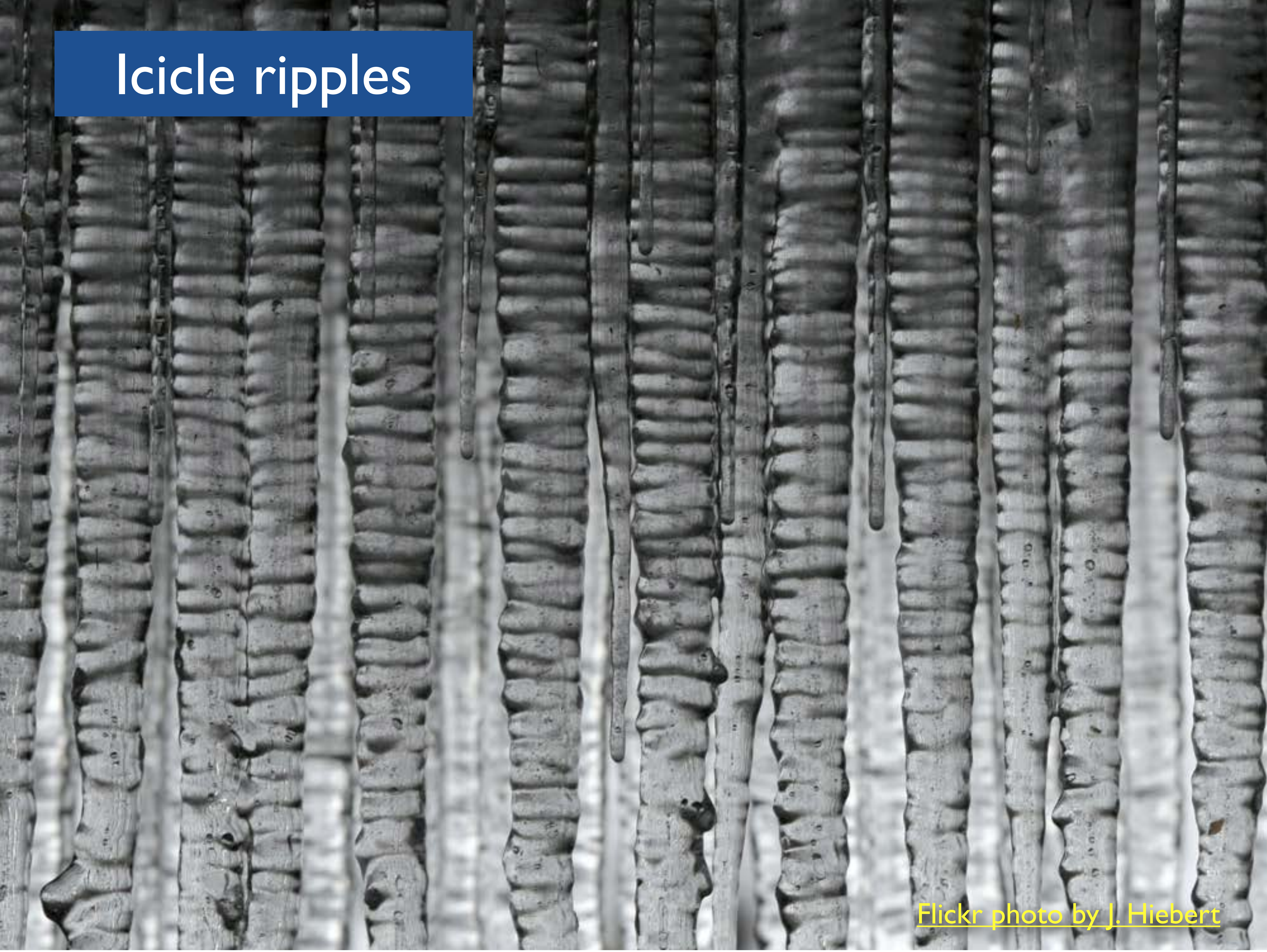
Heat flux due to radiation

All these are about the same size!

J. A. Neufeld, R. E. Goldstein and M. G. Worster, *JFM* **647**, 287 (2010).

Icicle ripples

Icicle ripples



Flickr photo by J. Hiebert

Icicle ripples

"Michelin Man" ring-like ridges are often seen on the surface of natural icicles.

Ripples are always observed to have a wavelength very close to $1\ \mathrm{cm}$, independent of flow rate, undercooling *etc.*

Ripples *not* observed on distilled water icicles, so they seem to depend on water purity.

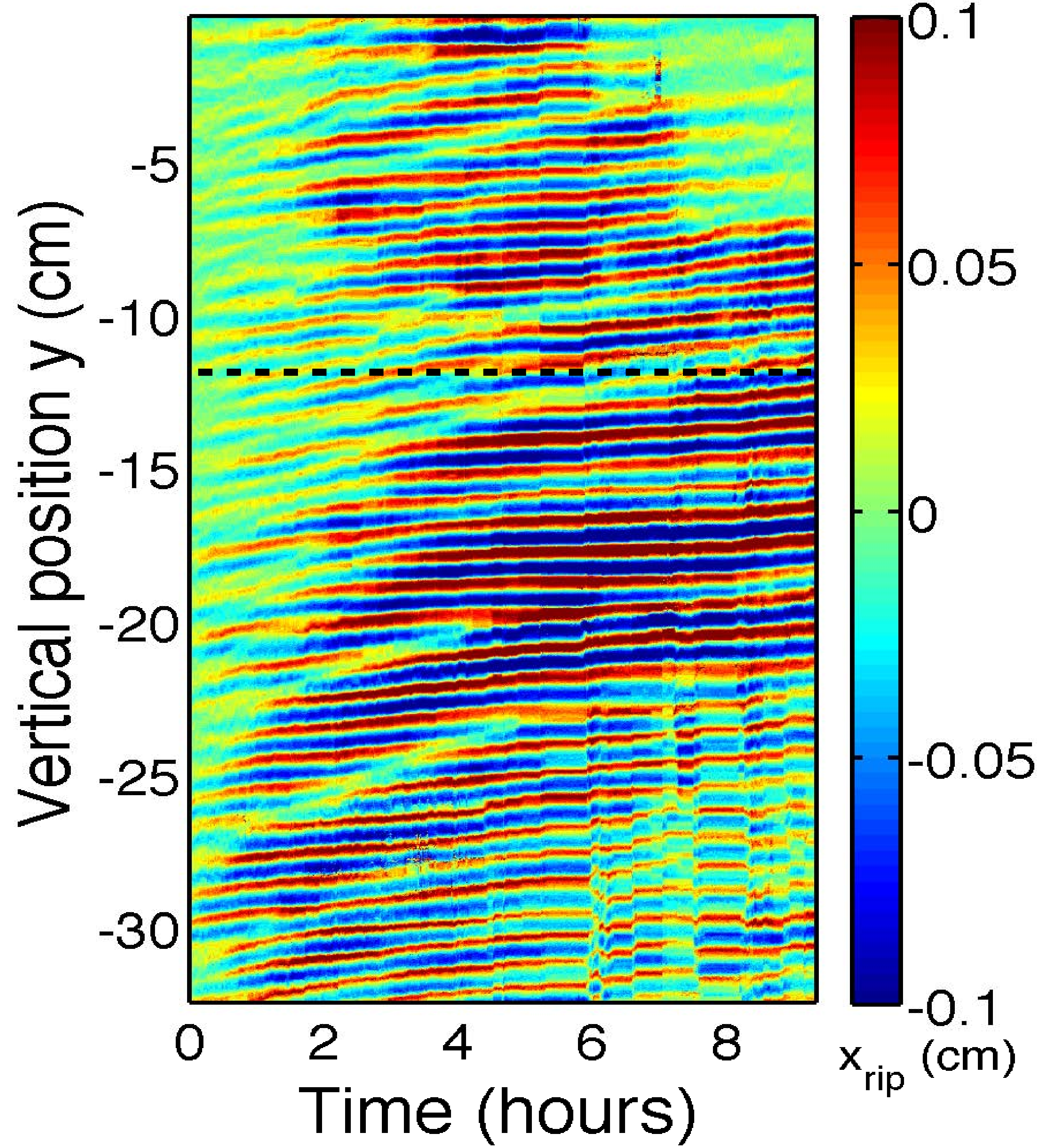
Rippling instability theory claims surface tension effects at air-water interface lead to ripples
*K. Ueno, Phys. Fluids **19**, 093602 (2007)*

Surface tension, impurity effects, radiation, and evaporation / condensation are all left out of the platonic icicle theory of Short *et al.*

Cave ripples: *crenulations*

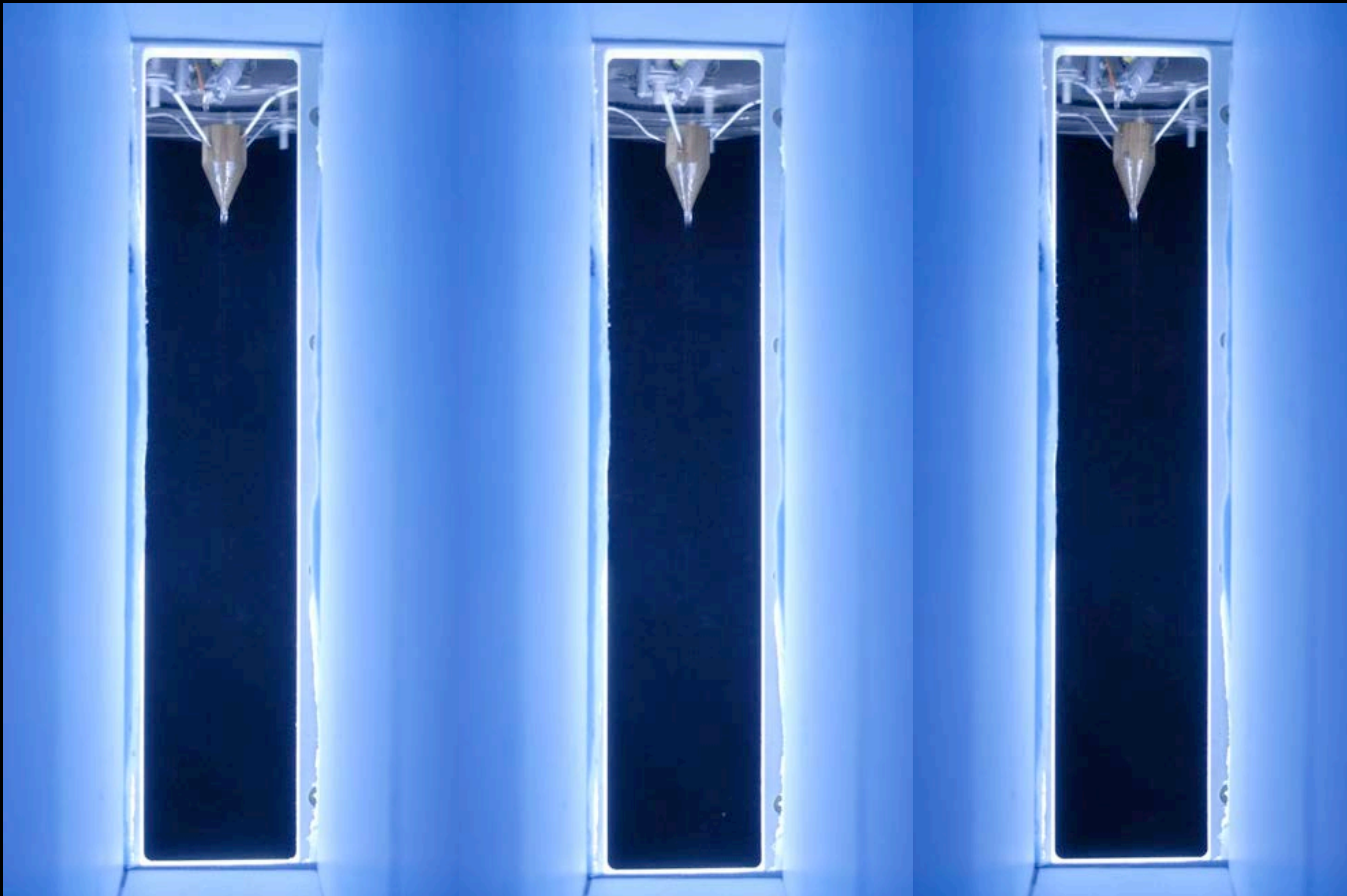
Ripples are also common in cave formations

C. Camporeale and L. Ridolfi, *Phys. Rev. Lett.*, **108**, 238501 (2012).

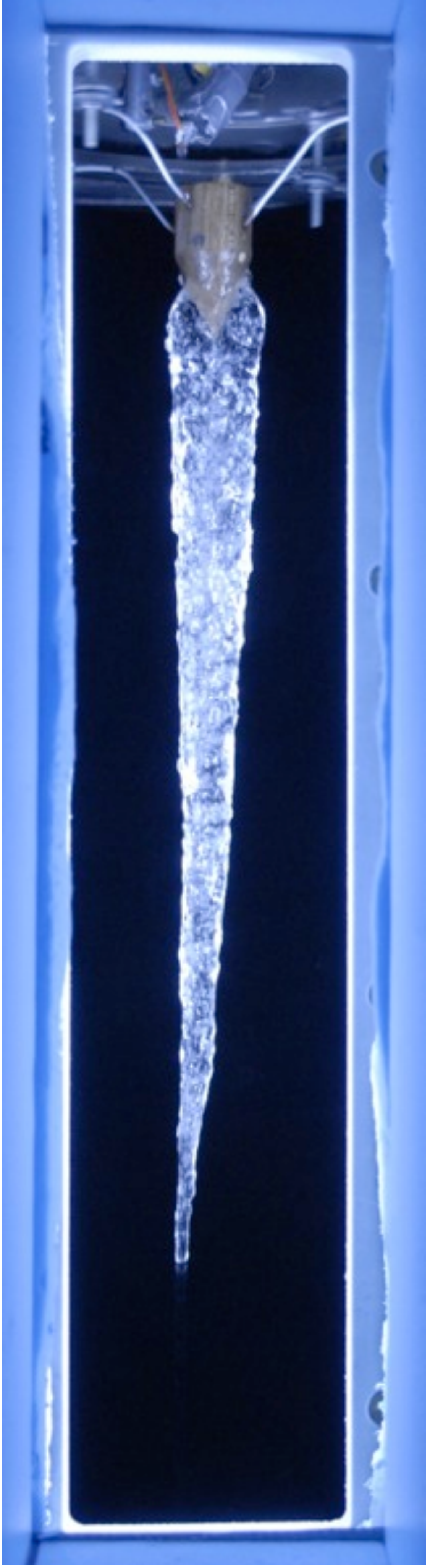


Ripples tend to move upward very slowly

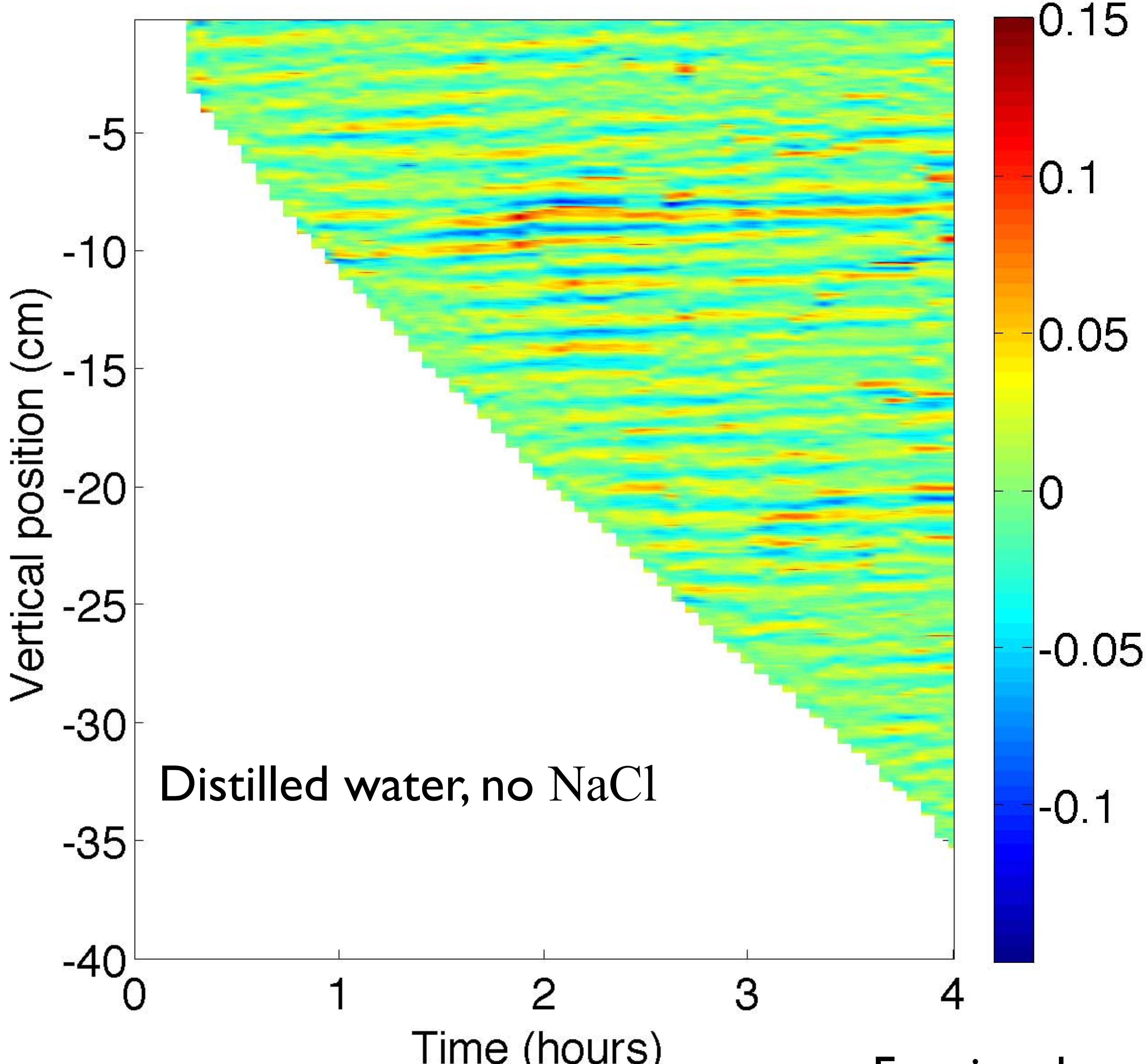
distilled water | **40 mg NaCl per kg H2O** | **1.28 g NaCl per kg H2O**



-12.3 deg C, 2.0 g/min

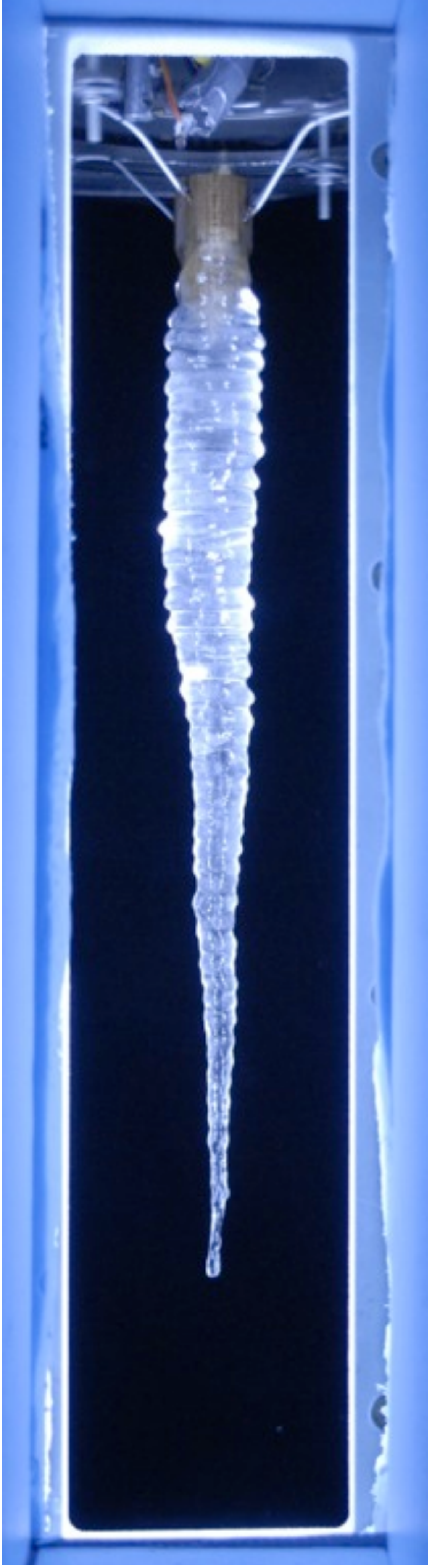


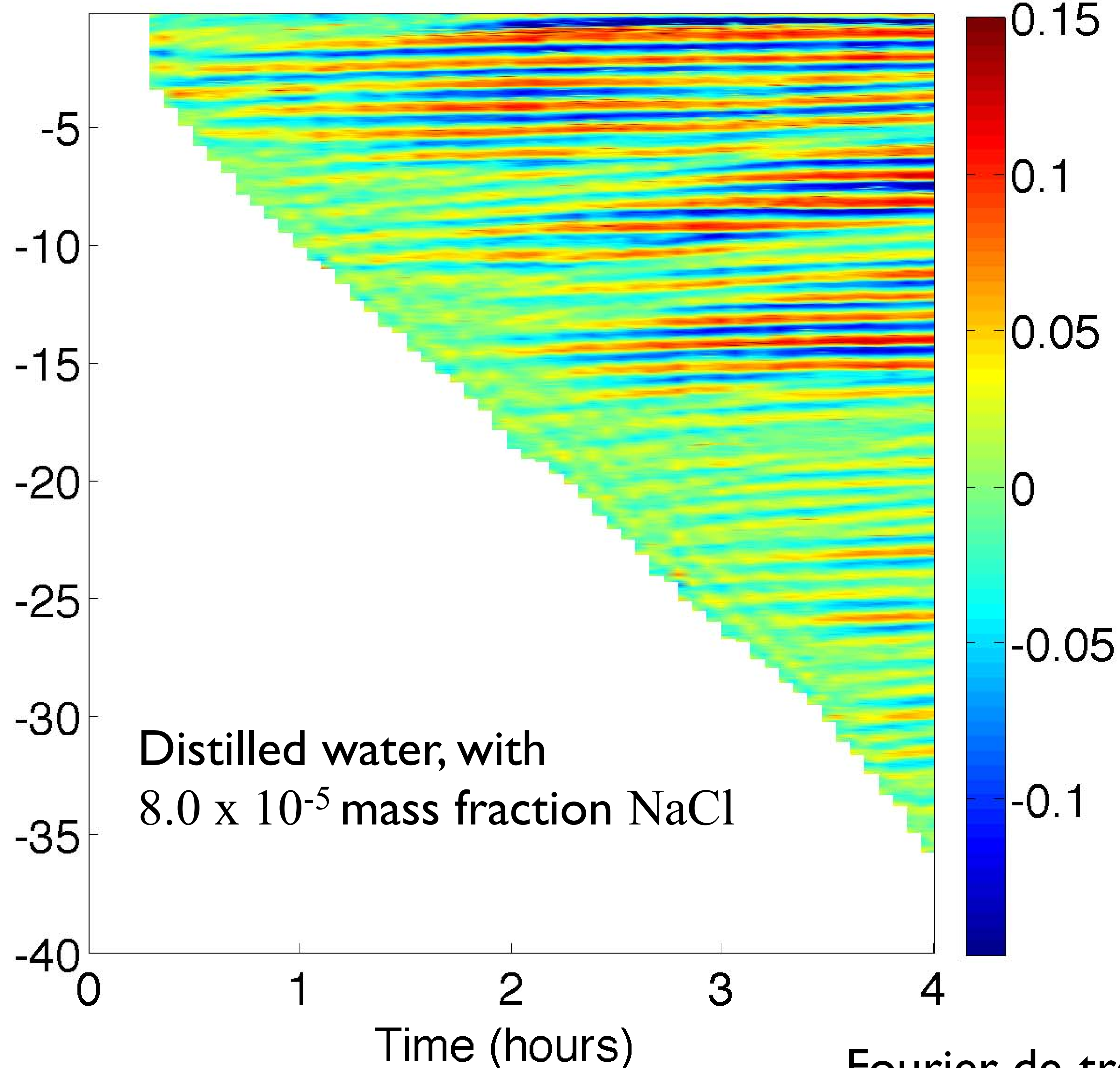
Icicle topography vs time



Fourier de-trended

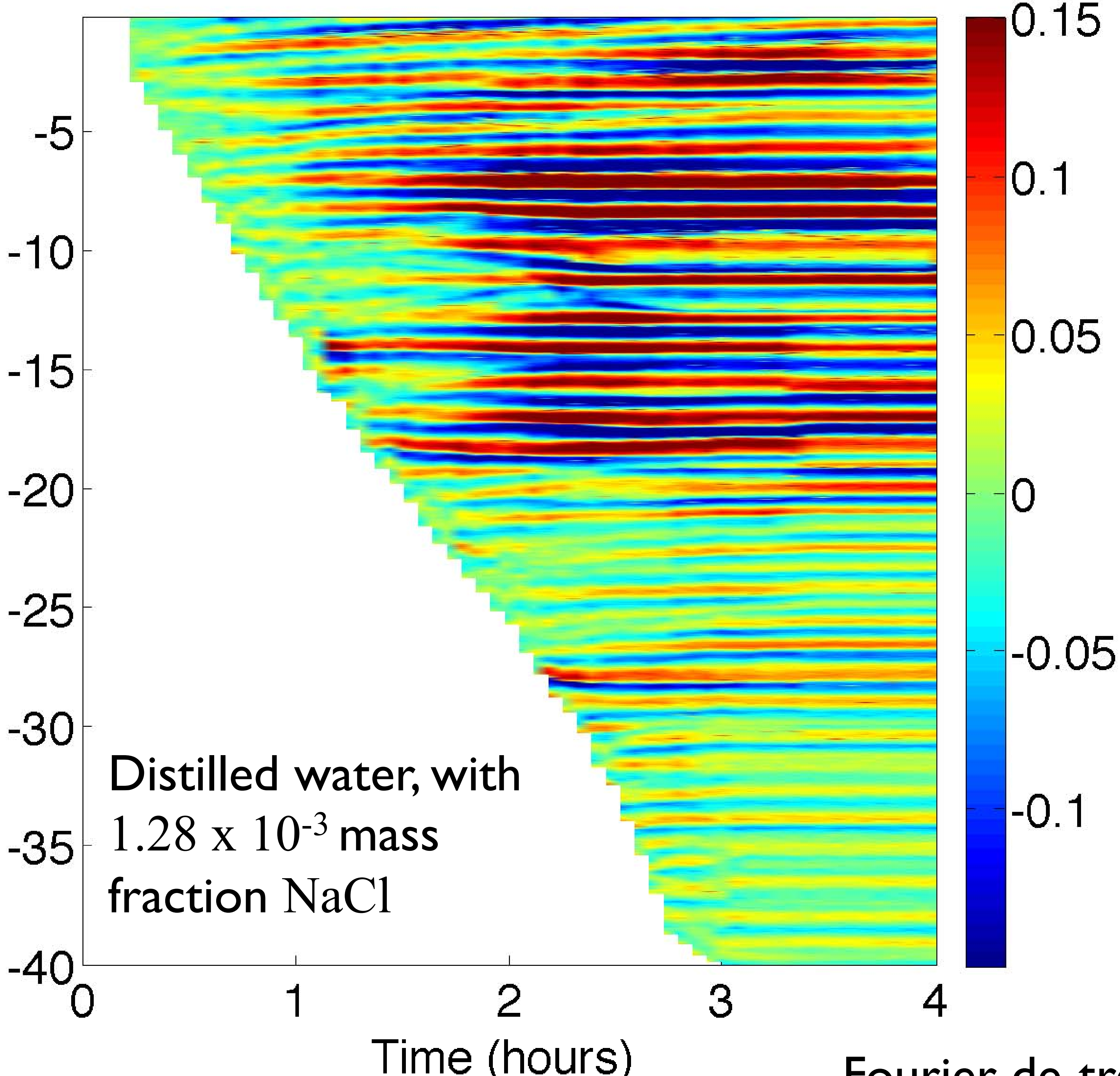
Icicle topography vs time





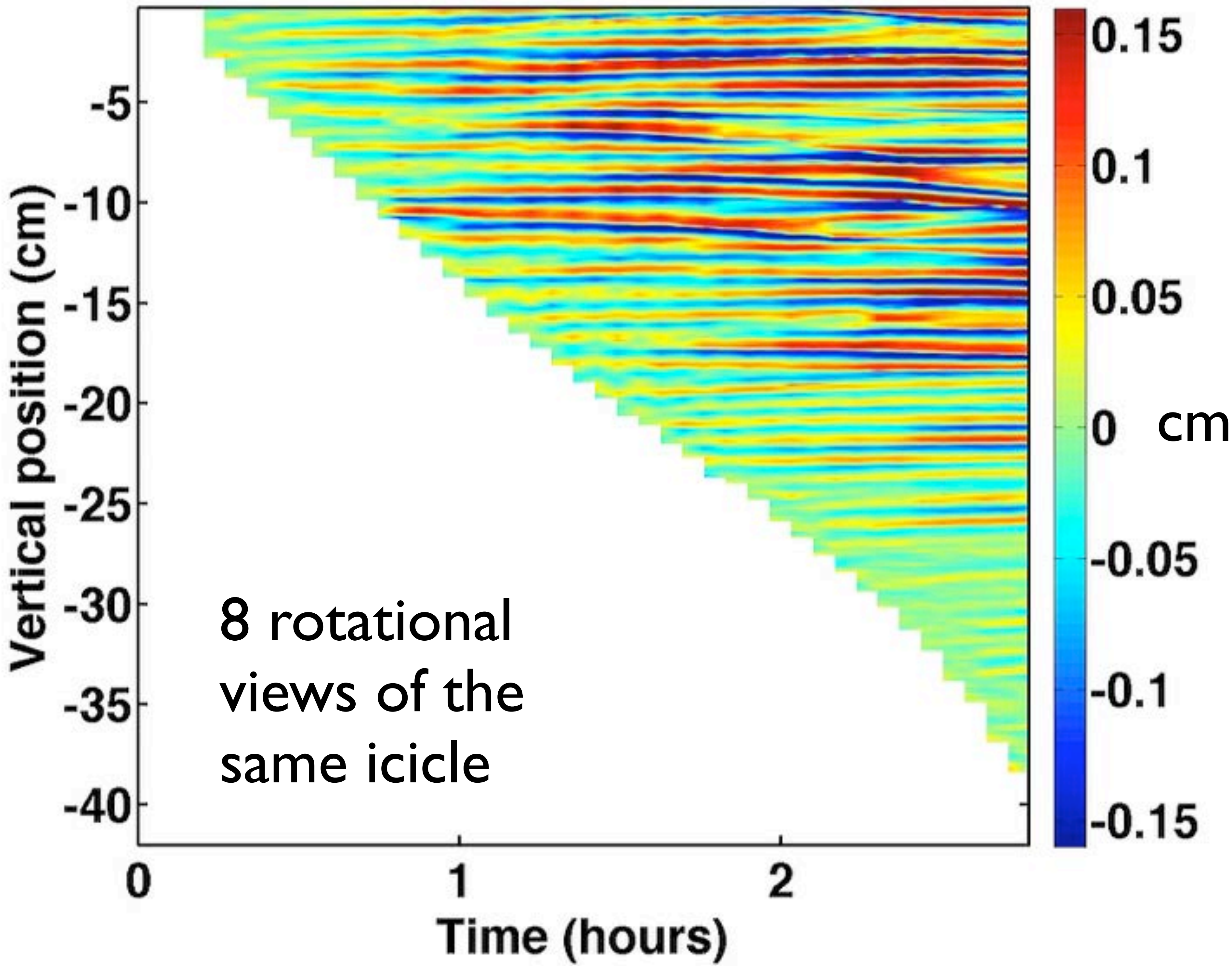
Fourier de-trended

Icicle topography vs time



Fourier de-trended

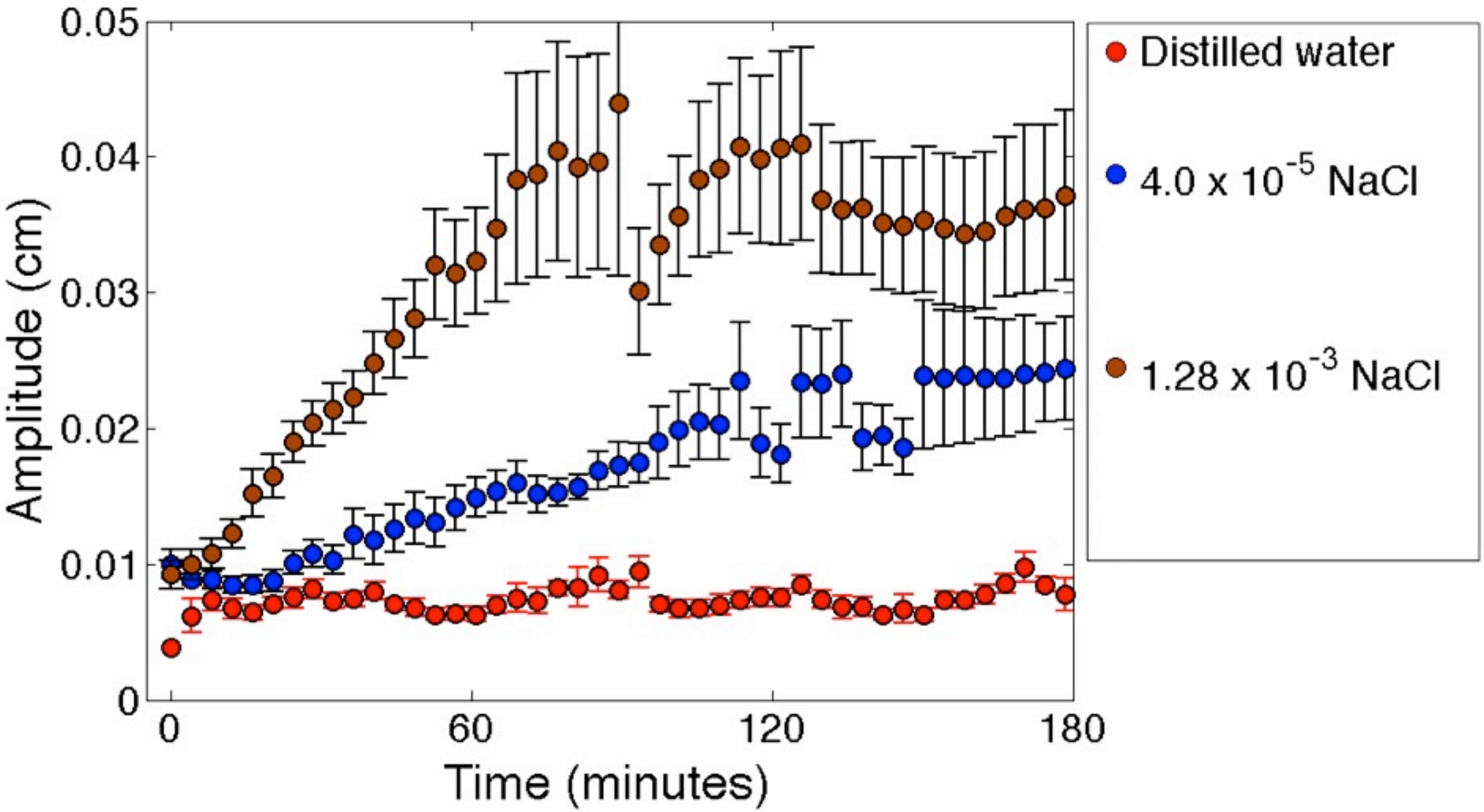
Icicle topography vs time



Distilled water, with 1.28×10^{-3} mass fraction NaCl

Fourier de-trended

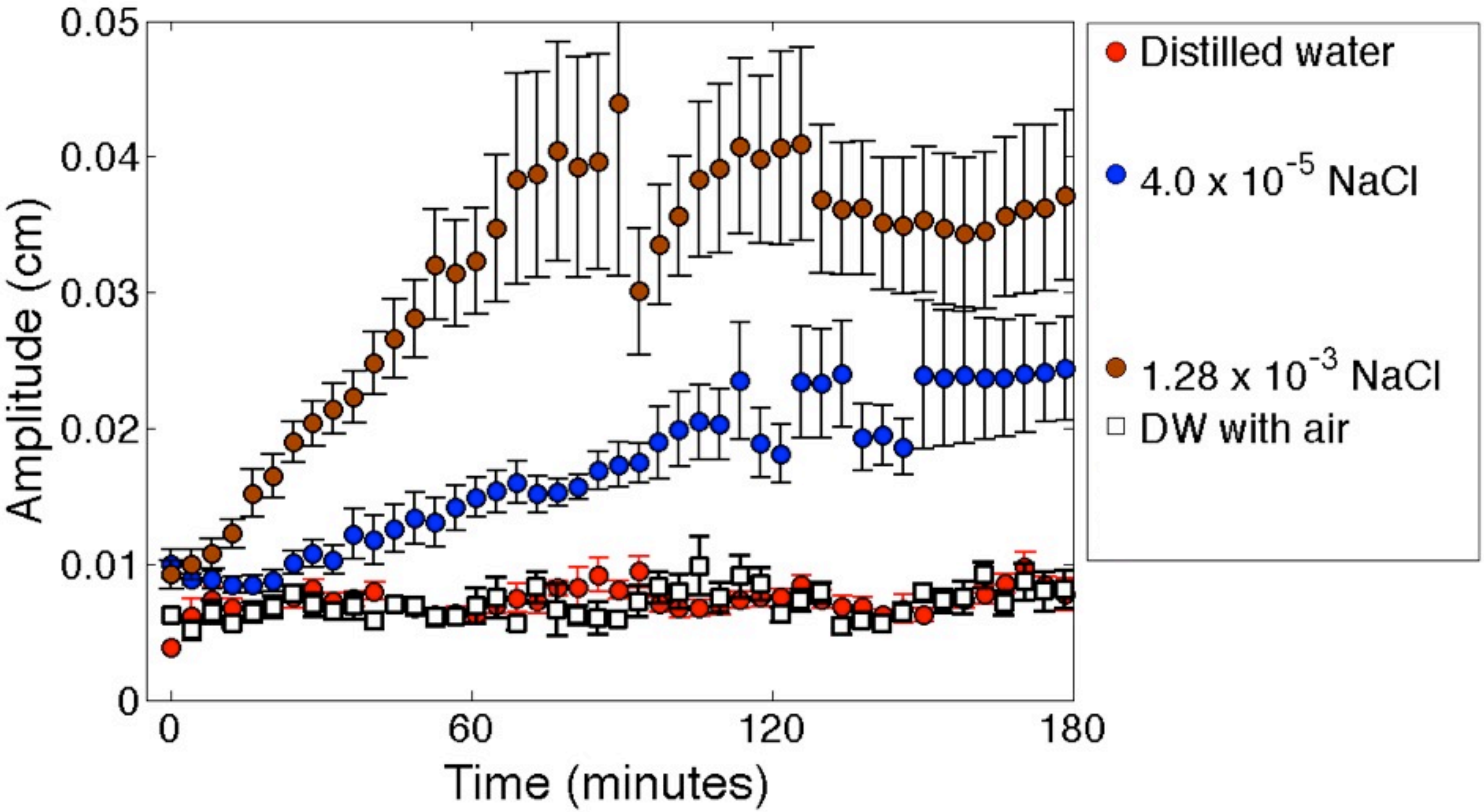
Growth and saturation of ripples vs time



Amplitudes averaged over 1 rotation of icicle and over repeated runs.

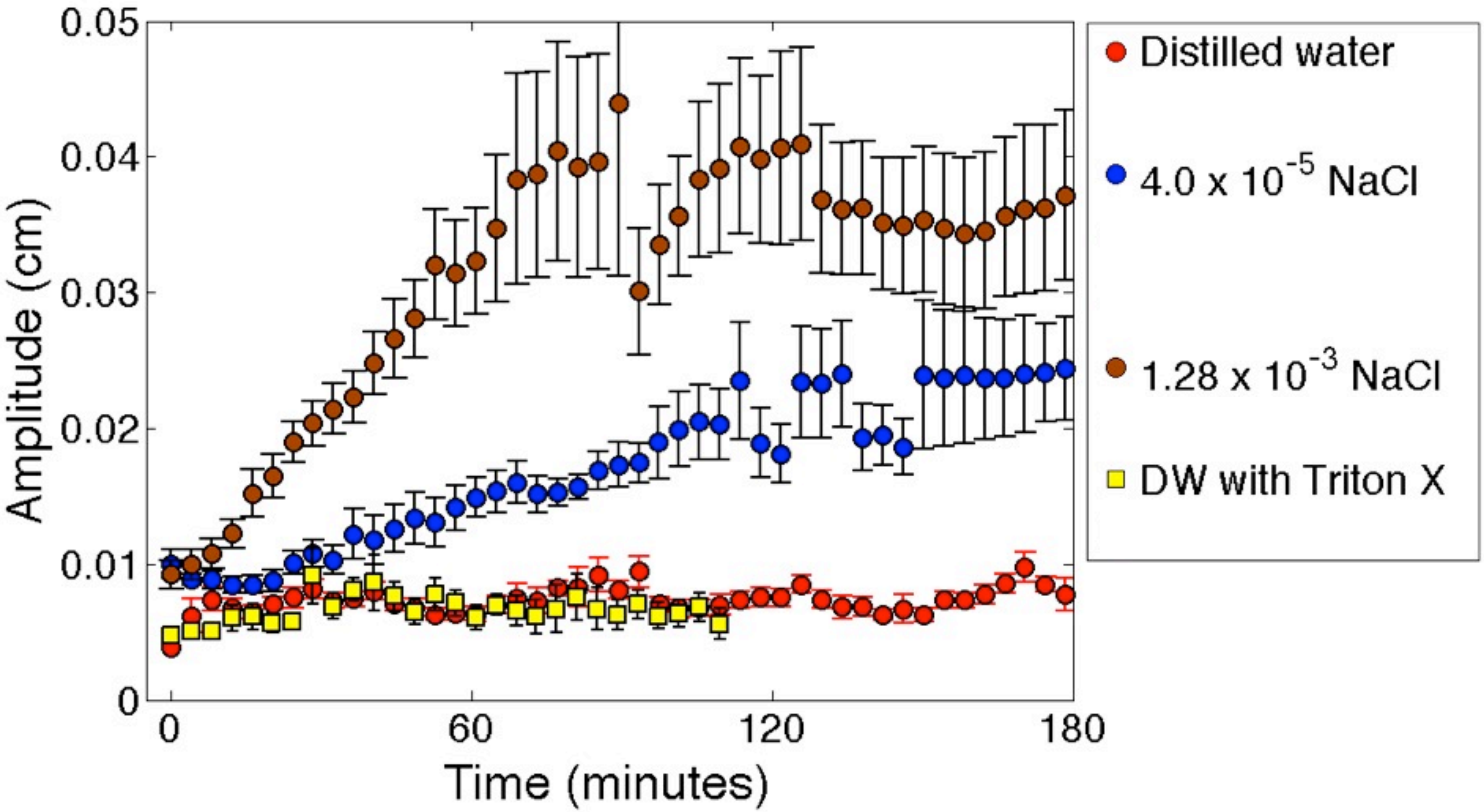
$t = 0$ when icicle reaches 10 cm, topography data from top 10 cm only.

Growth and saturation of ripples vs time



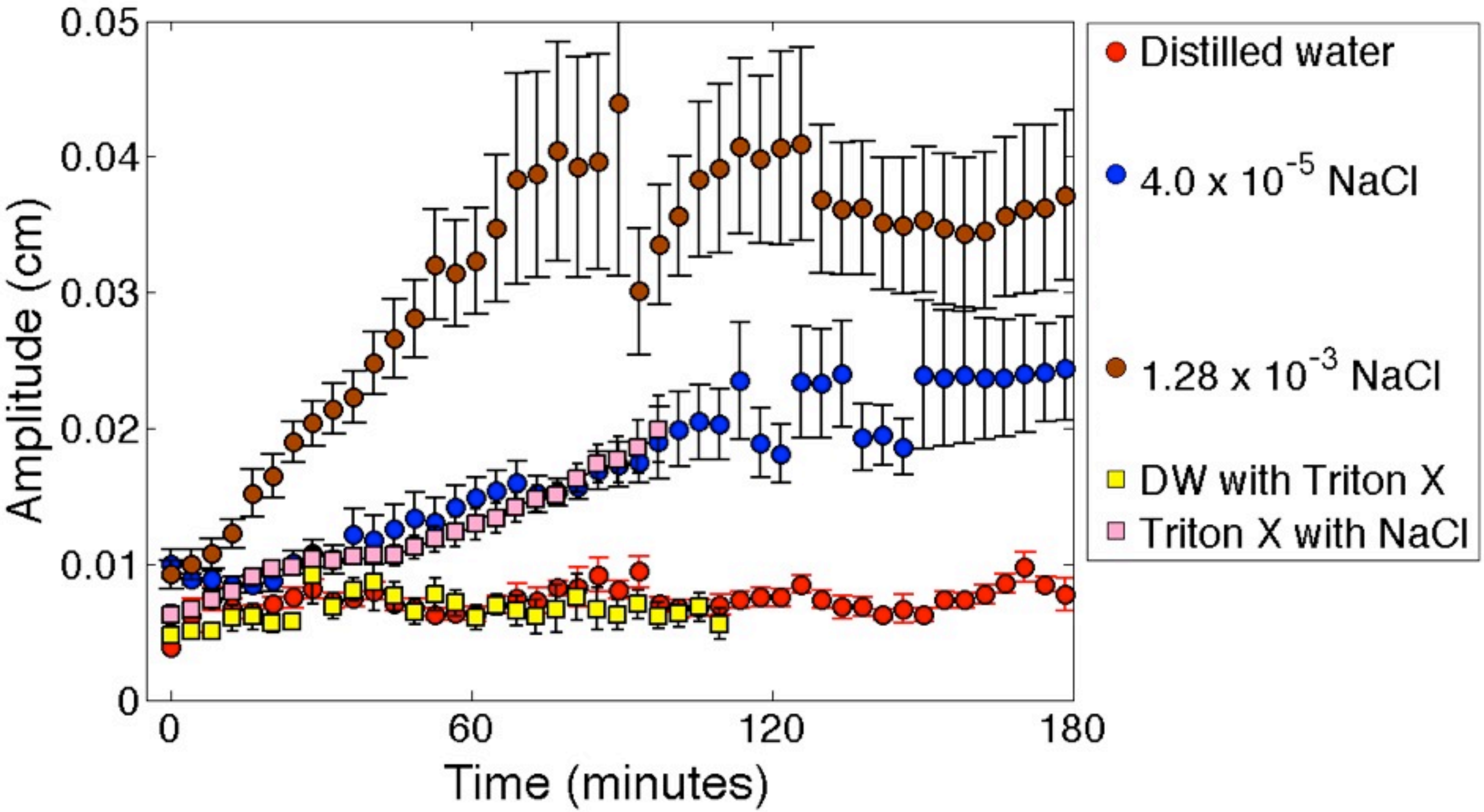
Bubbling air through the distilled water for a long time does **not** produce ripples. All samples are likely saturated with dissolved air.

Growth and saturation of ripples vs time



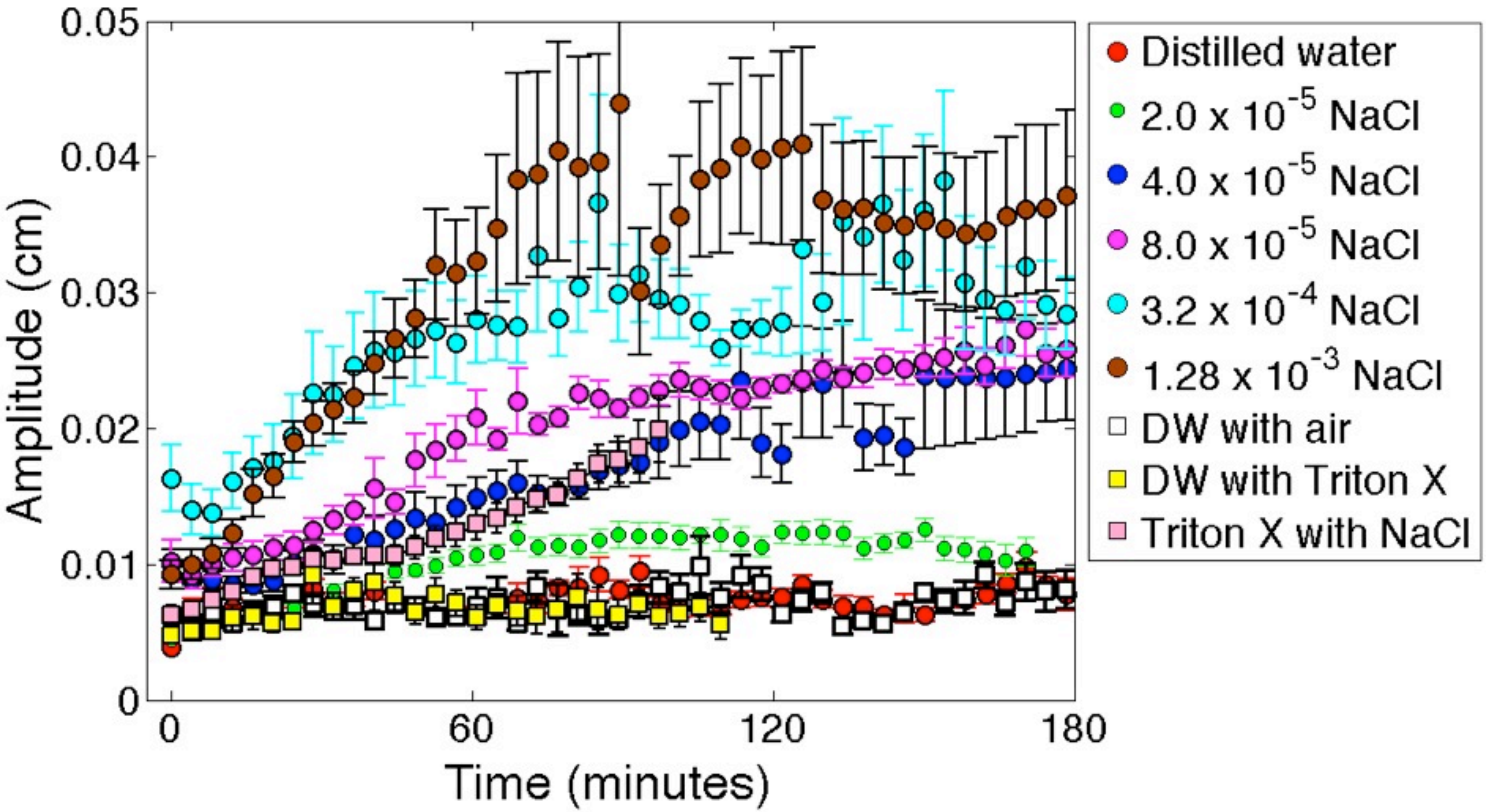
Adding a strong surfactant, Triton X, to distilled water does **not** produce ripples, even though the surface tension is reduced by 47%.

Growth and saturation of ripples vs time



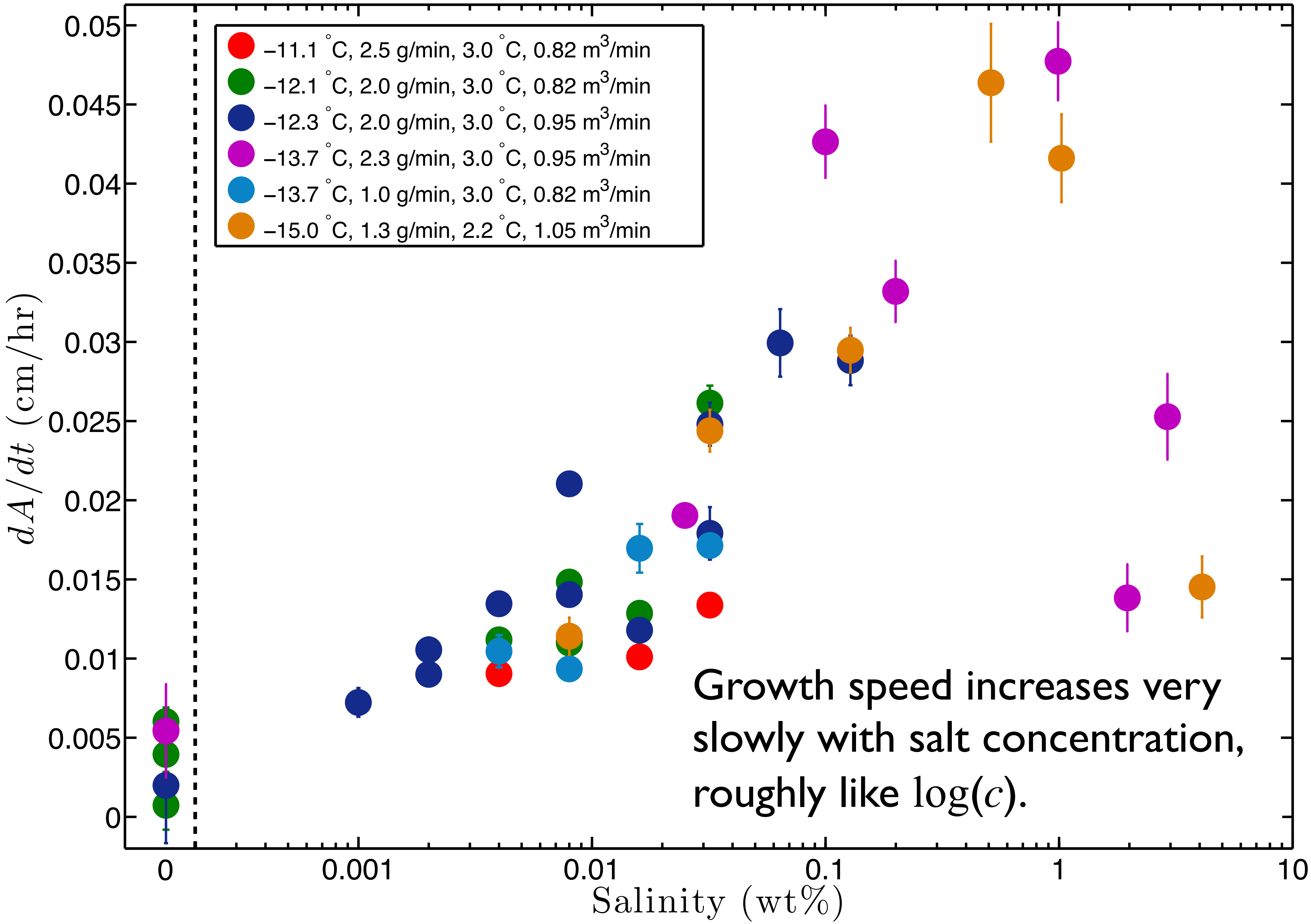
Adding a small concentration of salt *and* the Triton X, **does** produce ripples, but somewhat smaller than those with just the salt.

Growth and saturation of ripples vs time



Both ripple amplitude and growth rate increase with salt concentration, but surfactant and dissolved air do not produce ripples.

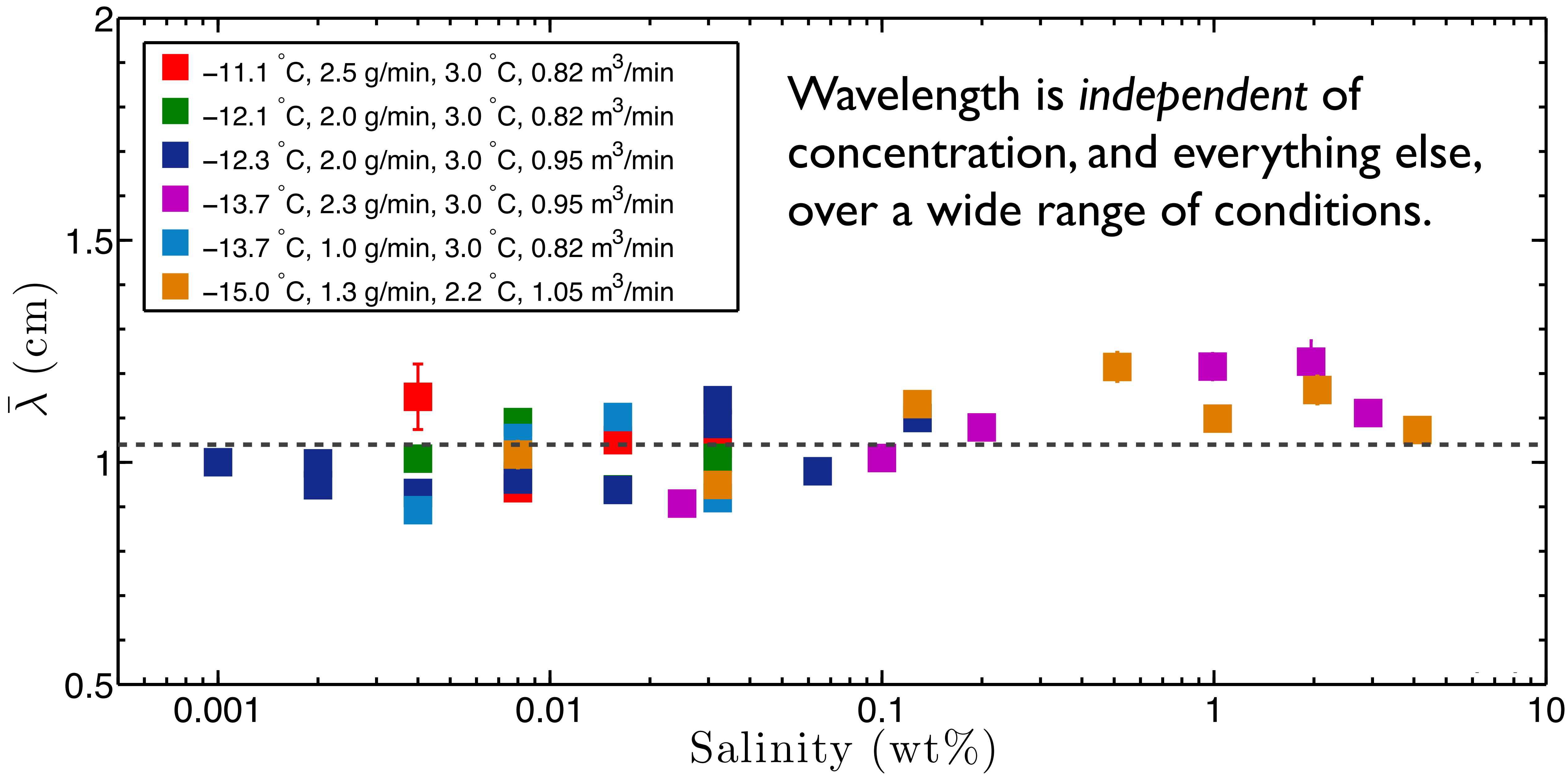
Growth speed of ripples vs concentration



Growth speed increases very slowly with salt concentration, roughly like $\log(c)$.

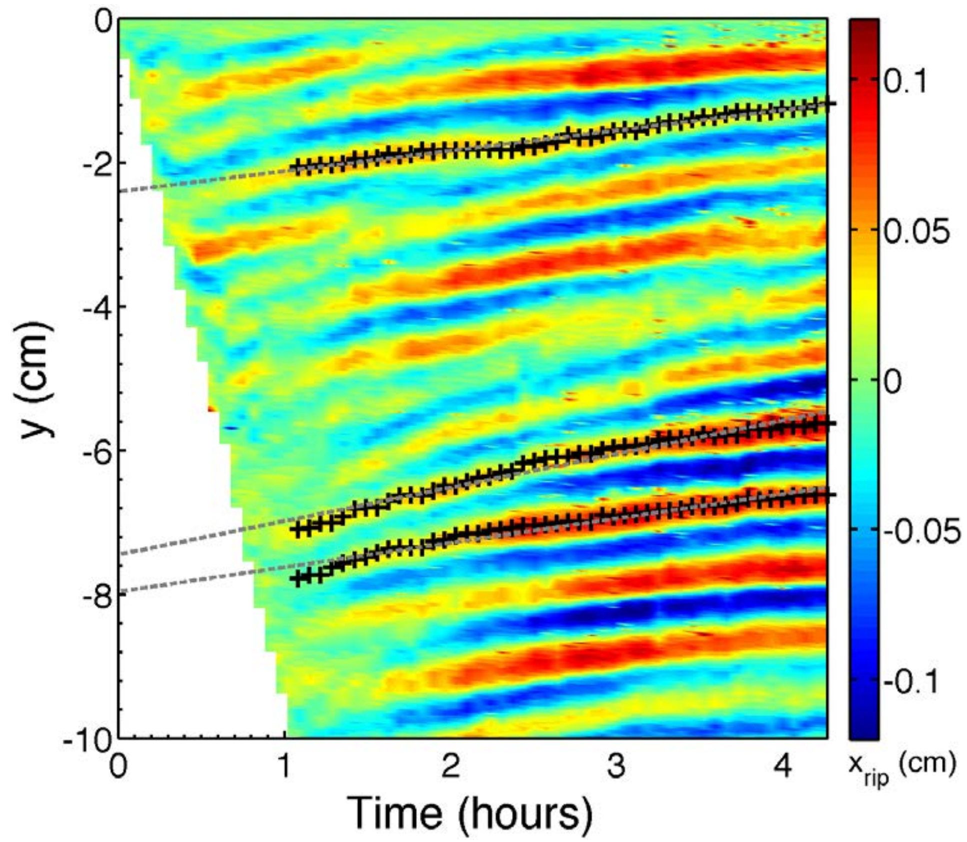
Averaged over time after 1 hour of growth, top 10 cm

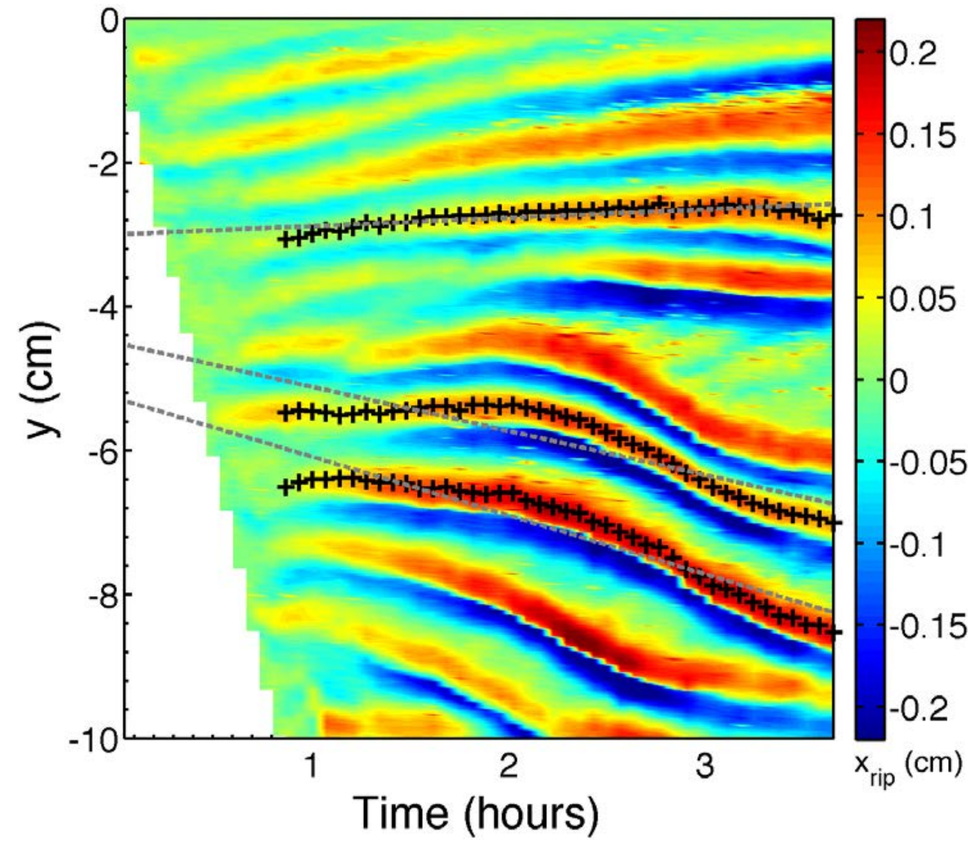
Mean wavelength of ripples vs concentration



Wavelength is *independent* of concentration, and everything else, over a wide range of conditions.

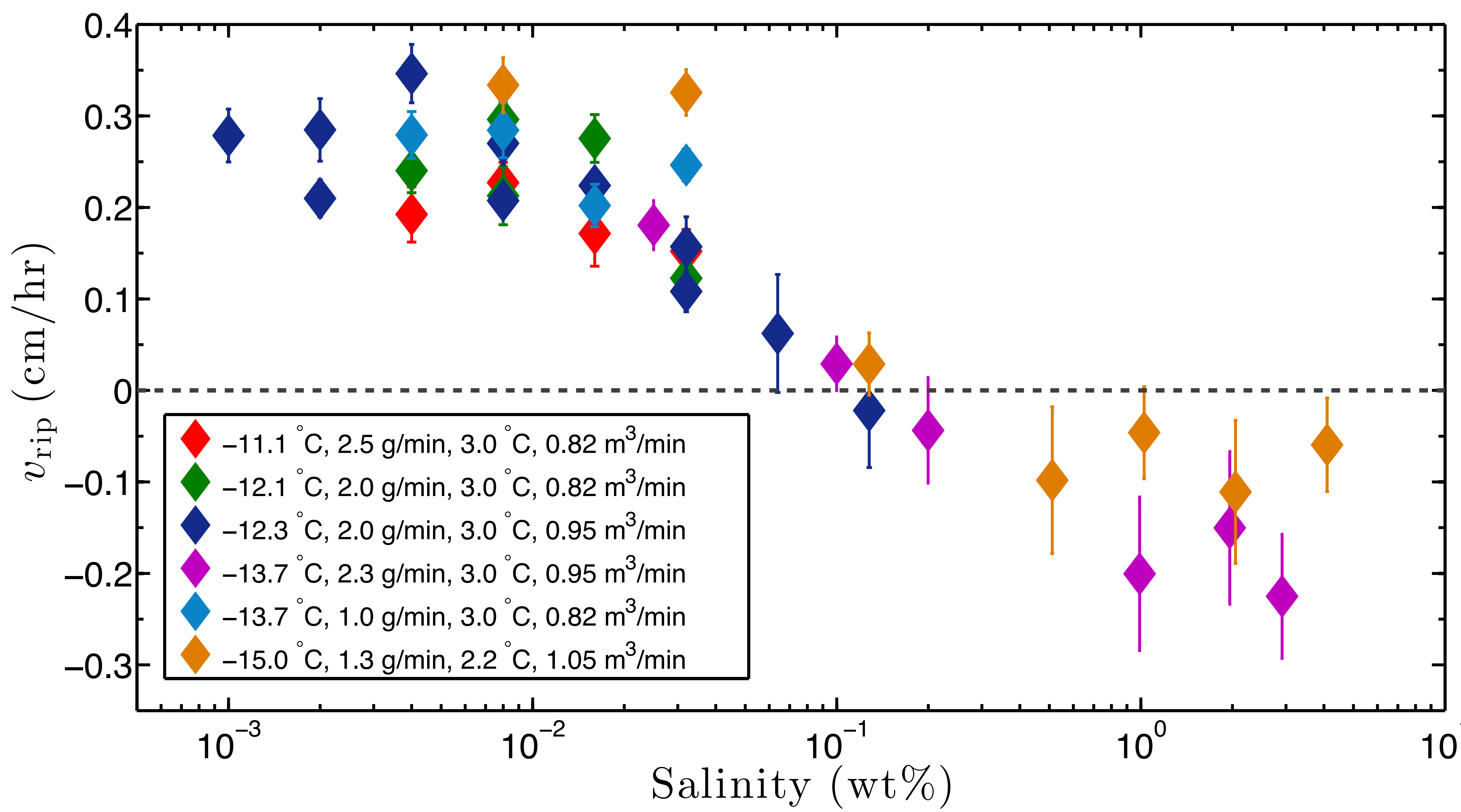
Traveling speed vs concentration

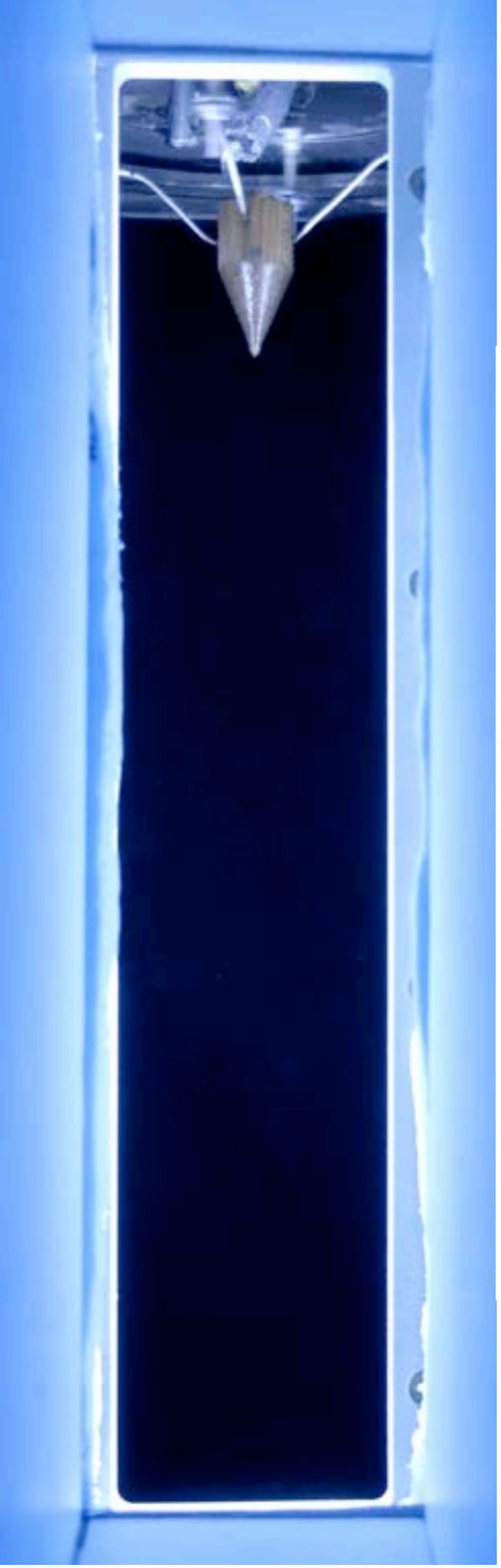




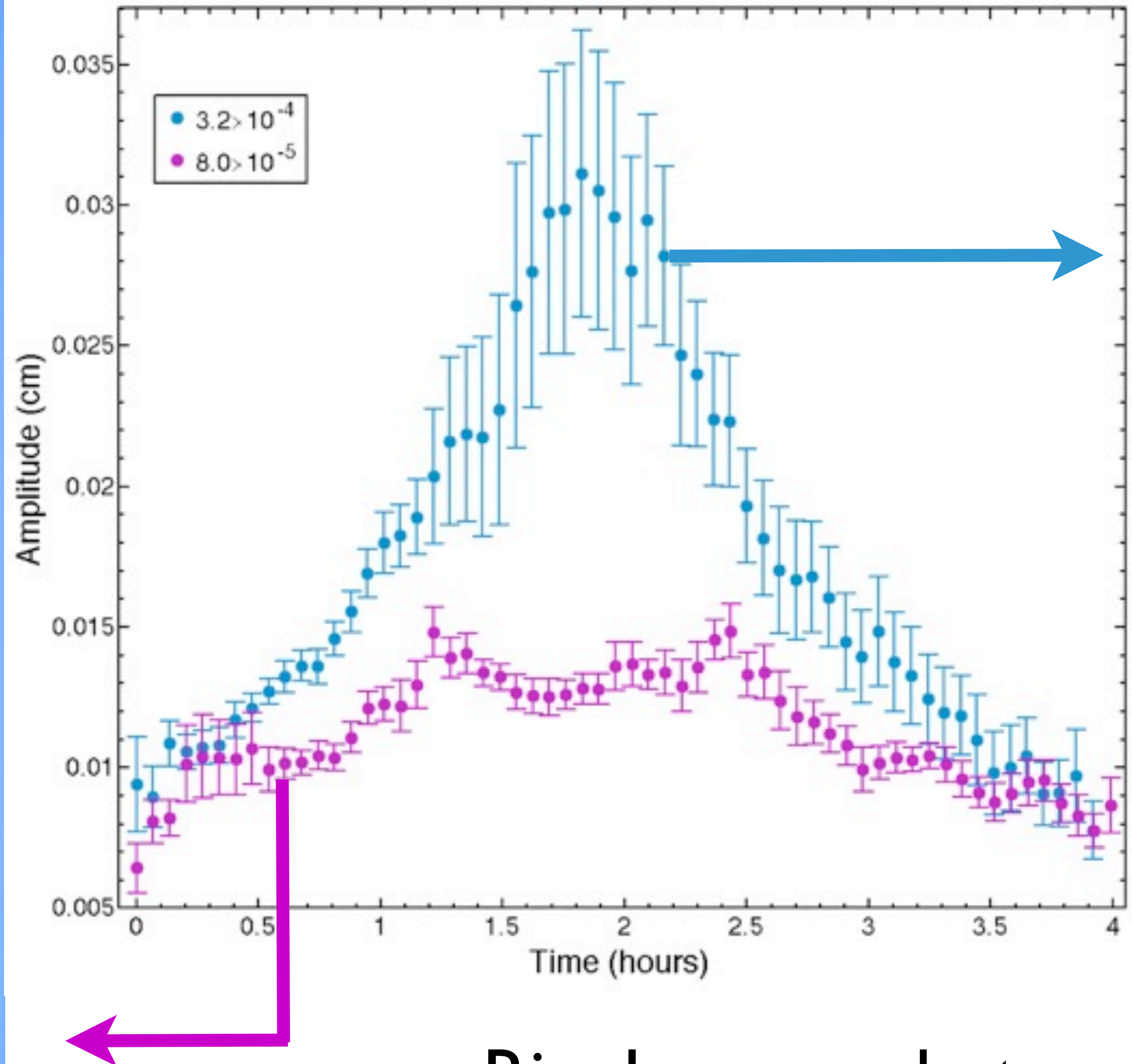
Traveling speed *changes sign* with salt concentration.

Positive is *up* the icicle, negative is *down*.

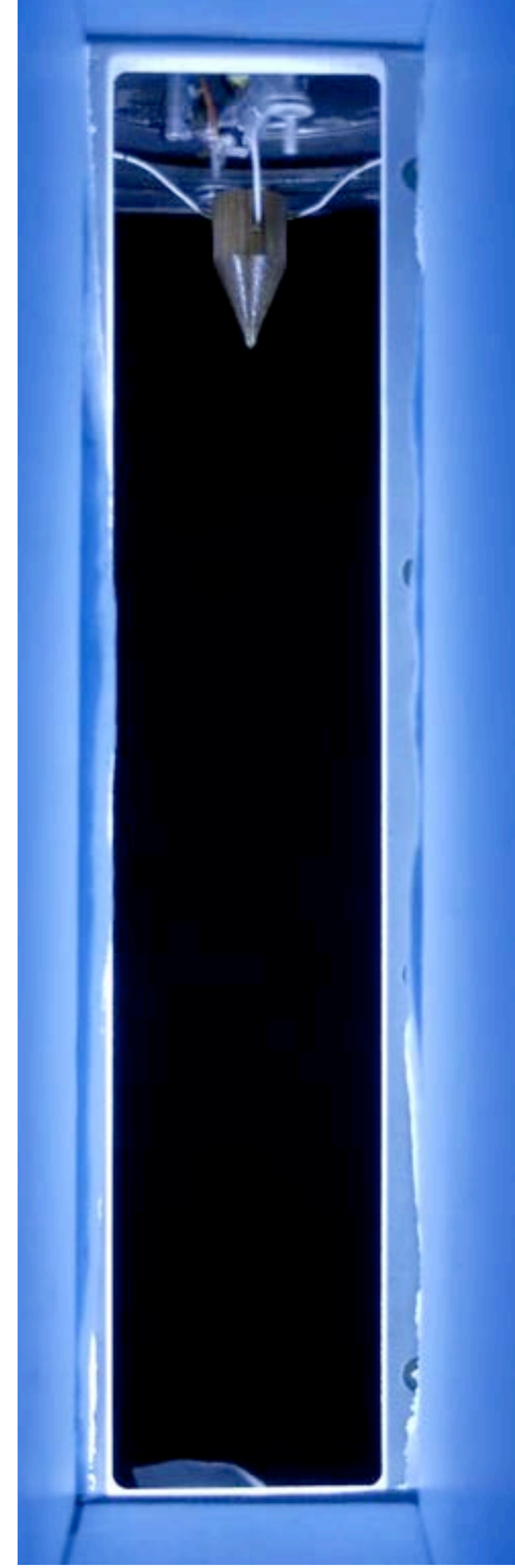




ripple evolution on cessating icicles



Ripples grow, but later smooth out



composition of natural icicles

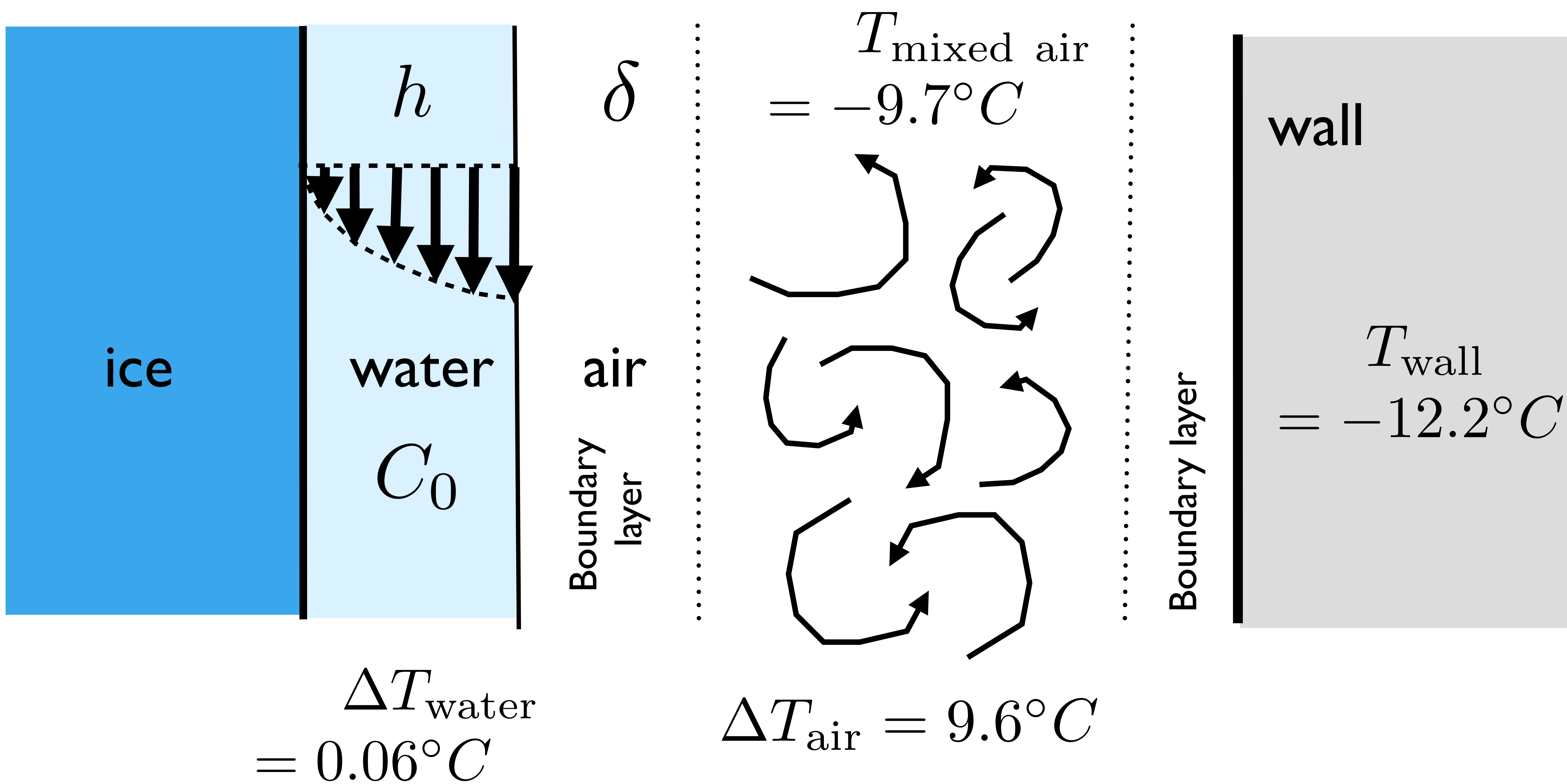
We collected about a dozen "wild type"natural icicles and measured their compositions. They are consistent with ripply laboratory icicles, perhaps somewhat cleaner.

	Conductivity
This icicle:	$19\mu S/cm$
Typical lab salty icicle:	$200\mu S/cm$
Ripple threshold:	$7\mu S/cm$
Distilled water:	$2\mu S/cm$

Toward a new theory of the ripple mechanism

Most of the temperature drop driving freezing happens in the air outside the icicle. The temperature drop across the thin water film is comparable to the freezing point depression for even a tiny amount of salt.

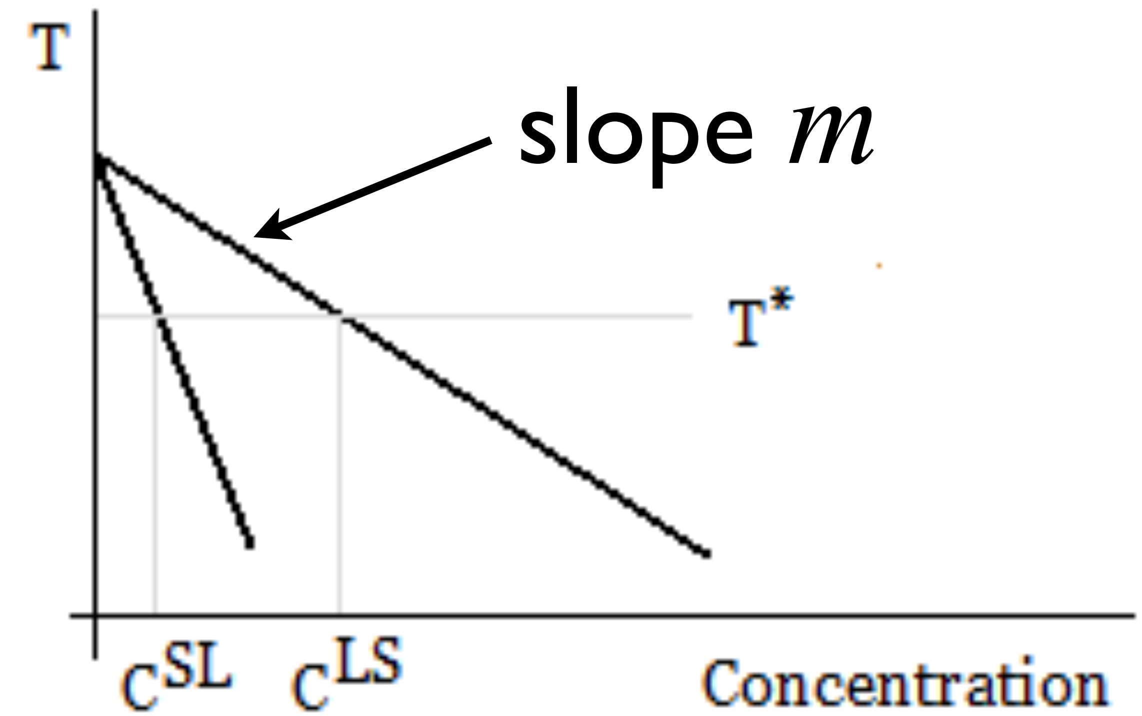
Compositional freezing point depression is probably crucial.

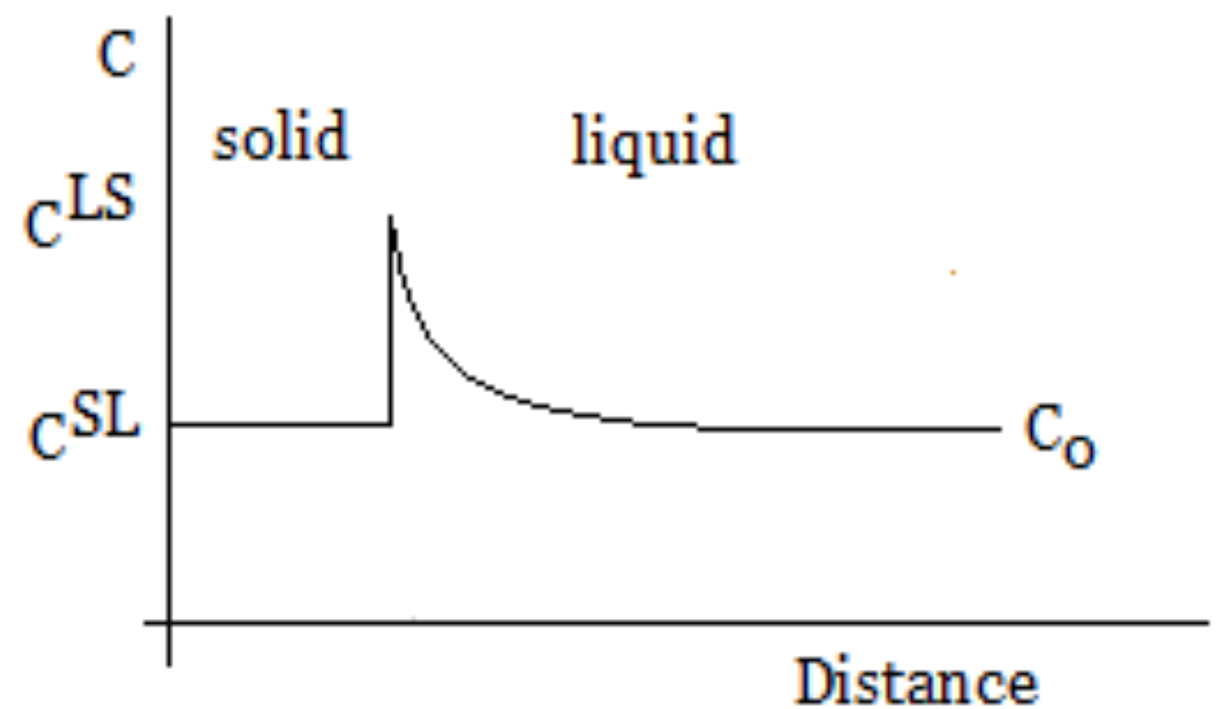


The water is all supercooled by a small amount

Compositional *vs.* kinetic freezing point depression

Freezing point decreases with salt concentration





Growth of ice excludes salty impurities, which build up ahead of the freezing front and depress the freezing temperature.

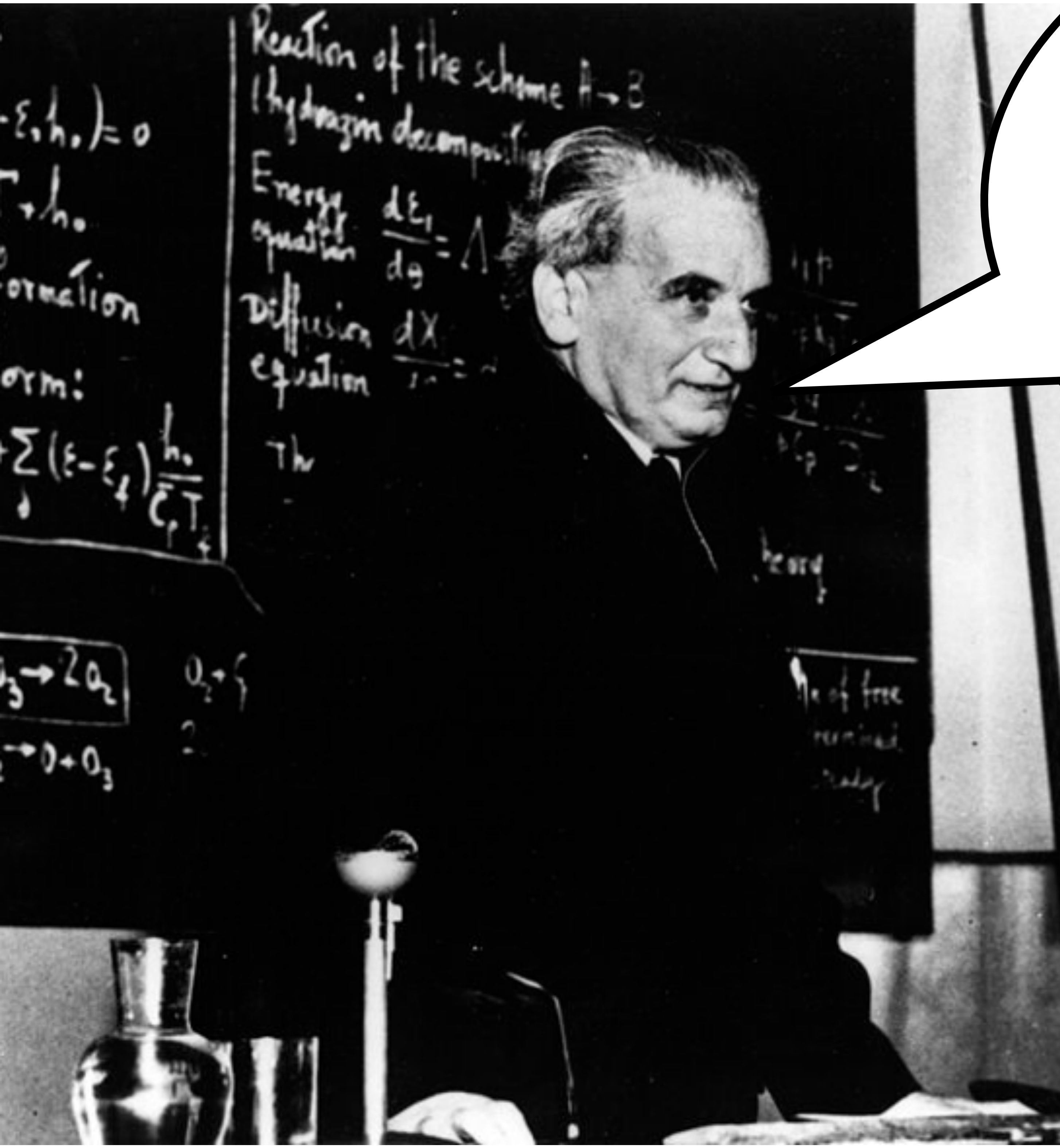
Growth can also be limited by kinetic effects which may further depress the freezing temperature.

Proposed boundary condition:

$$T_I = T_m - mC - KV$$

$$M = mC_0/\Delta T_{\text{water}} = 1.3$$

Boundary conditions



“There are only 3 problems in fluid mechanics:
Boundary conditions,
boundary conditions
and boundary conditions!”

Boundary conditions

4 fluid-dynamical boundary conditions +
2 conditions on the salt concentration, 1 linked to growth rate +
1 Stéfan condition linking growth rate to temperature gradient +
1 condition on the free surface temperature and its gradient +
1 condition linking salt concentration and ice surface temperature.
Total: 9

But only two are really difficult: the ice surface temperature

$$T_I = T_m - mC - KV$$

and the temperature continuity and temperature gradient condition at the free surface, which links

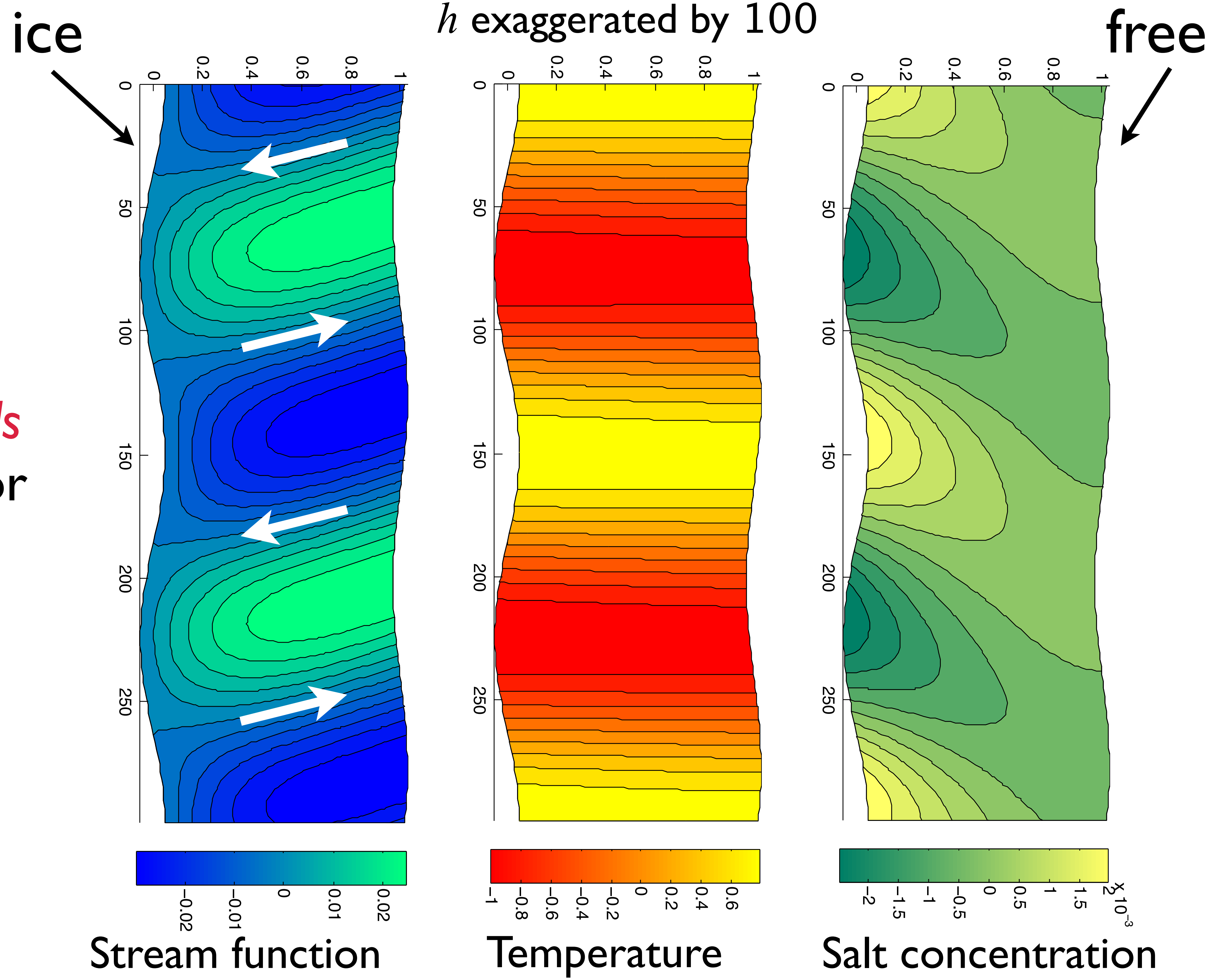
T and ∇T and involves modelling the air flow etc.

Linear stability theory

Attempt to account for both heat and salt and kinetic effects

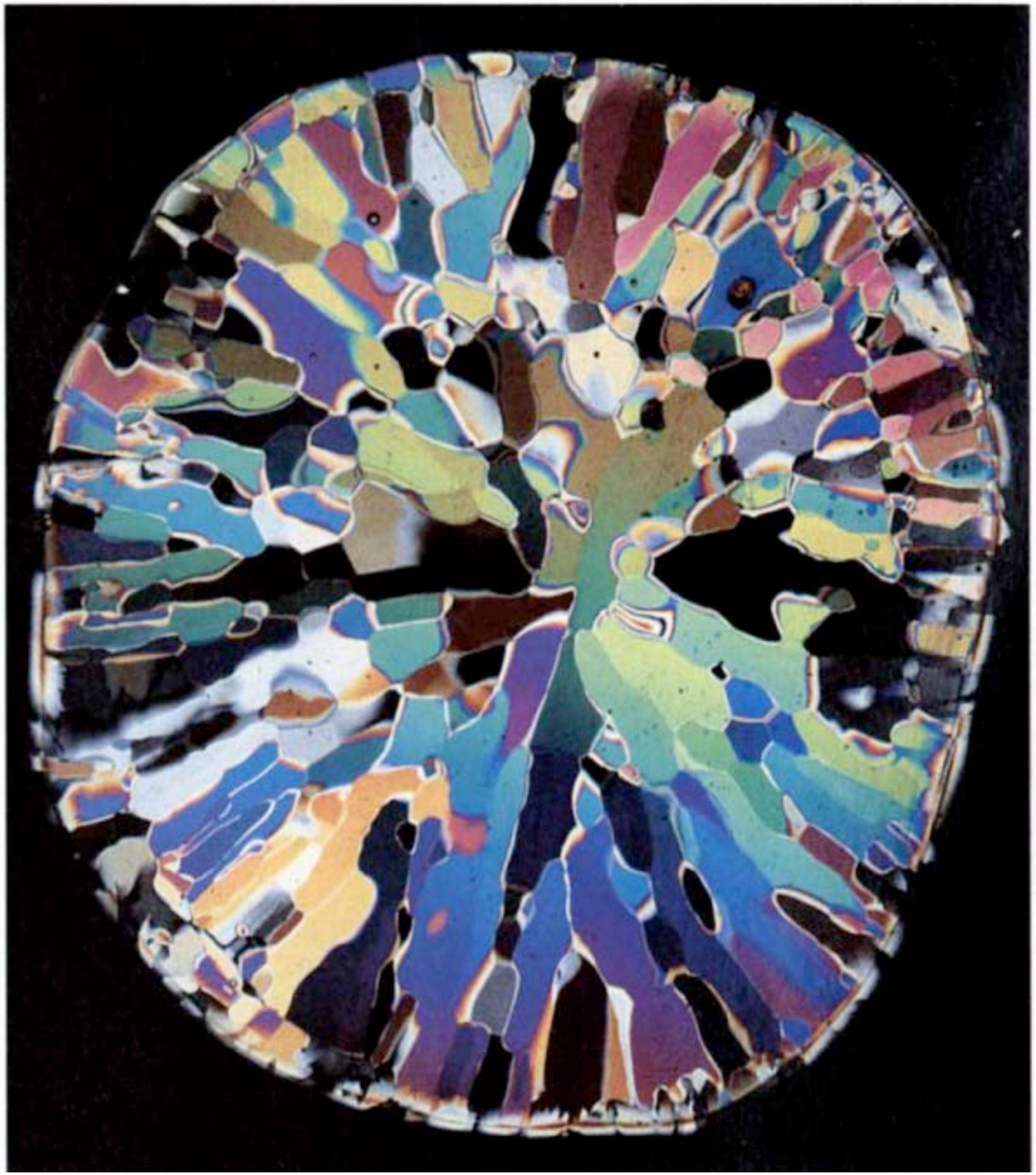
h exaggerated by 100

So far the theory *completely fails* to account for the effect of salt.



Crystal structure of icicles

Ignored by all theories so far



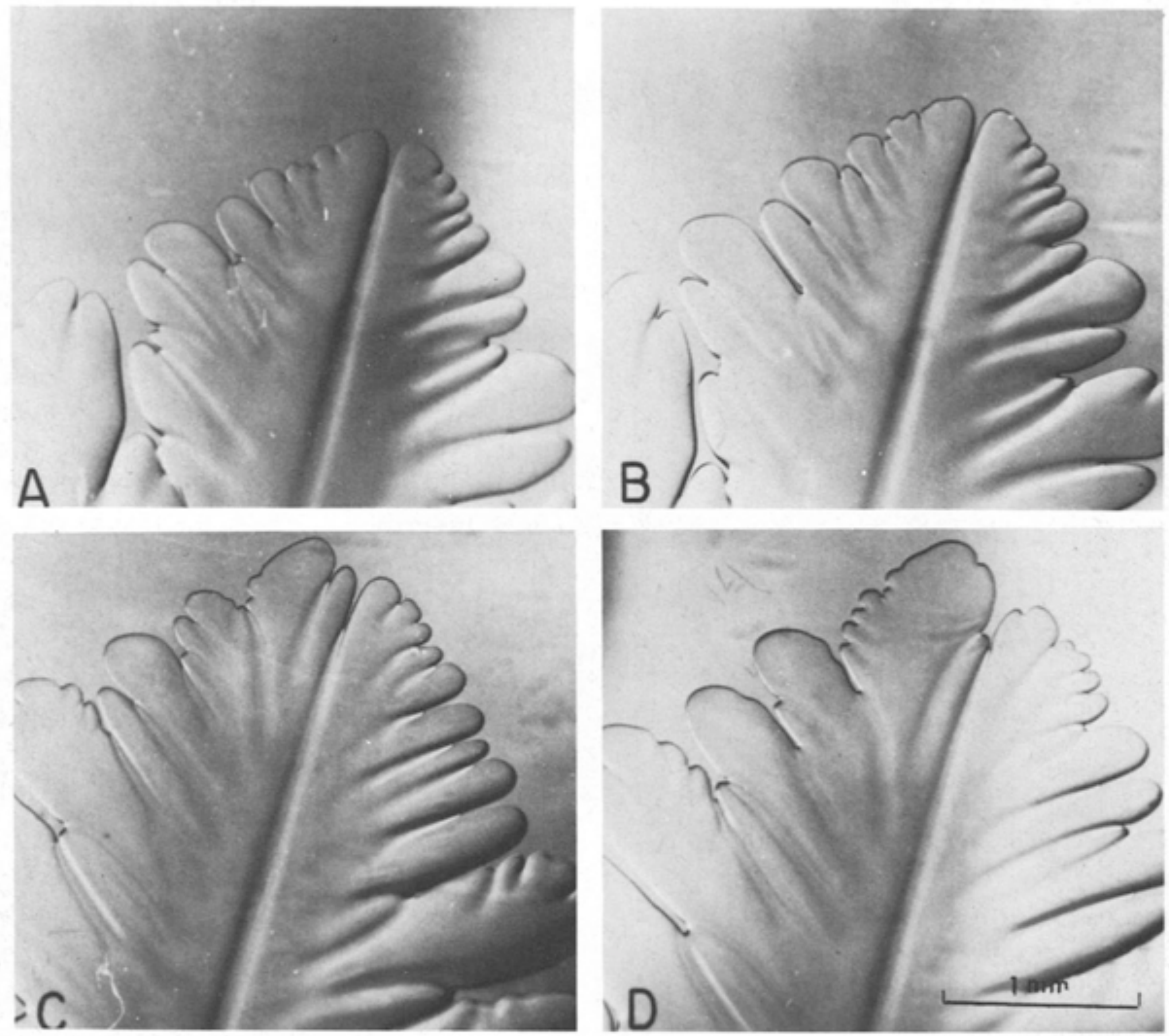
Jearl Walker, *The Amateur Scientist, Scientific American,* May 1988.

Linear stability theory

Attempt to account for both heat and salt and kinetic effects

No short wavelength cutoff at the ice-water surface.

The ice-water interface is not simple: *spongy* or *mushy* ice grows there ... via Mullins-Sekerka instability.



S. H. Tirmizi and W. N. Gill, J. Crystal Growth, 85, 488 (1987).

Conclusions:

- Pure water icicles can be close to the platonic theory shape
- Icicles can stop growing and otherwise deviate from platonic
- Ripples are due to ionic impurities and *not* to surface tension
- We don't have a good theory for much of this

The end