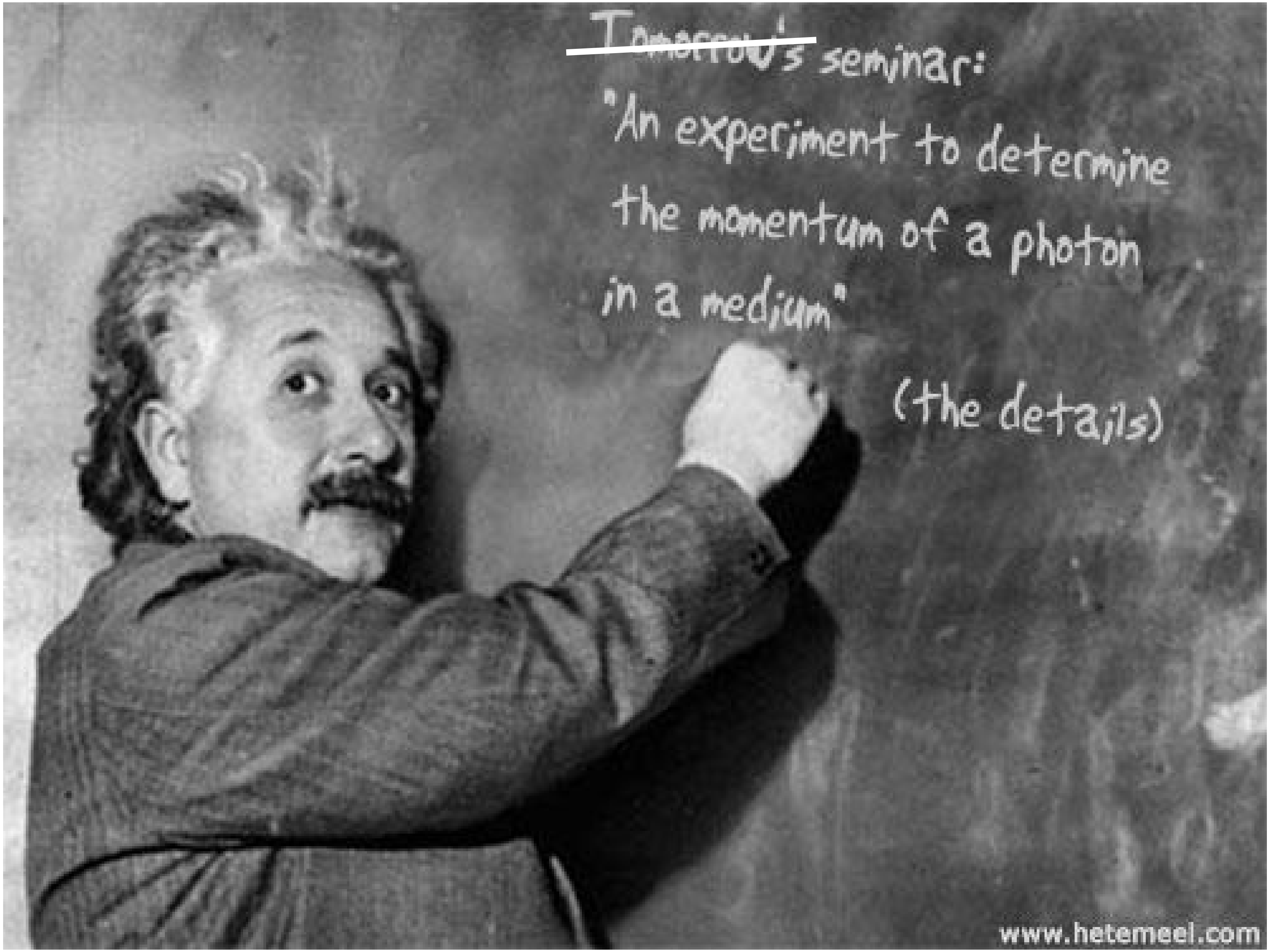

Tomorrow's seminar:
"An experiment to determine the momentum of a photon in a medium"
(the details)
www.hetemeel.com

The photon and its momentum

Paul Lett
National Institute of Standards and Technology
Gaithersburg, MD, USA

In this, the "World Year of Physics" and the 100th anniversary of Einstein's "miracle year" we take a look at the photon...

...and its momentum...

(and a few problems we are still having with it!)

to be fair, it’s not a big problem...

first, it’s not really the grand “WE” that admit to having problems...

second, it’s more of a conceptual, rather than a practical, problem...

and it’s a small one, so that testing for it is hard...

Q: What is the momentum of a photon in a medium?

An experiment to determine the "correct" energy-momentum tensor (?)

Paul Lett - NIST

Lijun Wang - U. Erlangen

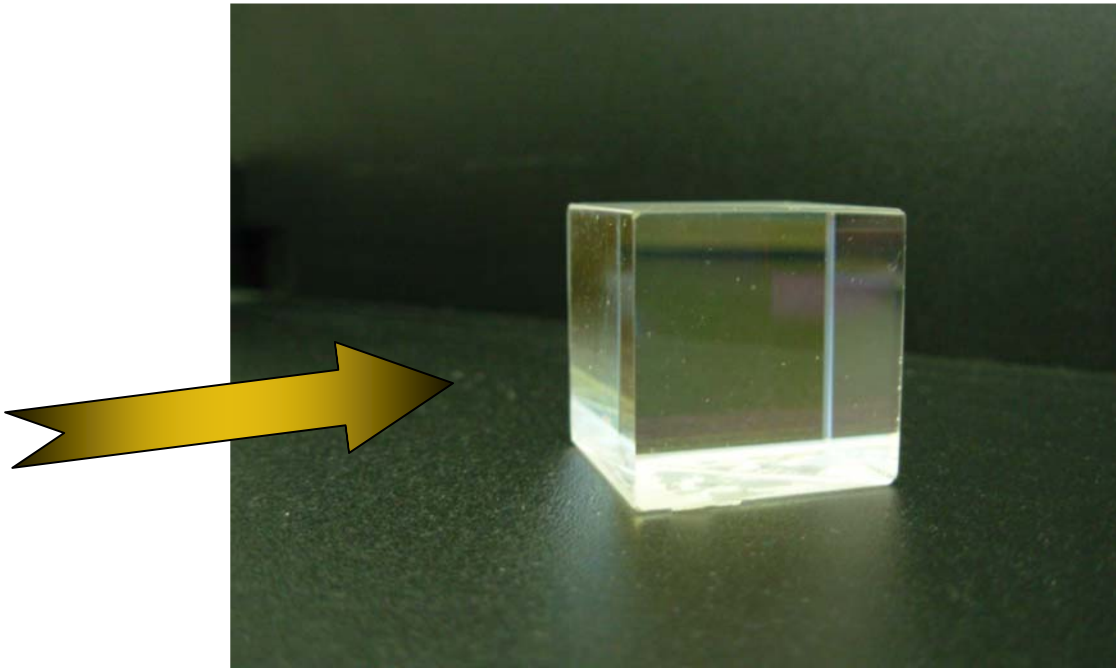
Kevin Jones - Williams College

with input from others at NIST,

U. Maryland, Oxford...

Funding from ONR, NIST

Consider a block of glass on a frictionless surface



and a pulse of light passing through... (no reflections)

...what happens to the block of glass?

Back to the stress-energy tensor... I'll bet you didn't even know that there was a controversy about this!

"tons" of "obscure" literature exists,

commented on by the likes of: Minkowski (1908);
Einstein and Laub (1908); Abraham (1909); Laue (1911);
Pauli (1958); Penfield and Haus (1967);
Peierls (1970's); Ginzburg (1970's);
Nelson (1991); Loudon, Allen and Nelson (1997)

multiple papers in 2004 & 2005 as well...

the “controversy”

Expressions have been proposed for the macroscopic energy-momentum tensor of a polarized medium in the presence of electromagnetic fields.

Splitting this tensor into “field” and “material” parts leads to much of the controversy on the “correct form” of the field part of the energy-momentum tensor.

The question is not well-posed unless the material bit is dealt with as well. The claim is made that only the sum of the two terms is physical, but certainly only a physical observable like the force on an object is.

Perhaps the force terms in the average over an oscillating optical field that are small, but not zero, can actually resolve this...

- summary of work following de Groot and Suttorp (1972)

early work

The discussion of the energy-momentum tensor (often just a discussion of the field part, neglecting the matter part of the tensor) goes back to the non-relativistic work of Lorentz.

H. A. Lorentz (Enc. Math. Wiss. V 2, fasc. 1 (Teubner, Leipzig 1904) 200) considered the forces on a polarized medium at rest.

A. Einstein and J. Laub (Ann. Physik. **26**, 541 (1908)) were the first to give a relativistic form for the force density in a polarized medium; they use the same electric dipole terms as Lorentz and postulated analogous magnetic terms.

They considered the force density in a material at rest and did not consider the material part of the force density at all.

H. Minkowski (1908)

Minkowski (H. Minkowski, Nachr. Ges. Wiss. Gottingen, 53 (1908); Math. Ann. **68,** 472 (1910)) suggested an expression for the field energy-momentum tensor on the formal grounds that it should be form-invariant in all Lorentz frames.

This implies that the tensor should depend on the fields, but not on the 4-velocity of the medium with respect to the observer.

The material part of the tensor was not considered.

note: covariance does not necessarily imply form-invariance.

M. Abraham (1909)

Abraham gave up form-invariance but assumed that the field tensor is symmetric in all Lorentz frames, even for anisotropic media. (M. Abraham, R. C. Circ. Mat. Palermo **28, 1** (1909); **30,** 33(1910))

Abraham's field tensor contains a field momentum density which is proportional to the field energy flow.

It thus obeys the "law of inertia of energy" (M. Planck) and is used as an argument for the Abraham form (Balazs, ...).

Others argue that this should only apply to a closed system, and thus is only applicable to the total energy-momentum tensor, and not just the field tensor.

further arguments

Von Laue argued in favor of Minkowski's form (Z. Phys. **128**, 387 (1950)) following A. Scheye, Ann. Phys. **30**, 805 (1909)) that the energy transport velocity (energy flow divided by energy density) should transform such that the addition rule for 4-velocities should hold.

I.e., a light wave in a moving body should transform like a particle velocity under Lorentz transformations.

Minkowski's form is not the only form of the field tensor that satisfies this criterion, and when you add the material part of the tensor the total tensor does not have this property.

and more arguments

Abraham’s tensor gives a positive energy density for all macroscopic velocities, consequently it always results in a positive photon energy when the field is quantized.

Minkowski’s form does not; it sometimes leads to negative-energy photons. (i.e., in the case of Čerenkov radiation)

These arguments again only deal with the “field” portion of the tensor and not the total energy-momentum tensor.

(Artifically dividing the tensor into “field” and “matter” components does not actually make these subsystems separable...)

The choice of stress-energy tensor

(or, here, the resultant force on a dielectric medium)

Minkowski (1908)

$$f^M = -\tfrac{1}{2}E^2\nabla\varepsilon - \tfrac{1}{2}H^2\nabla\mu$$

Abraham (1909)

$$f^A = f^M + \frac{\varepsilon\mu - 1}{c^2}\frac{\partial}{\partial t}(\vec{E}\times\vec{H})$$

plus other choices given by a bunch of other authors,

including Einstein & Laub, Peierls, Nelson,...

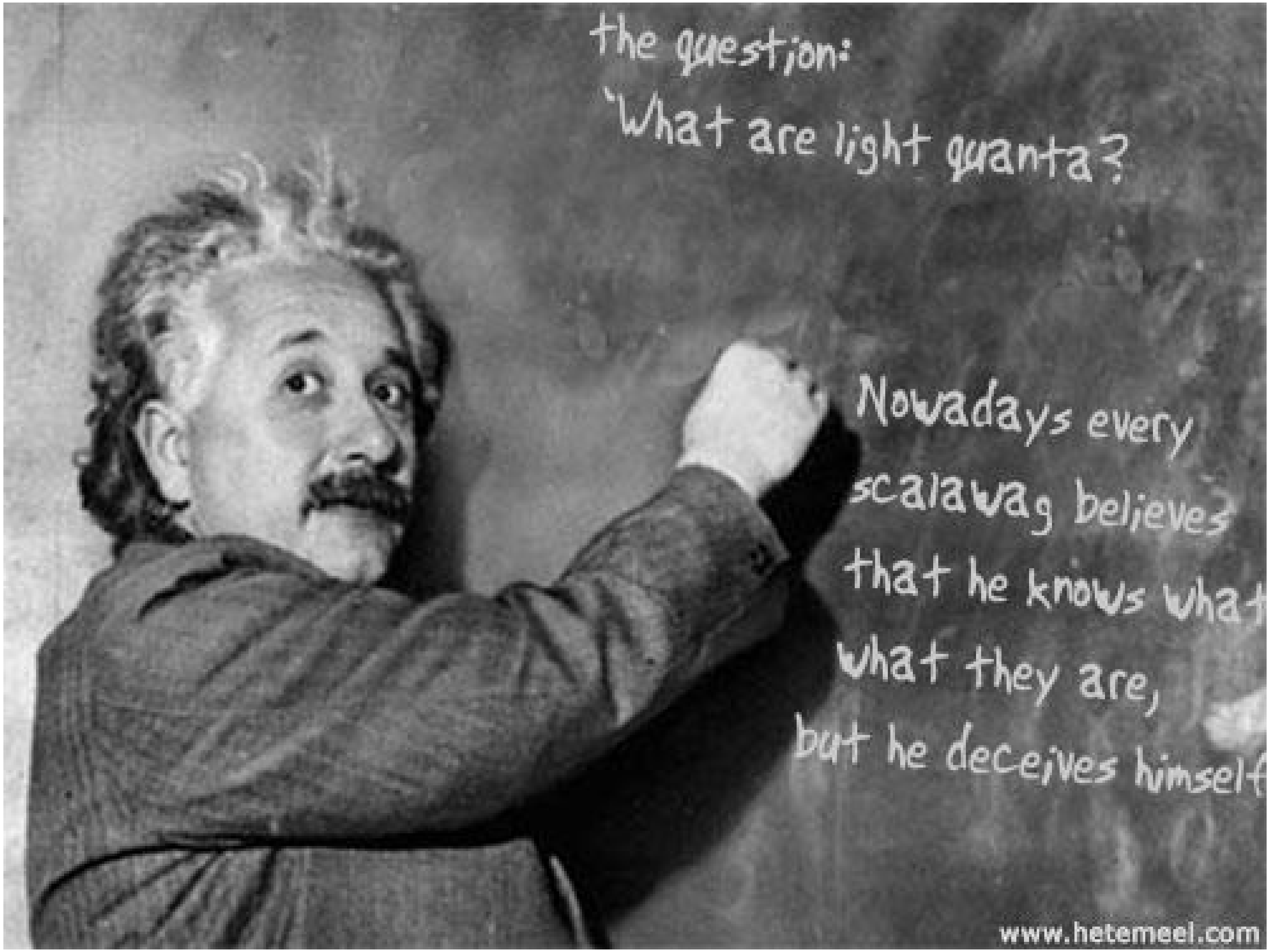
the "Abraham force"

$$f^{A} = f^{M} + \frac{\varepsilon\mu - 1}{c^{2}} \frac{\partial}{\partial t}(\vec{E} \times \vec{H})$$

This term is known as the "Abraham force."

It is generally small and has only been observed under "quasi-static" field conditions.

Note that: $\vec{S} = \vec{E} \times \vec{H}$ is the Poynting vector.

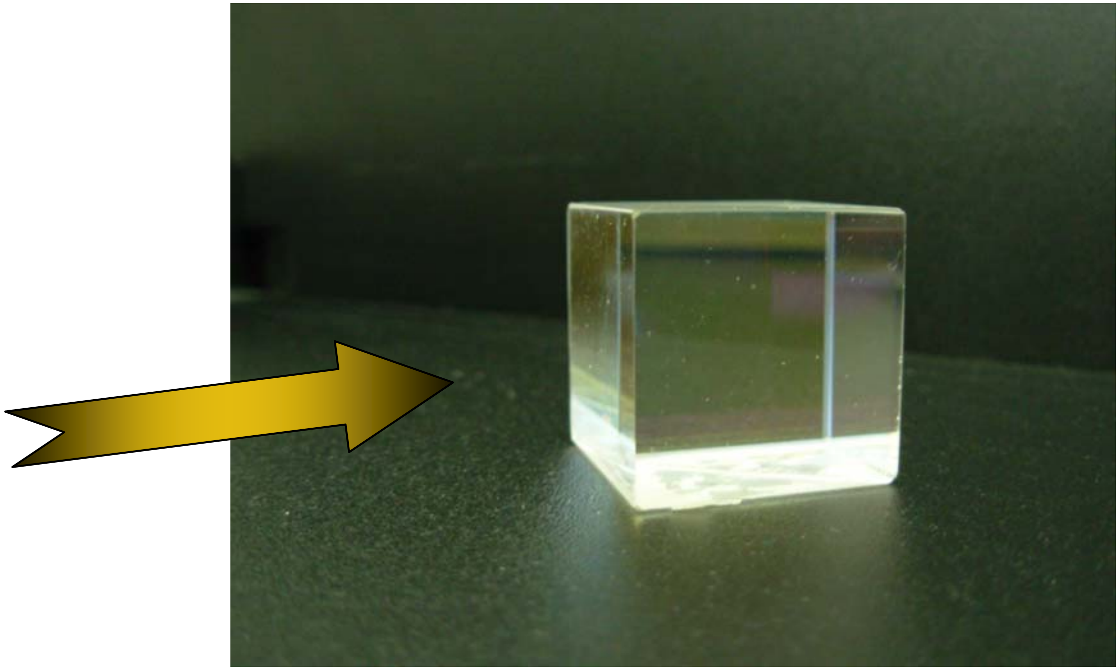

the question:
'What are light quanta?
Nowadays every
scalawag believes
that he knows what
what they are,
but he deceives himself
www.hetemeel.com

what is the momentum of a photon in a medium?

it’s a little tricky…

…so let me mislead you through it…

Consider a block of glass on a frictionless surface



and a pulse of light passing through... (no reflections)

What happens to the block of glass?

Or, what is the momentum of a photon in the medium?

What happens to the momentum of the light as it passes into a piece of glass?

you have 3 choices:

1. **it gets larger**
2. **it gets smaller**
3. **it stays the same**

Note that options 1 and 2 necessarily imply that we give a kick to the piece of glass when the photon enters!

A fairly straightforward question. You'd think it would have been answered by now!

In a slightly different and more complicated form, a good number of famous physicists have weighed-in on the topic.

The two main camps are those of Abraham and Minkowski.

Einstein (with Jakob Laub) weighed-in on this too.

Some related experiments (not with a block of glass) were performed here at the University of Toronto...

what is the momentum of a photon in a medium of index n?

naive approach #1:

note that the frequency of oscillation, ν, is a constant

vacuum:

$$\lambda = \frac{c}{\nu}$$

$$p = \hbar k = \frac{h}{\lambda} = \frac{h\nu}{c}$$

medium:

$$\lambda = \frac{c}{n\nu}$$

$$p = \hbar k = \frac{nh}{\lambda} = \frac{nh\nu}{c}$$

what is the momentum of a photon in a medium of index n?

naive approach #2:

vacuum:

$$p = m\text{v}$$

$$m = \frac{E}{c^2}$$

$$p = \frac{E}{c^2}c = \frac{h\nu}{c}$$

medium:

$$p = m\text{v}$$

$$\text{v} = \frac{c}{n}$$

$$p = \frac{E}{c^2}\frac{c}{n} = \frac{h\nu}{nc}$$

So, which is it?

$$p = \hbar k = \frac{nh}{\lambda} = \frac{nh\nu}{c}$$

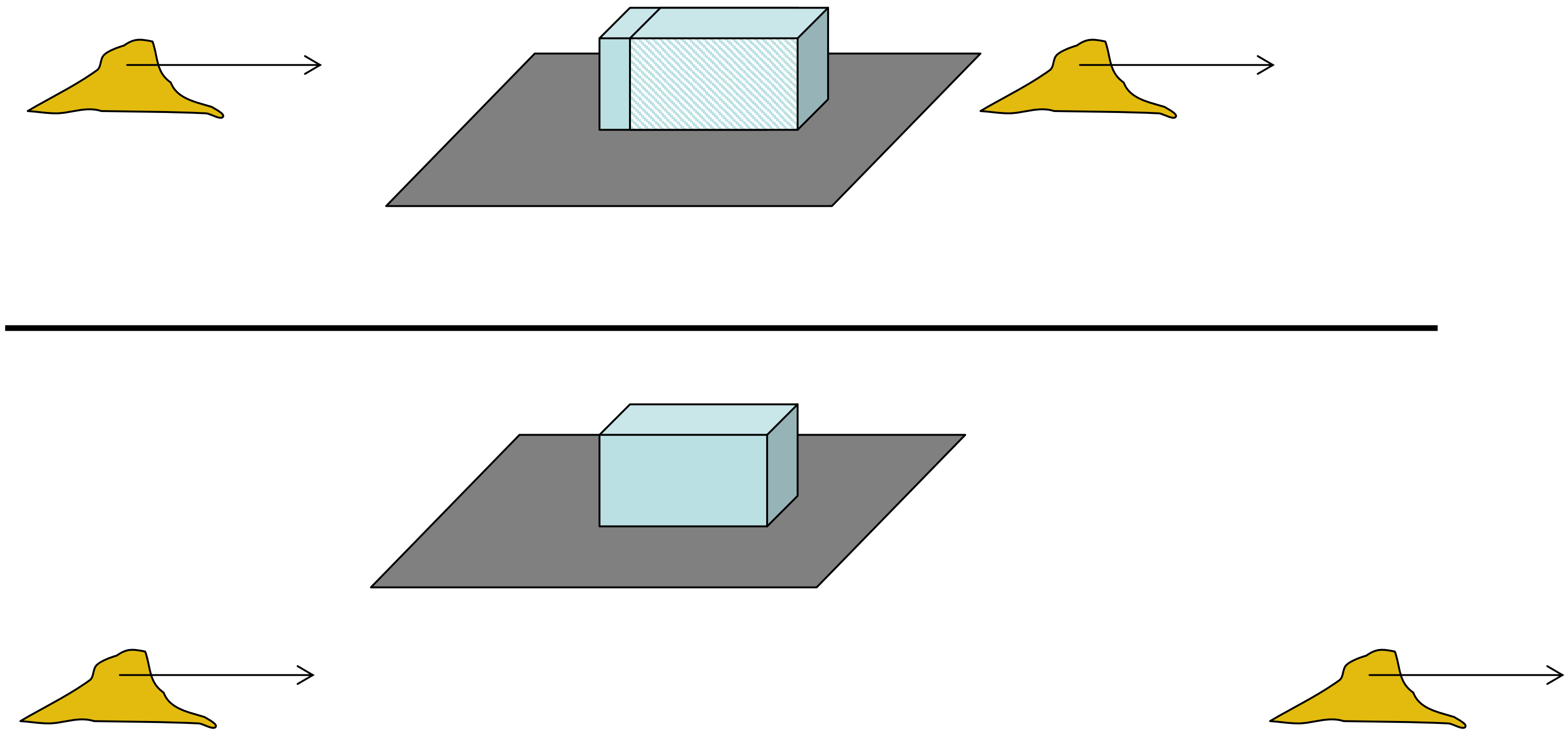
or

$$p = mv = \frac{E}{c^2}\frac{c}{n} = \frac{h\nu}{nc}$$ **?**

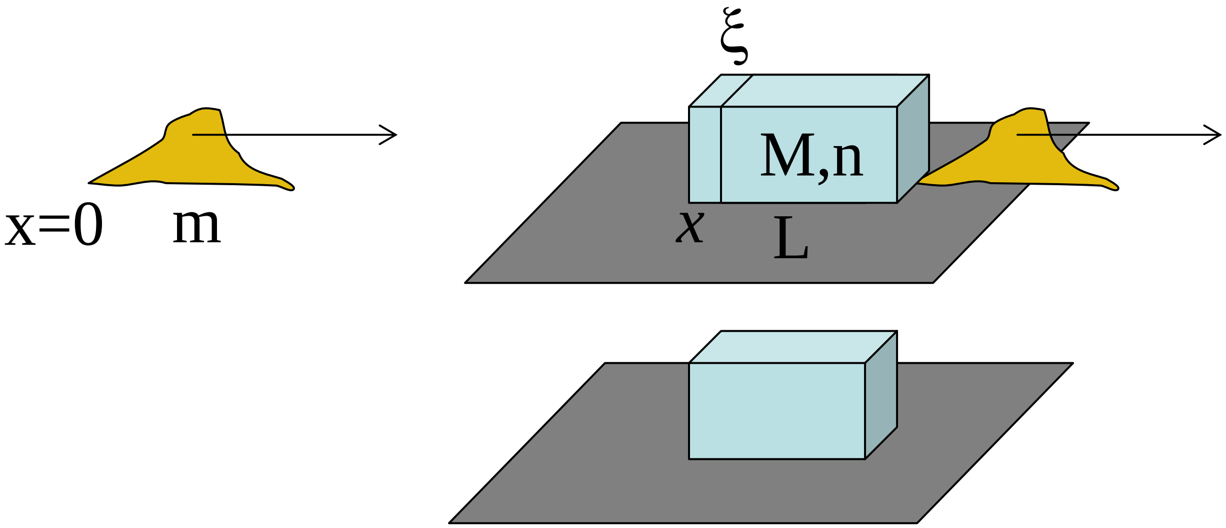
Minkowski **or** **Abraham ?**

a gedanken experiment

Balazs, 1953



Consider the center of mass motion



$$t = \frac{x}{c} + \frac{L+\xi}{c/n}$$

$$mct = m(x+\xi+L)+M\xi$$

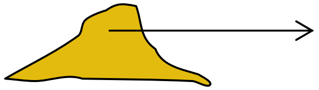
$$m(x+n(L+\xi)) = m(x+\xi+L)+M\xi$$

$$m(n-1)(L+\xi) = M\xi$$

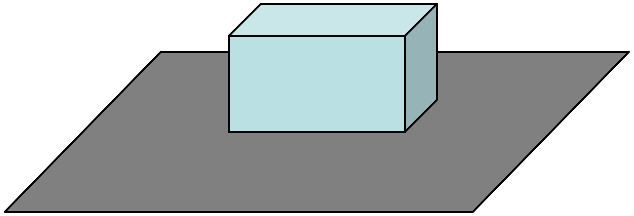
$$m(n-1)L = \xi[M-m(n-1)]$$

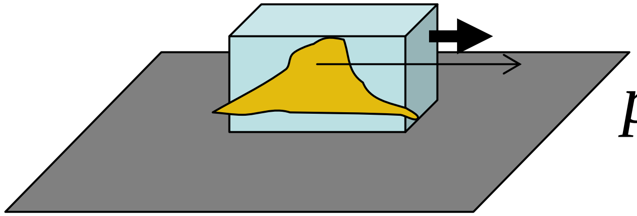
$$\xi = \frac{m(n-1)L}{M-m(n-1)}$$

Consider conservation of momentum



$$p = \frac{h\nu}{c}$$





$$p' = \frac{h\nu}{nc}$$

(Abraham's answer)

If the block is going to move "forward," and momentum is going to be conserved while the pulse is in the medium,...

Then the photons must have less momentum in the medium than in vacuum.

interpretation...

$$\xi = \frac{m(n-1)L}{M - m(n-1)}$$

m(n-1) acts as the “effective mass” of the light pulse

this mass can become comparable to the mass M, of the “slab” if the slab is as light as possible; for instance, a collection of about 10^6 atoms in a BEC

if M = m(n-1) then the displacement becomes infinite!
(this ignores effects of the velocity of the medium on the light propagation)

if $n < 0$ you get “faster than light” propagation
(or if $M < m(n-1)$) and the displacement changes direction!

Abraham vs. Minkowski on the topic of the displacement of the block

Look at the time-averaged forces due to an optical field.

Abraham gives the expression from above:

$$\xi^A = \frac{m(n-1)L}{M}$$

approximated for M >> m(n-1)

Minkowski gives an expression with the opposite sign!

$$\xi^M = \frac{m(1-n)nL}{M} = -n\xi^A$$

note, however, that interpretation is straightforward only if n =n_{phase}!
(explicitly not what I want!)

A real experiment that will not answer this question.
(… but might answer a related one!)

You need a very light block of glass, with a very high index of refraction, on a completely frictionless surface… not so easy…

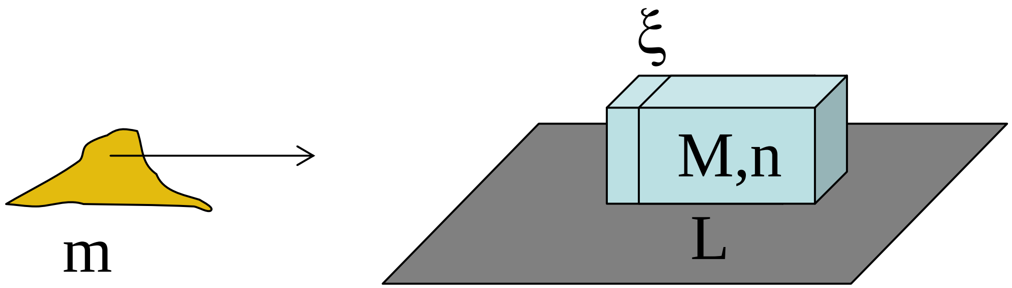
A BEC can, under some circumstances, behave like a block of glass (sort of), can be given a huge index of refraction (sort of), and can sit on a frictionless surface (no, really - that part is easy!).

The proposed experiment

Pulse:

3 x 10^6 photons/1 μs
= 1 μW at 589 nm
1 ms duration
$m = E/c^2 = 10^{-26}$ kg

"Block of glass" = BEC
3×10^6 sodium atoms
$M = 1.2 \times 10^{-19}$ kg
L = 100 μm



$$\xi = \frac{m(n-1)L}{M - m(n-1)}$$

index n

$n_g = d\omega/dk$
EIT / "slow light"
$n = 10^6$ c/n = 300 m/s
transit time* = 100μm/300m/s = 0.3μs

"effective mass" $m(n-1) = 10^{-20}$ kg

ξ ~ 0.1 L = 10 μm

Einstein and Laub

Well, ok, this is something of a stretch to get Einstein in…

Less than two weeks after Minkowski's theory of the electrodynamics of moving media appeared in print (in 1908), Einstein wrote to his wife with great news: on the basis of Jakob Laub's calculations, he had found an error concerning the definition of ponderomotive force density. Together, Einstein and Laub came up with an alternative definition.

Einstein devised arguments in defense of their formula in 1910, but lost interest in it some time later. In a letter to Walter Dallenbach in 1918, Einstein candidly remarked that it had been known for some time that the expression he and Laub devised was false.

"The mathematical education of the young physicist [Albert Einstein] was not very solid, which I am in a good position to evaluate since he obtained it from me in Zurich some time ago." -H. Minkowski

Einstein-Laub (cont.)

Not really considered a viable option any longer, but nonetheless, their formula is still discussed…

Observation of Electromagnetic Angular Momentum within Magnetite

D. G. Lahoz and G. M. Graham
Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada
(Received 27 November 1978)

A hollow cylinder of magnetite at 4 K has been subjected to the simultaneous action of constant axial magnetic field $\vec{B}$ and low-frequency radial electric field $\vec{E}$. The reaction torque, which appears in the magnetite as the electromagnetic angular momentum changes, has been observed for the first time. Preliminary results favor the Livens' reaction-force density $-\partial(\epsilon_0\vec{E}\times\vec{B})/\partial t$ and contradict the alternative $-\partial(\epsilon_0\mu_0\vec{E}\times\vec{H})/\partial t$ of Einstein and Laub which was followed by Abraham and is widely accepted by modern authors.

Phys. Rev. Lett. 42, 1137 (1979)

it’s a Canadian problem…

(or at least it gets worked on a lot in Canada)

The Abraham force: comments on two recent experiments

PAUL LORRAIN

Département de Physique, Université de Montréal, Montréal (Qué.), Canada H3C 3J7

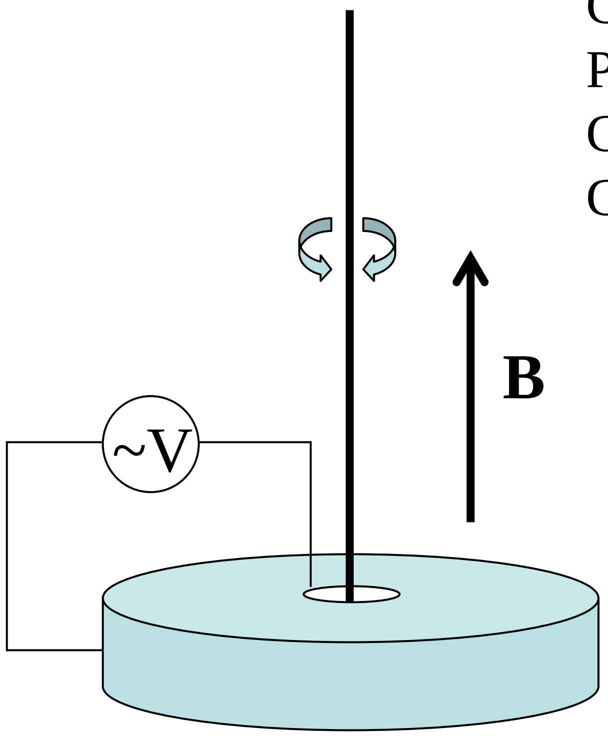
Received September 27, 1979

The Abraham force is discussed briefly. It is recalled that the old controversy concerning the Minkowski and Abraham tensors for an electromagnetic field in matter, as well as efforts at observing the Abraham force, are pointless because it is now well established that both tensors are equally satisfactory, as long as the various subsystems are treated correctly. If one ascribes to the electromagnetic field in matter a momentum density of $\boldsymbol{E} \times \boldsymbol{H}/c^2$, like Abraham, then Lahoz and Graham have observed the magnetic force density on a polarization current, or only one of the six components of the Abraham force. Their interpretation of their experiments is erroneous.

Can. J. Phys. **58**, 683 (1980)

torsion balance experiments

G. B. Walker, D. G. Lahoz, and G. Walker, Can,. J. Phys. **53**, 2577 (1975);
G. B. Walker and D.G. Lahoz, Nature **253**, 339 (1975);
G. B. Walker and G. Walker, Nature **263**, 401 (1976);
265, 324 (1977).



barium titanate disk
$\varepsilon \sim 3620 \quad \mu \sim 1$

non-propagating field:

static magnetic field and
low-frequency oscillating electric field
(0.4 Hz)

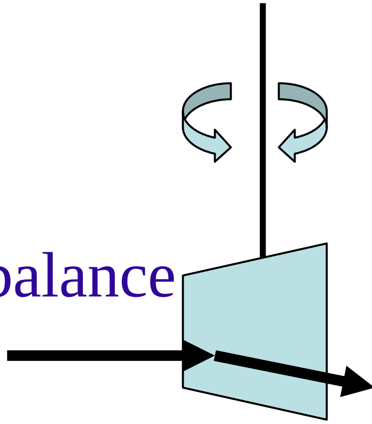
verified existence of "Abraham force" term to 5%, or approximately 20 σ

R. P. James,
Proc. Natl. Acad. Sci **61**, 1149 (1968)

ferrites, $\varepsilon_r\mu_r \sim 100$, different geometry
~10 kHz piezoelectric resonance

The Jones-Richards and Jones-Leslie experiments

R. Jones, and J. Richards, Proc. Roy. Soc. Lond. A 221, 480 (1954),
R. Jones, and B. Leslie, Proc. Roy. Soc. Lond. A 360, 347 (1978)



bounce a light beam off of a mirror on a torsion balance suspended in air or in a liquid dielectric

Result: momentum transfer increases with the index of refraction : $n\hbar k$

Momentum transfer goes as the phase index n_ϕ, not the group index n_g. (Verified to 0.05% and 23 σ.)

(perhaps not that well - see Brevik, 1979)

Unfortunately, these experiments say very little about the problem: Both the Abraham and Minkowski tensors give the same answer (and agree with the experiment).

Jones-Leslie continued...

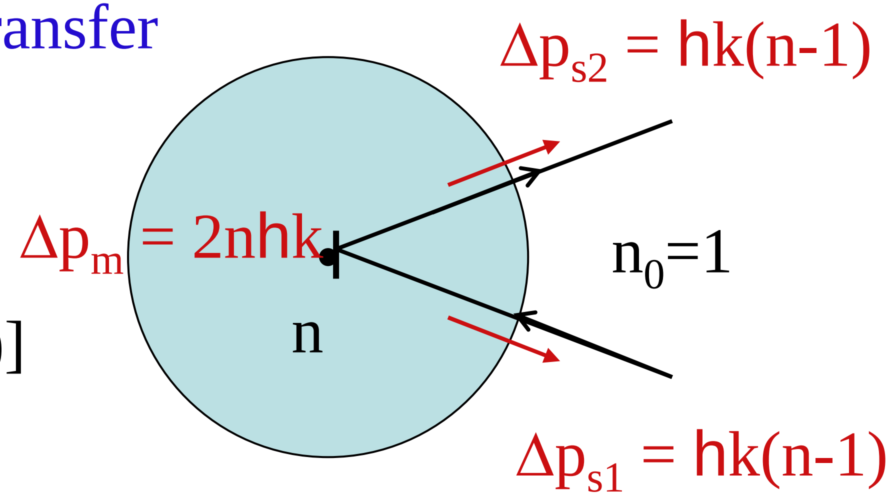
The experiment also addresses the Peierls tensor which predicts a variation with angle of incidence (i.e., plane of polarization); The experiment finds no variation of momentum transfer with angle.

Conclusion: The momentum of a beam increases in direct proportion to the refractive index of the medium into which the beam enters.

How do we understand this???

Look at the total momentum transfer to the system (from above):

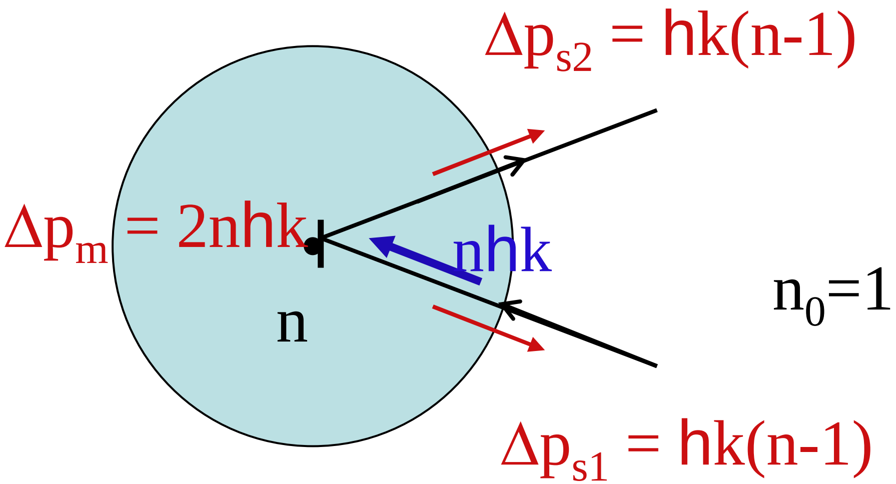
$$\Delta p = \Delta p_{s1} + \Delta p_m + \Delta p_{s2}$$
$$= hk[-(n-1) + 2n - (n-1)]$$
$$= 2hk$$



more on Jones-Leslie...

Minkowski picture

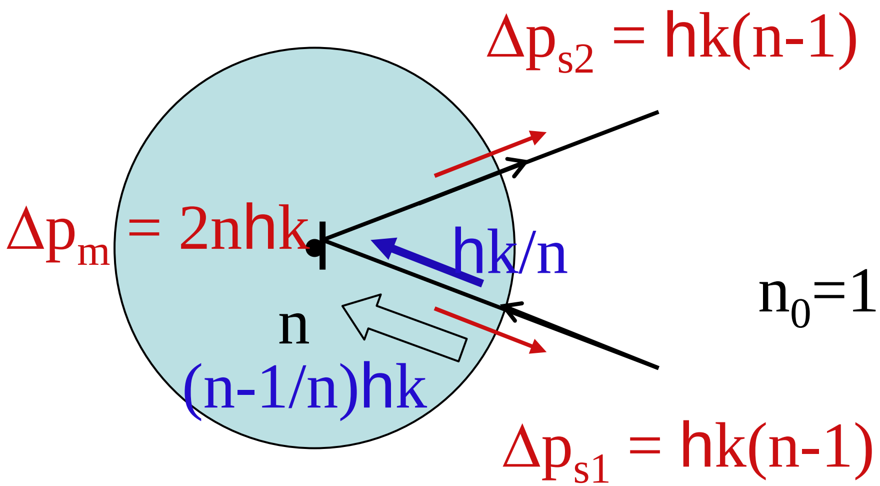
each photon in the medium
has $n\hbar k$ of momentum



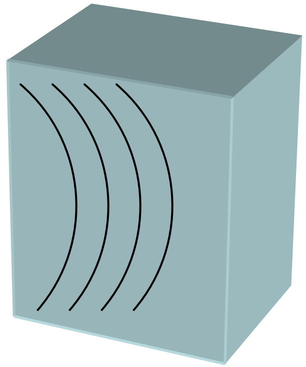
Abraham picture

each photon in the medium
has $\hbar k/n$ of momentum

the medium also carries
$(n-1/n)\hbar k$ of momentum



analogy with sound waves



an acoustic wave propagates in a crystal

it carries no mass along with it, thus,
since $p=mv$, it carries no momentum either

$\hbar k$ is the pseudomomentum of a phonon in the medium

sound, being entirely a material-based field, has only pseudomomentum

light has both momentum and pseudomomentum in the medium

note: this does not address the Abraham/Minkowski controversy!

The question,
"What is the momentum recoil of a photon absorber/reflector in a medium?"
while interesting, is not the same question as
"What is the momentum carried by a photon in the medium?"

Both Minkowski and Abraham answer the first question the same, but differ on the second one.

absorber/reflector experiments

A number of different situations give the same result here: the question is essentially,

hk or nhk?   where n is the phase index

- “mirror-in-a-bucket” (Jones-Richards, Jones-Leslie)
- atom recoil in a gas (Haugan-Kowalski, Campbell/Ketterle)
- photon drag experiments (Gibson, et al.)
- atom recoil in an evanescent wave
  (Westbrook/Aspect, Spreeuw/vLvdH)

answer well-established experimentally

What is the momentum of the photon in the medium?
vs.
What recoil does an absorber in the medium receive?

To see what momentum the photon carried you have to let the photon only interact with the medium.

The block-of-glass problem, and variants, seem to be the only sorts of experiments that address the question of the momentum of the photon in the medium (and lead to different answers from Abraham and Minkowski).

The proposed experiment

Pulse:

3×10^6 photons/1 μs

= 1 μW at 589 nm

1 ms duration

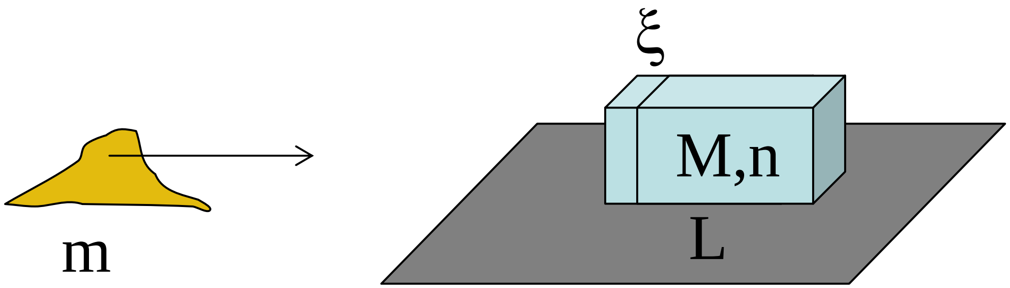
$m = E/c^2 = 10^{-26}$ kg

"Block of glass" = BEC

3×10^6 sodium atoms

$M = 1.2 \times 10^{-19}$ kg

L = 100 μm



$$\xi = \frac{m(n-1)L}{M - m(n-1)}$$

index n

$n_g = d\omega/dk$

EIT / "slow light"

$n = 10^6$ c/n = 300 m/s

transit time* = 100μm/300m/s = 0.3μs

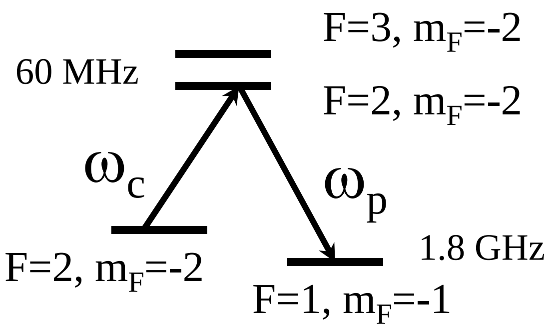
"effective mass" $m(n-1) = 10^{-20}$ kg

ξ ~ 0.1 L = 10 μm

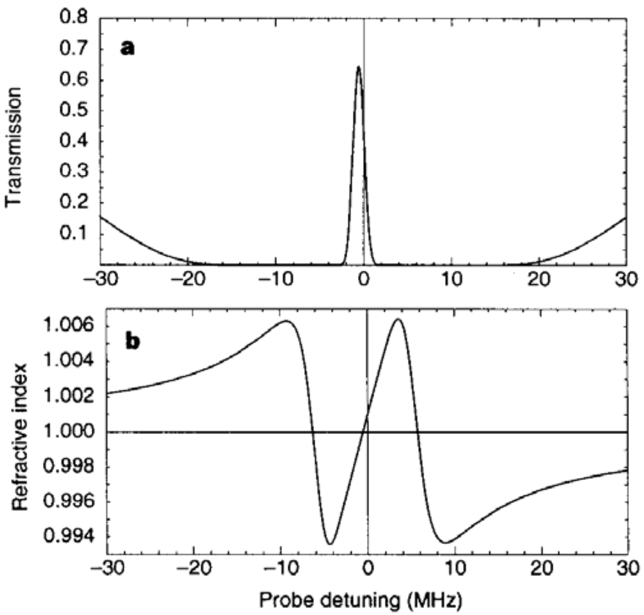
“EIT” and “slow light”

To get the large values of the group velocity index that are necessary, we need to employ "slow light" techniques, or "electromagnetically-induced transparency".

An extra beam "dresses" the atoms in such a way that the pulse of light sees an apparently large dispersion:



plot of transmission and dispersion



17 m/s or $n_g \sim 2\text{x}10^7$

unfortunately, transmission isn't amazing

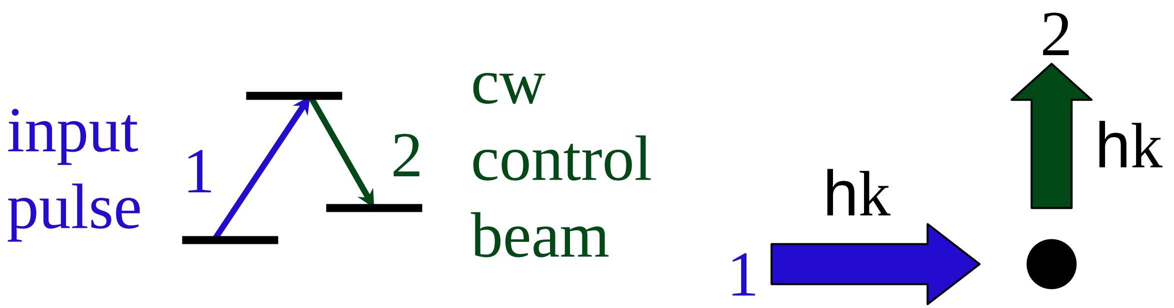
from L. Hau, et al., Nature 397, 594(1999)

a photon-by photon description

we convert photons in vacuum to polaritons in the medium

photons move at velocity **c**

polaritons (coupled atom-field entities) move at $\mathbf{c/n_g}$



The “real” momentum that is associated with an atom (polariton?) depends on the angle between the two Raman beams required for producing the EIT effect.

If the beams are orthogonal then the analysis of momentum conservation along the direction of the pulse is the same as for a “passive” medium like glass.

polaritons, (cont.)

3x10^6 atoms and 1 μW (3x10^6 photons/μs)
with a transit time of 0.3μs
gives us about 10^6 polaritons in steady state (1/3 of the atoms)

For orthogonal beams each polariton carries 1 $\hbar k$ of momentum along the direction of the beam (and 1 $\hbar k$ orthogonal as well).

For sodium the recoil velocity is 3 cm/s and on average 1/3 of the atoms carry this recoil.

A 1 cm/s average velocity
times a 1 ms duration pulse → **10 μm displacement**

just as expected

Abraham vs. Minkowski with a dispersive medium

Abraham gives: $$\xi^A = \frac{m(n_g - 1)L}{M}$$ approximated for M >> m(n-1)

Minkowski still gives an expression with the opposite sign:

$$\xi^M = -\frac{m(n-1)n_g L}{M}$$

note: the <u>Abraham version</u> is independent of the phase index

in the example discussed above,
for n = 1.001, $n_g = 10^6$; $\xi^A = 10\ \mu m$; $\xi^M = -0.01\ \mu m$

spontaneous light scattering

residual absorption in "EIT" process is a severe problem

-Hau paper had only 65% transmission

assume that:

- we have our control beam at 90 degrees

-3 x 10^6 atoms in the "block"

-one spontaneous scattering event removes the atom

-we can afford to lose ~30% of the atoms and still see the effect

$1\mu W = 3 \times 10^6$ photons/μs

x 1 ms pulse = 3 x10^9 photons

in this scenario we can afford only 0.1% absorption

(the need for a small angle for the control beam makes this worse)

a BEC as a block of glass

If you drive a Raman transition with an $\hbar k$ of recoil, an individual atom will be kicked out of the condensate - the BEC needs to act as a "solid."

If the recoil is about 0.1 $\hbar k$ or less, then the recoil is less than the speed of sound in the medium and the condensate should recoil as a whole, generating phonons, but not ejecting atoms.

The Raman beam that "dresses" the atoms for the EIT effect must then be at a "small" angle and the momentum exchange along this direction must be accounted for; **this complicates the interpretation**, and makes the transparency requirement more stringent.

can it be done?

If the EIT effect can really be made transparent...

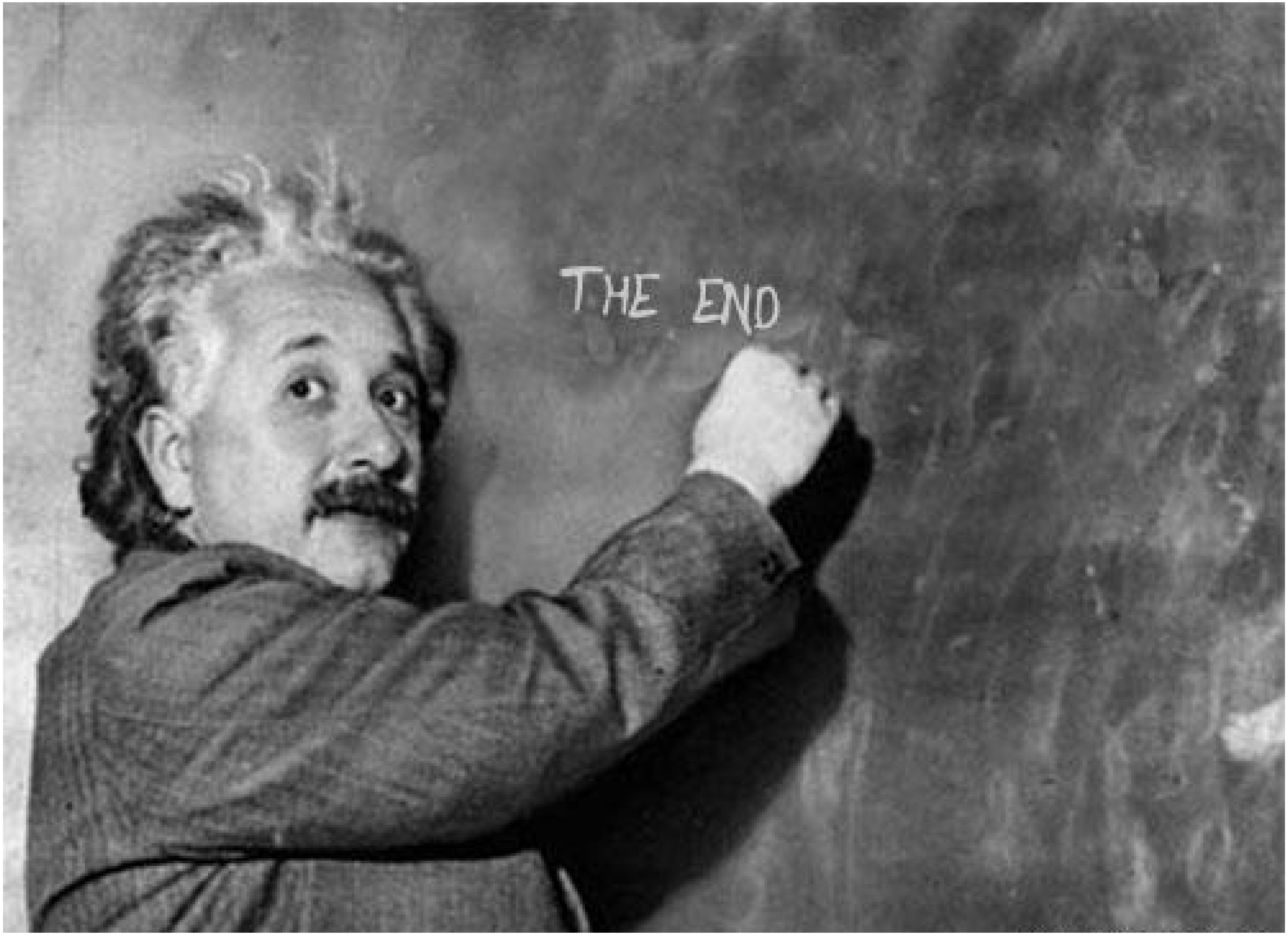
-This can be a “big” effect and can be done as a qualitative, rather than quantitative experiment and still distinguish between the Abraham and Minkowski choices.

- The experiment also gives a graphic demonstration of “faster-than-light” effects and interesting things that happen with negative index of refraction materials

So, what went wrong?

$$p = \hbar k = \frac{nh}{\lambda} = \frac{nh\nu}{c}$$ **vs.** $$p = mv = \frac{E}{c^2}\frac{c}{n} = \frac{h\nu}{nc}$$ **?**

answer: You cannot assume that p=ħk in the medium.
This assumes that the photon carries all the momentum and that the medium carries none (as does Minkowski).


THE END