Turbulent Liquid Metal Dynamo Experiments

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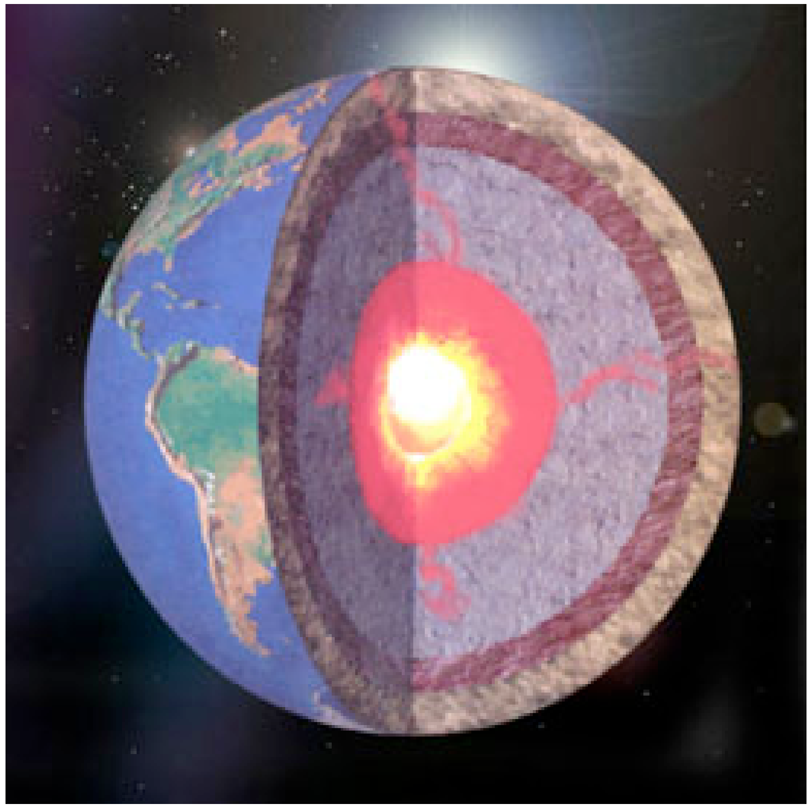
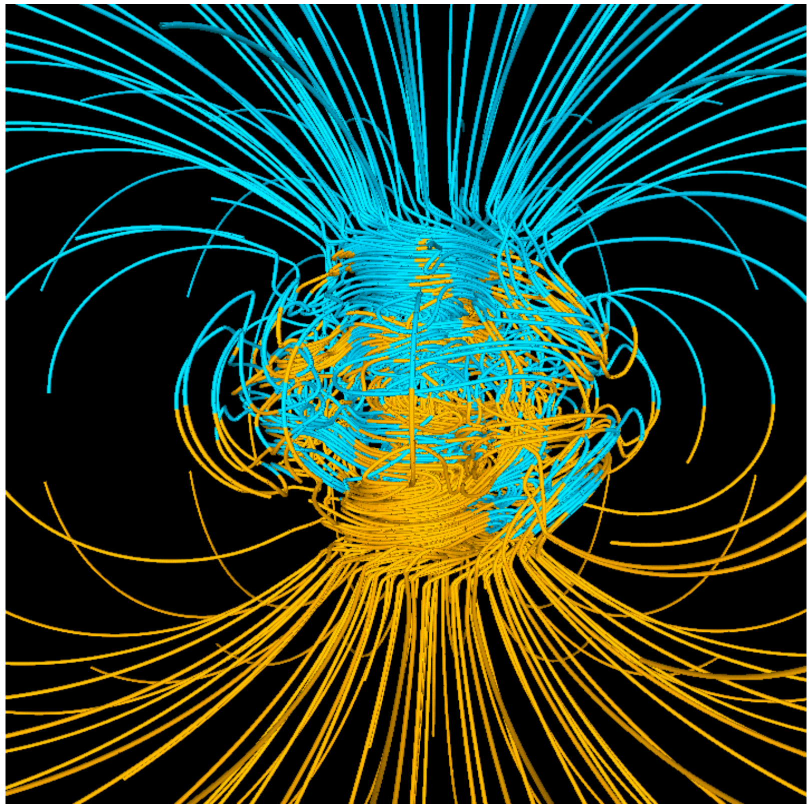
November 30, 2006

Acknowledgements: Adam Bayliss, Roch Kendrick, Craig Jacobson, Mark Nornberg, Carlos Parada, Erik Spence, Zane Taylor,

The Earth and other planets have magnetic fields

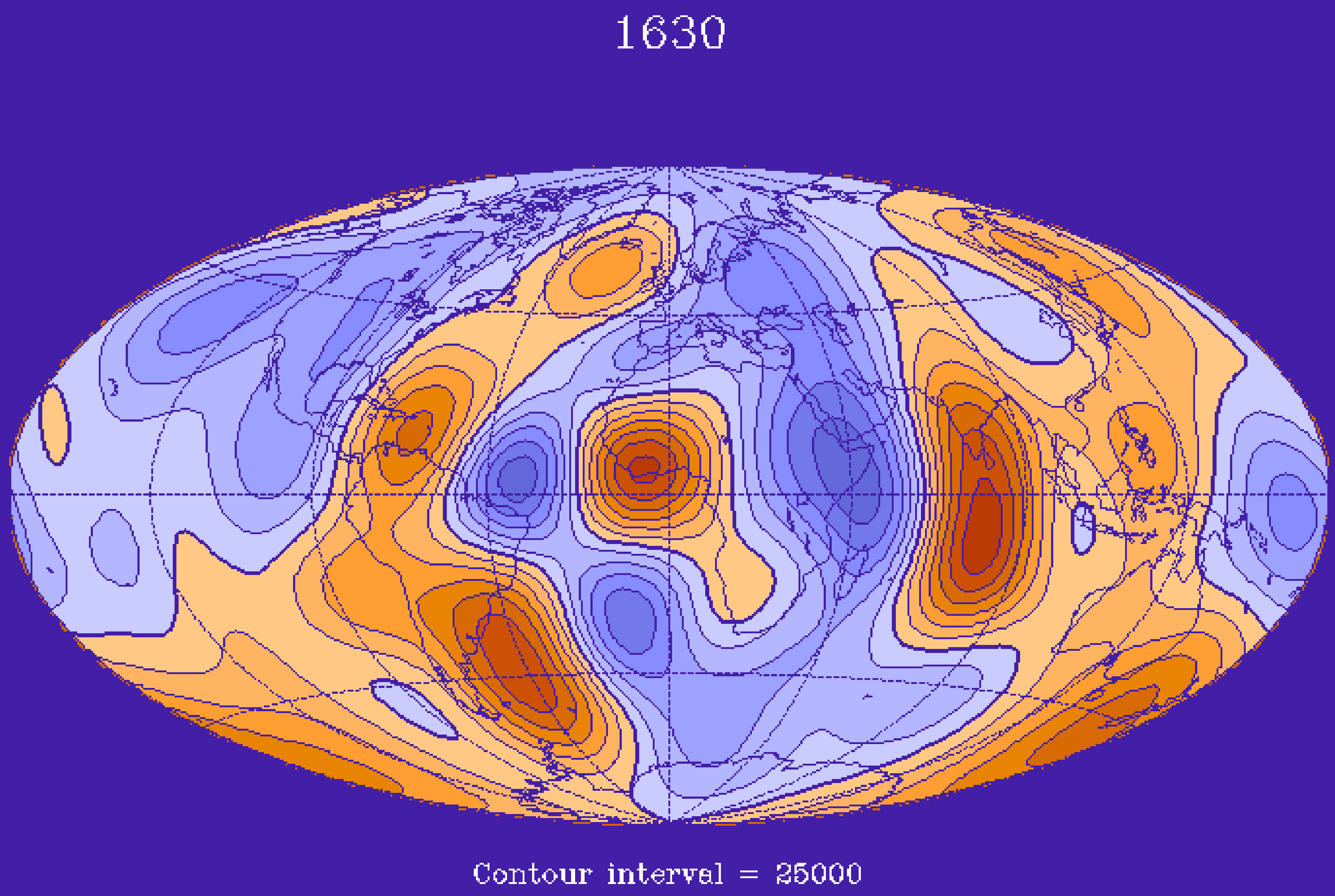




Glatzmaier and Roberts, *A three-dimensional self-consistent computer simulation of a geomagnetic field reversal*, Nature **377** 203 (1995).

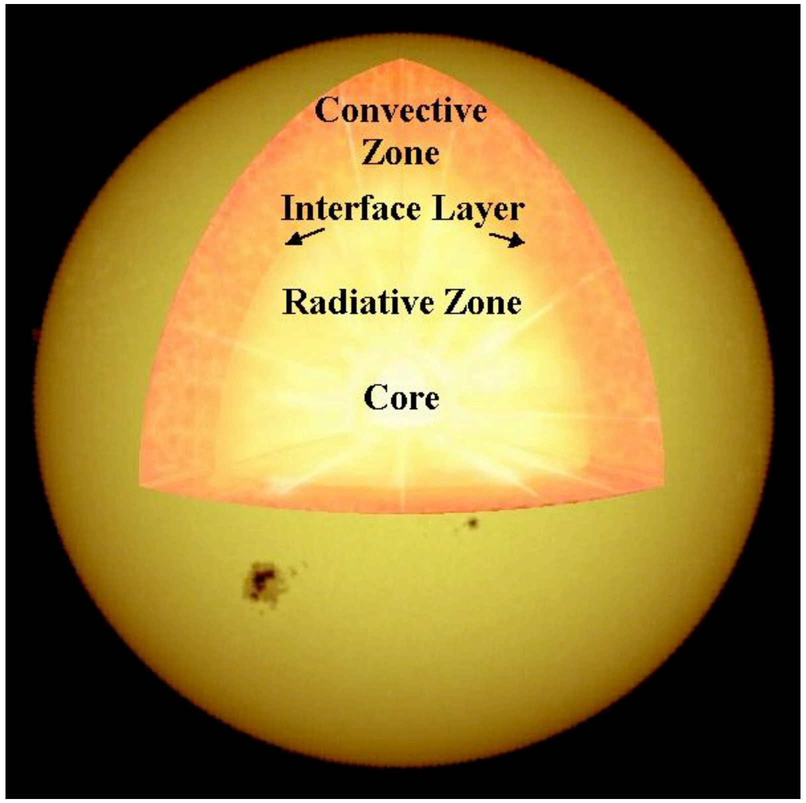
- The Earth's Dynamo
  - Primarily a dipole, aligned with rotation axis
  - liquid metal core is turbulent ($Re \sim 10^6$)

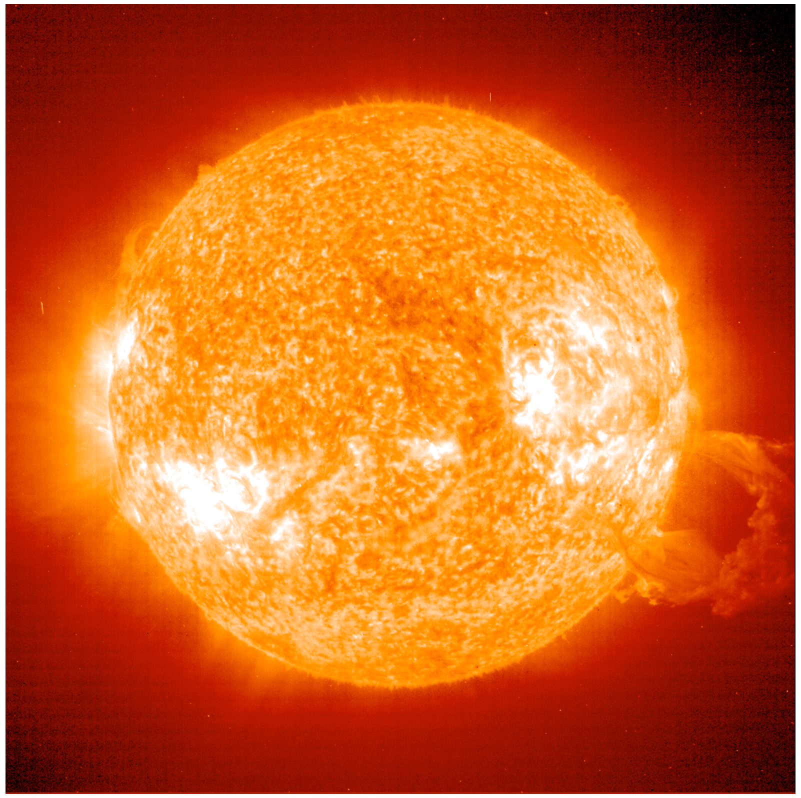
The Earth's dynamo is spatially complex and dynamic



Jackson, Jonkers and Walker, *Four centuries of geomagnetic secular variation*, Phil. Trans. R. Soc. Lond. A **358** 957 (2006).

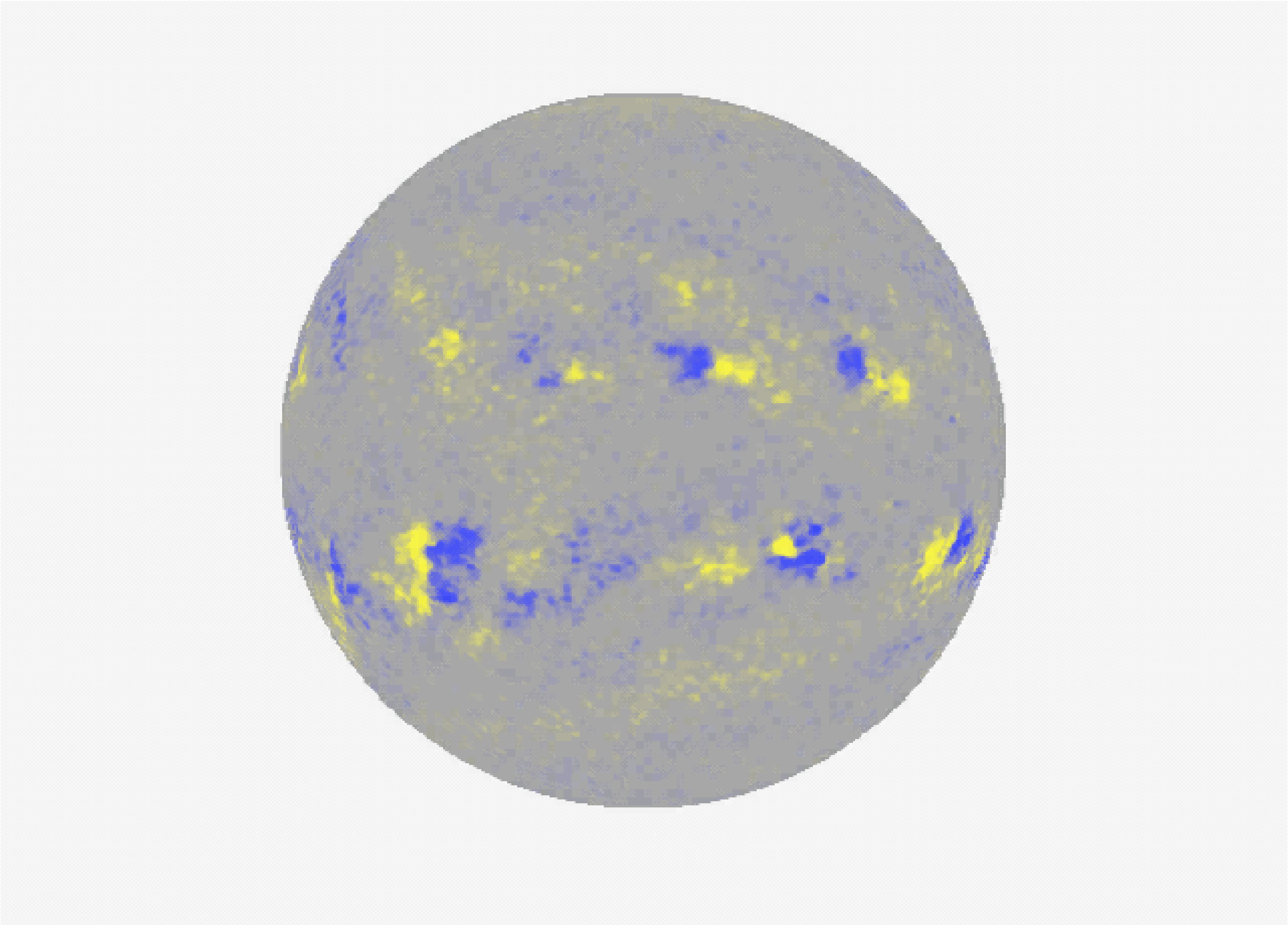
The Sun's magnetic field is even more complex



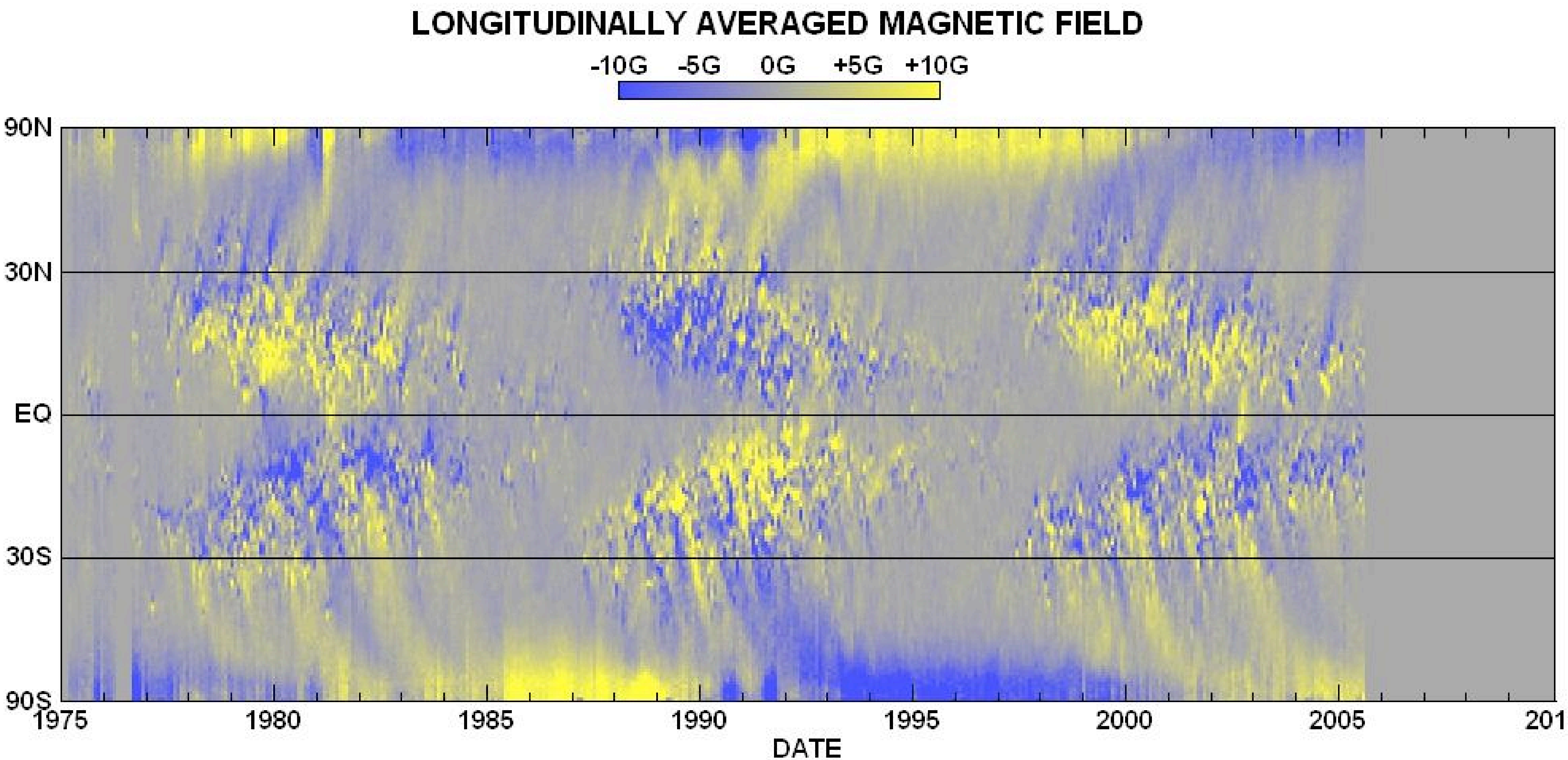


- The Sun's magnetic field
  - Free surface, rigid inner core
  - Large scale dipole with higher order multipoles
  - Periodic (22 year cycle)

Magnetic field of Sun is dynamic



Solar magnetic field alternates polarity



Hathaway http://science.msfc.nasa.gov/ssl/pad/solar/dynamo.htm (2005)

How are these magnetic fields generated?

" . . . possible for the internal cyclic motion to act after the manner of the cycle of a self-exciting dynamo, and maintain a permanent magnetic field from insignificant beginnings, at the expense of some of the energy of the internal circulation."

J. Larmor, *How could a rotating body such as the Sun become a magnet?* Br. Assoc. Adv. Sci. **159** (1919).

Outline

- The magnetic fields of planets and stars
- Self-excited dynamos
- The Standard Model of the MHD self-excited dynamo
- Self-exciting dynamo experiments
- Experiments on turbulent dynamos

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What is a self-exciting dynamo?

Faraday's Law of Induction

$$\vec{J} = \sigma \left(\vec{E} + \vec{V} \times \vec{B} \right)$$

feedback

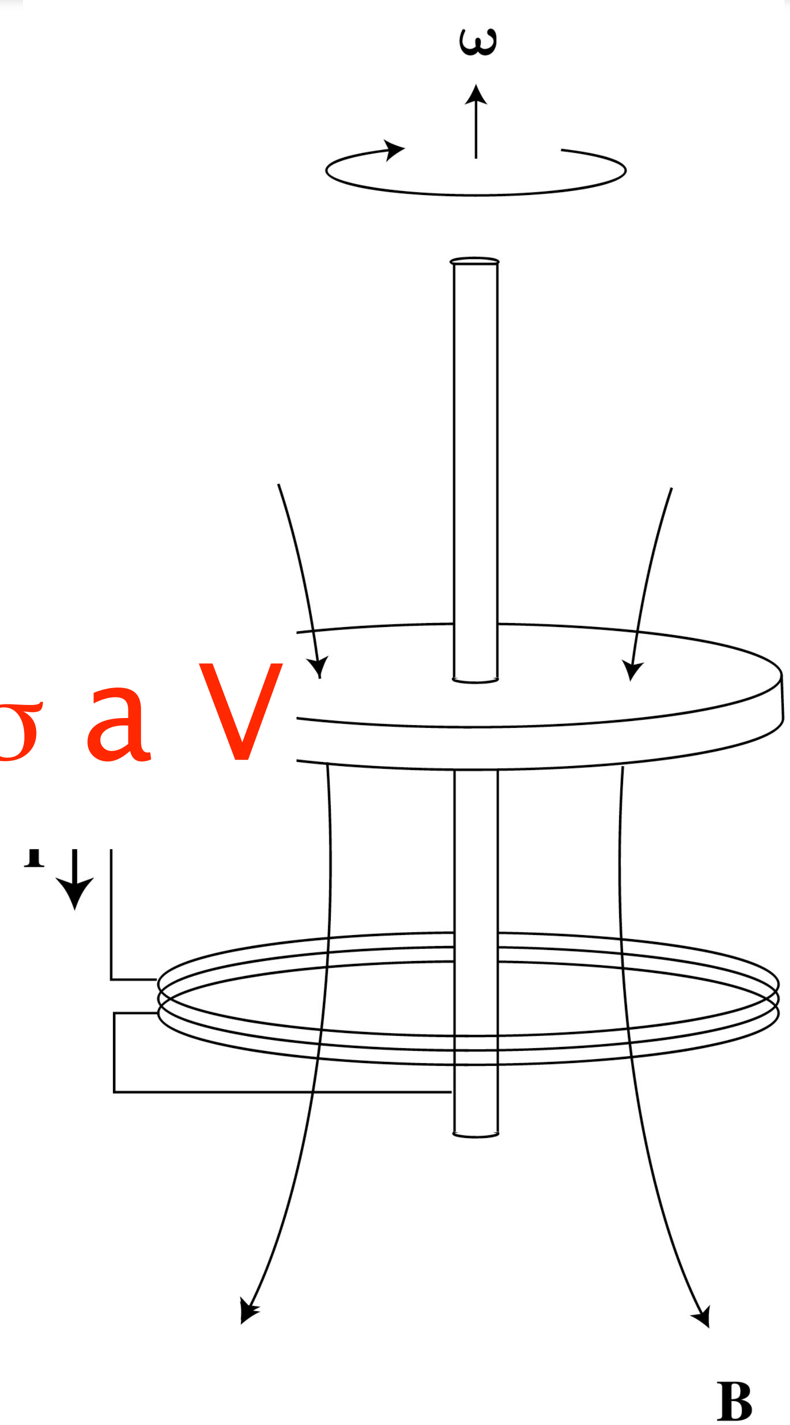
Induction Equation

$$\frac{\partial I}{\partial t} = \left(\frac{\omega L}{2\pi N R} - 1 \right) \frac{I}{\tau_{L/R}}$$

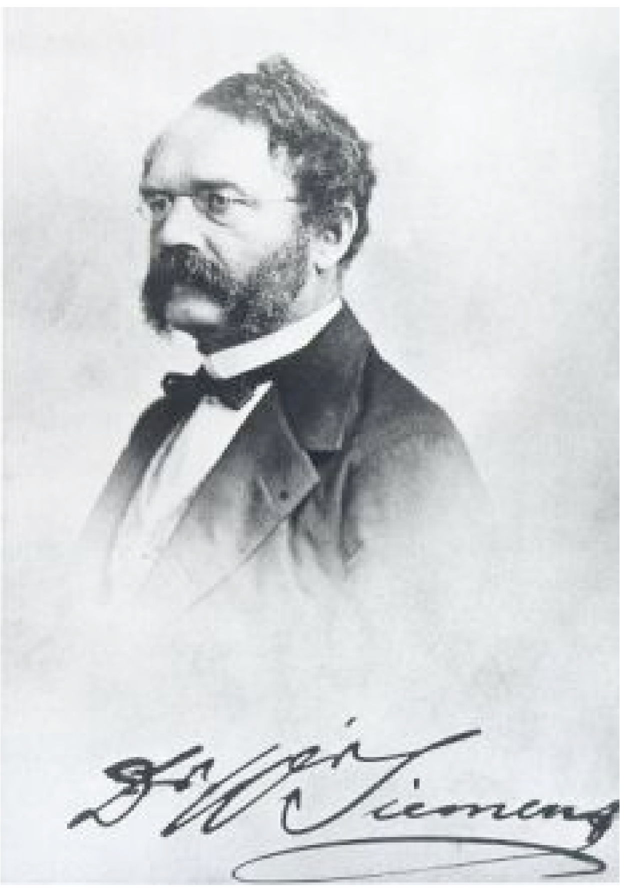
$\sim \mu_0 \, \sigma \, a \, V$

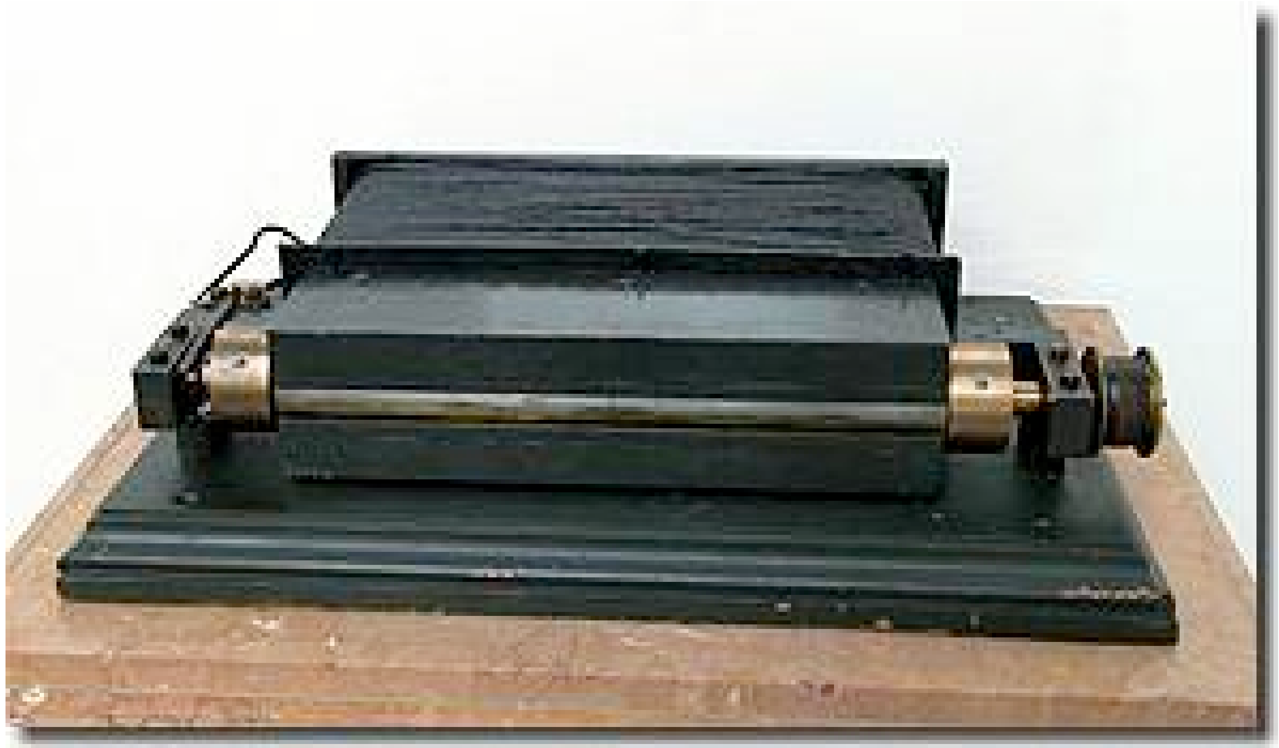
Equation of Motion

$$\frac{\partial \omega}{\partial t} = \frac{T_{ext}}{\mathcal{I}} - \frac{L}{2\pi N R \mathcal{I}} I^2$$



The self-excited generator of Werner von Siemens (1866)





The "dynamo electric principle"

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- The magnetic fields of planets and stars
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MHD equations describe well the magnetic field evolution in a liquid metal or plasma

Ohm's law and Pre-Maxwell's Equations

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{V} \times \mathbf{B}\right)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

<u>Induction Equation</u>

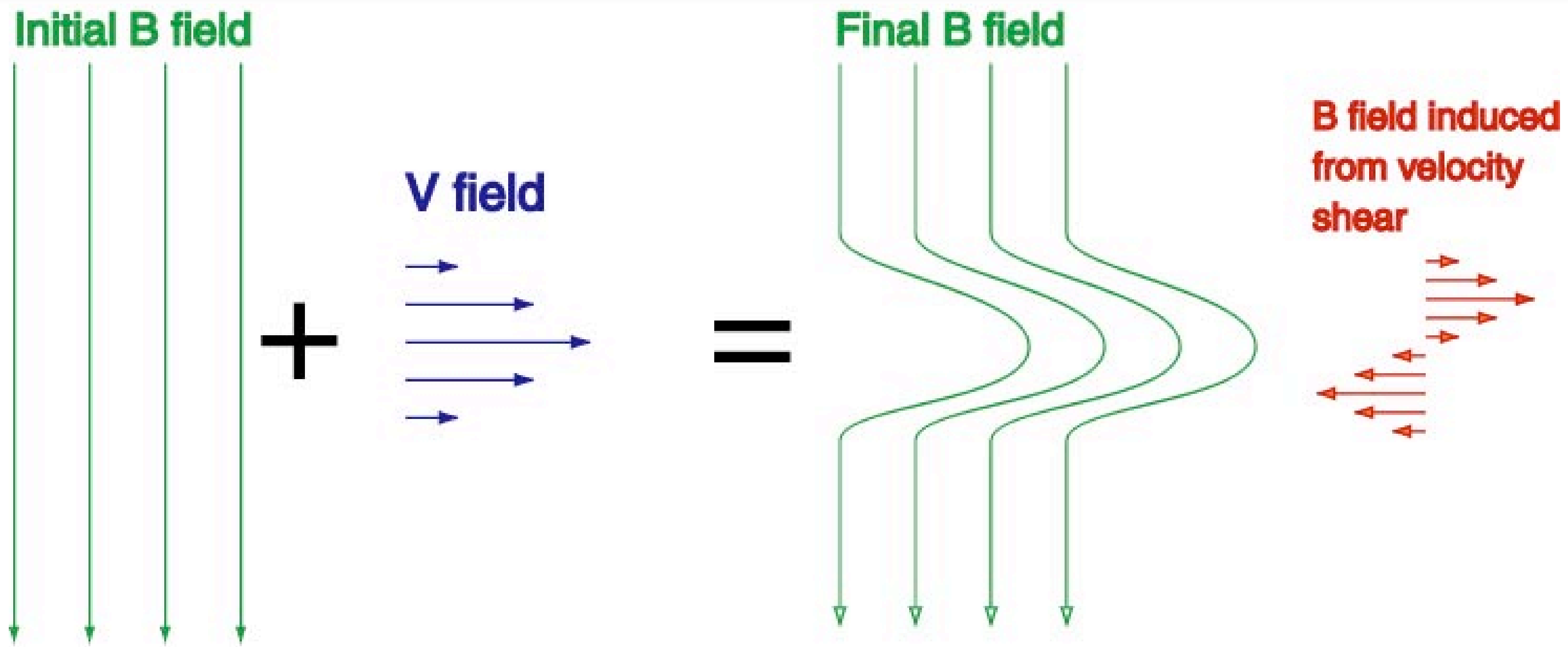
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

$$\frac{\nabla \times V \times B}{\frac{1}{\mu_o \sigma} \nabla^2 B} \sim \mu_O \sigma L V_0 \equiv Rm$$

<u>Equation of Motion</u>

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V}\right) = -\nabla p + \mathbf{J} \times \mathbf{B} + \mu \nabla^2 \mathbf{V} + F_{prop}$$

Fluid flow can amplify and distort magnetic fields when the magnetic Reynolds number is large



- transverse component of field is generated, amplifying the initial field
- finite resistance leads to diffusion of field lines

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

0

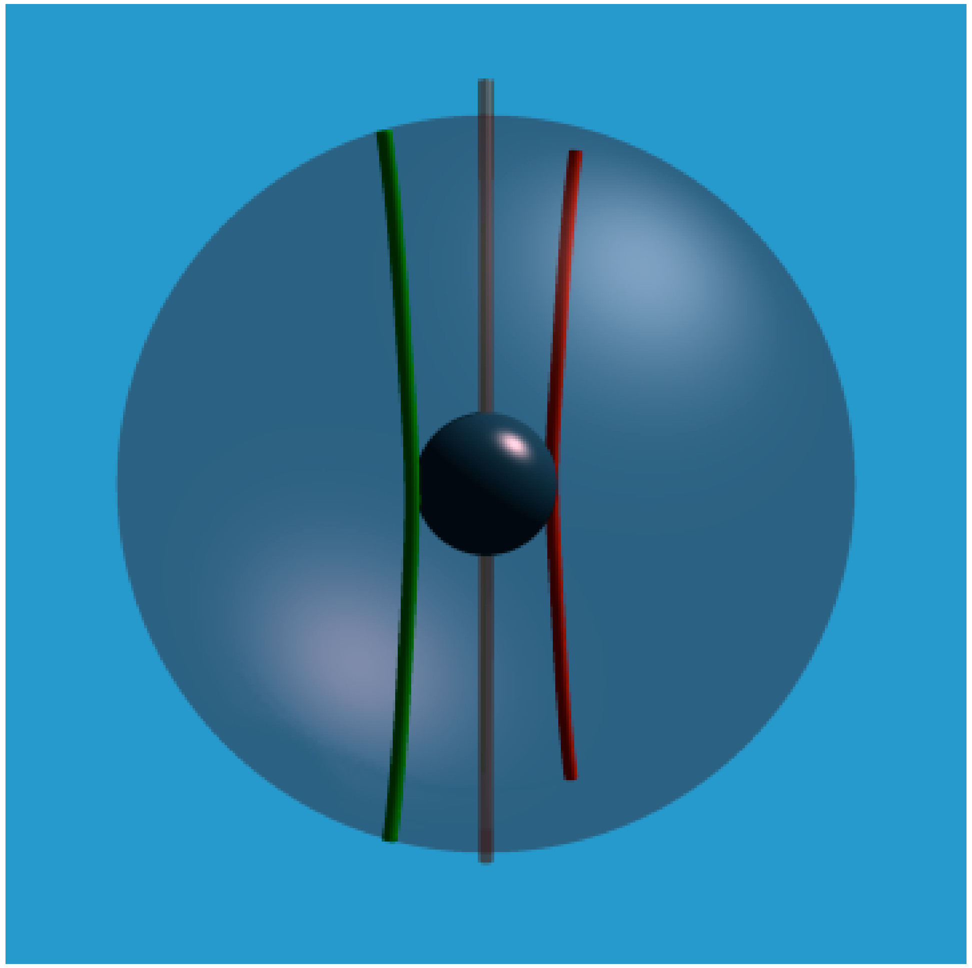
Standard Model of an MHD dynamo

Step 1: dipole field can be converted into strong toroidal field

Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

$$\frac{\partial B_\phi}{\partial t} = \mathrm{Rm}\, r \mathbf{B}_P \cdot \nabla \Omega + (\nabla^2 - r^{-2}) B_\phi$$



The "Ω effect"

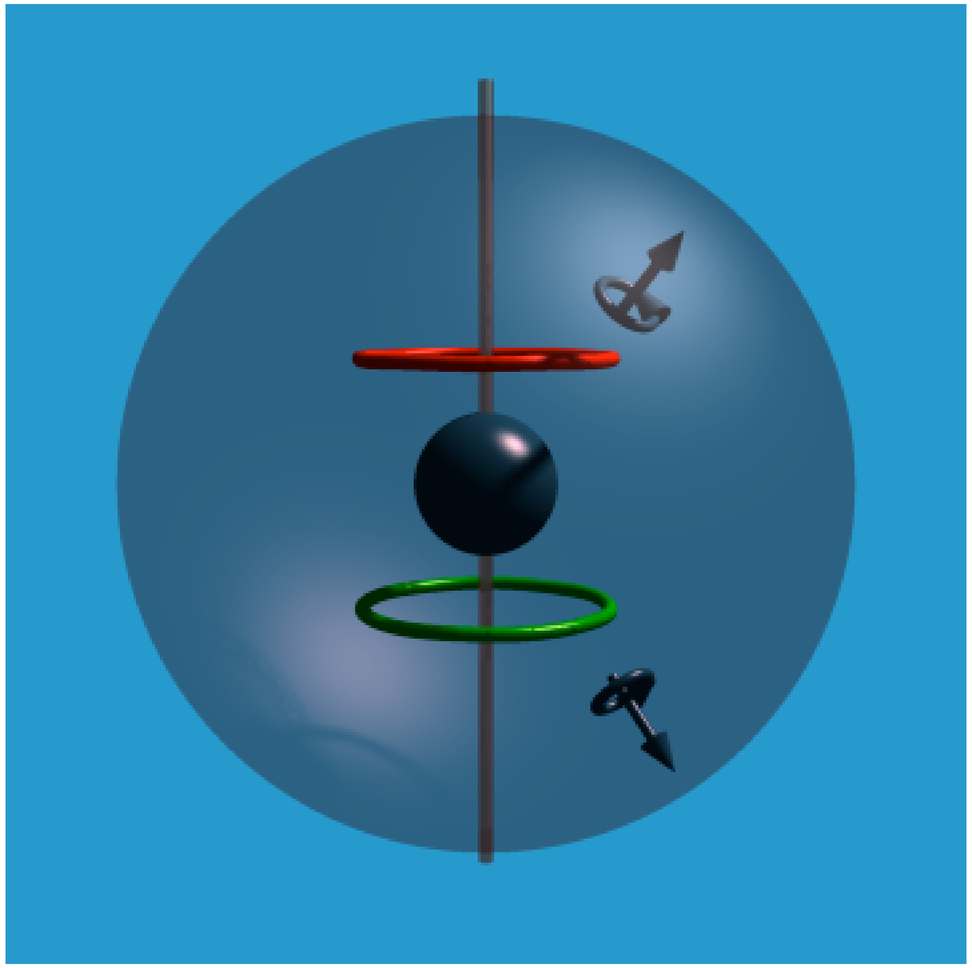
Cowling's Anti-Dynamo Theorem

When the magnetic field and the fluid motions are symmetric about an axis...no stationary dynamo can exist.

T.G. Cowling, *The magnetic fields of sunspots*, Monthly Notices Roy. Astron. Soc. **94** 39 (1933).

Standard Model of an MHD dynamo

Step 2: Nonaxisymmetric, helical flows convert toroidal field back into dipole



The "α effect"

<u>Mean Field Electrodynamics</u>

$$\boldsymbol{B} = \langle \boldsymbol{B} \rangle + \widetilde{\boldsymbol{b}}, \quad \boldsymbol{V} = \langle \boldsymbol{V} \rangle + \widetilde{\boldsymbol{v}}$$

$$\langle \boldsymbol{J} \rangle = \sigma \left(\langle \boldsymbol{E} \rangle + \langle \boldsymbol{V} \rangle \times \langle \boldsymbol{B} \rangle + \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle \right)$$

$$\mathcal{E} = \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle = \alpha \langle \boldsymbol{B} \rangle + \beta \nabla \times \langle \boldsymbol{B} \rangle$$

E.N. Parker, *Hydromagnetic dynamo models*, Astrophys. J. **122** 293 (1955)

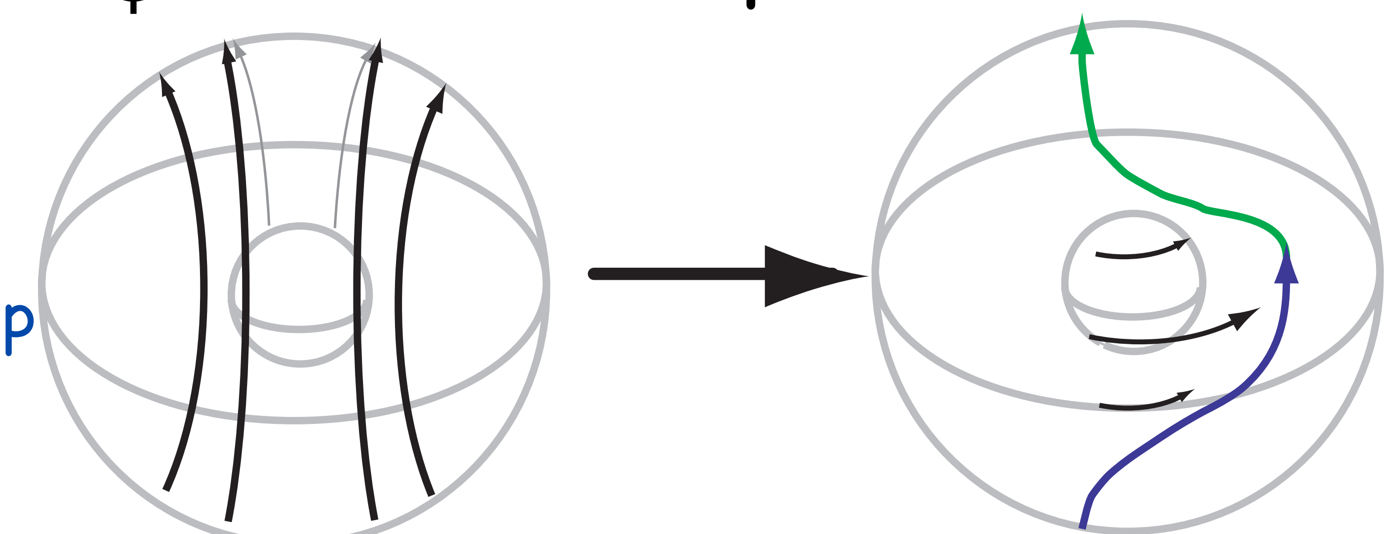
The Standard Dynamo Model: The $\alpha\Omega$ Dynamo

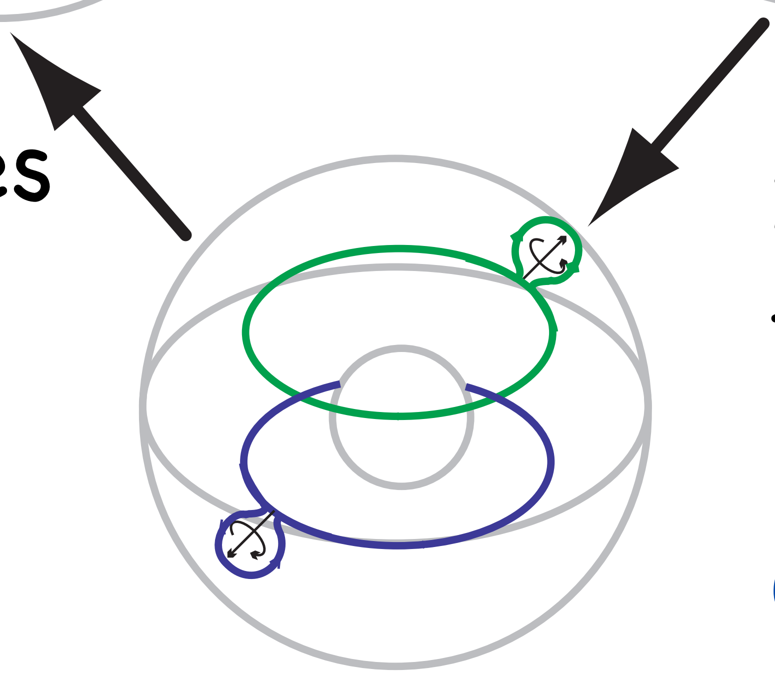
1. Ω-Effect: differential, axisymmetric rotation generates B_φ from seed dipole

$B_\varphi \propto Rm\ B_p$

Amplification depends on $Rm=\mu_0\sigma aV$

seed B_p





J_φ reinforces seed field

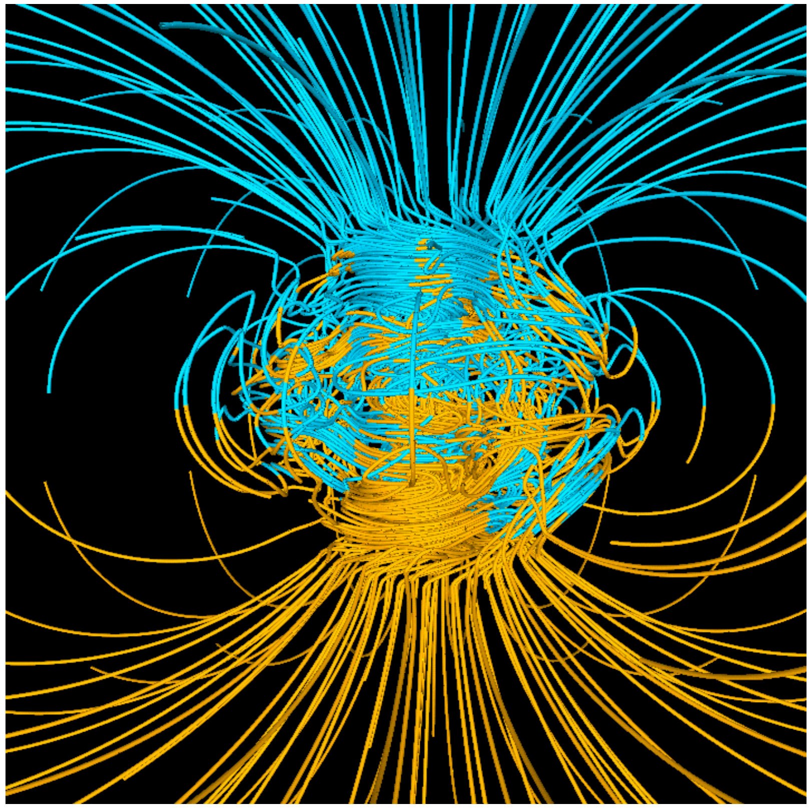
2. symmetry breaking helical fluctuations generate toroidal current: $J_\varphi = \sigma\ \alpha B_\varphi$

αB is turbulent EMF= $\langle \tilde{V} \times \tilde{B} \rangle$

Current state of theory is to solve the non-linear MHD equations numerically

- Induction equation

$$\frac{\partial \mathrm{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathrm{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathrm{B}$$



- Equation of Motion

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \mathbf{J} \times \mathbf{B} + \mu \nabla^2 \mathbf{V} + F_{prop}$$

Why do Experiments?

…in magnetohydrodynamics one should not believe the product of a long and complicated piece of mathematics if it is unsupported by observation.

Enrico Fermi

Why do experiments when we can simulate self-exciting dynamos?

- Simulations are limited in resolution and speed
- To resolve resistive dissipation scale requires a 3D grid of Rm^3
  - easy for Earth where Rm=300-600
  - hard for Sun where $Rm=10^7$
- To resolve viscous dissipation scale requires a 3D grid of Re^3
  - Pm = Rm/Re is a property of the medium
  - for liquid metals and solar plasma $Pm=10^{-5}$
- $Re>10^7$ in Earth and Liquid metal experiment
  - Flows are very turbulent
  - Can't be simulated accurately

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Which flows are self-exciting dynamos?

The kinematic dynamo problem:

- choose geometry (e.g. a sphere)
  - uniform conductivity σ
  - radius a
  - surrounded by an insulating region
- Find V(r) which leads to growing B(r,t)
- Ignore back-reaction

The kinematic dynamo problem

- Solve induction equation:

$$\begin{aligned} \frac{\partial \boldsymbol{B}}{\partial \tau} &= \mathrm{Rm} \nabla \times \hat{\boldsymbol{V}} \times \boldsymbol{B} + \nabla^2 \boldsymbol{B} \\ \tau &= t/\mu_o \sigma a^2, \;\; \hat{\boldsymbol{V}} = \boldsymbol{V}/V_{max} \\ \mathrm{Rm} &= \mu_o \sigma a V_{max} \end{aligned}$$

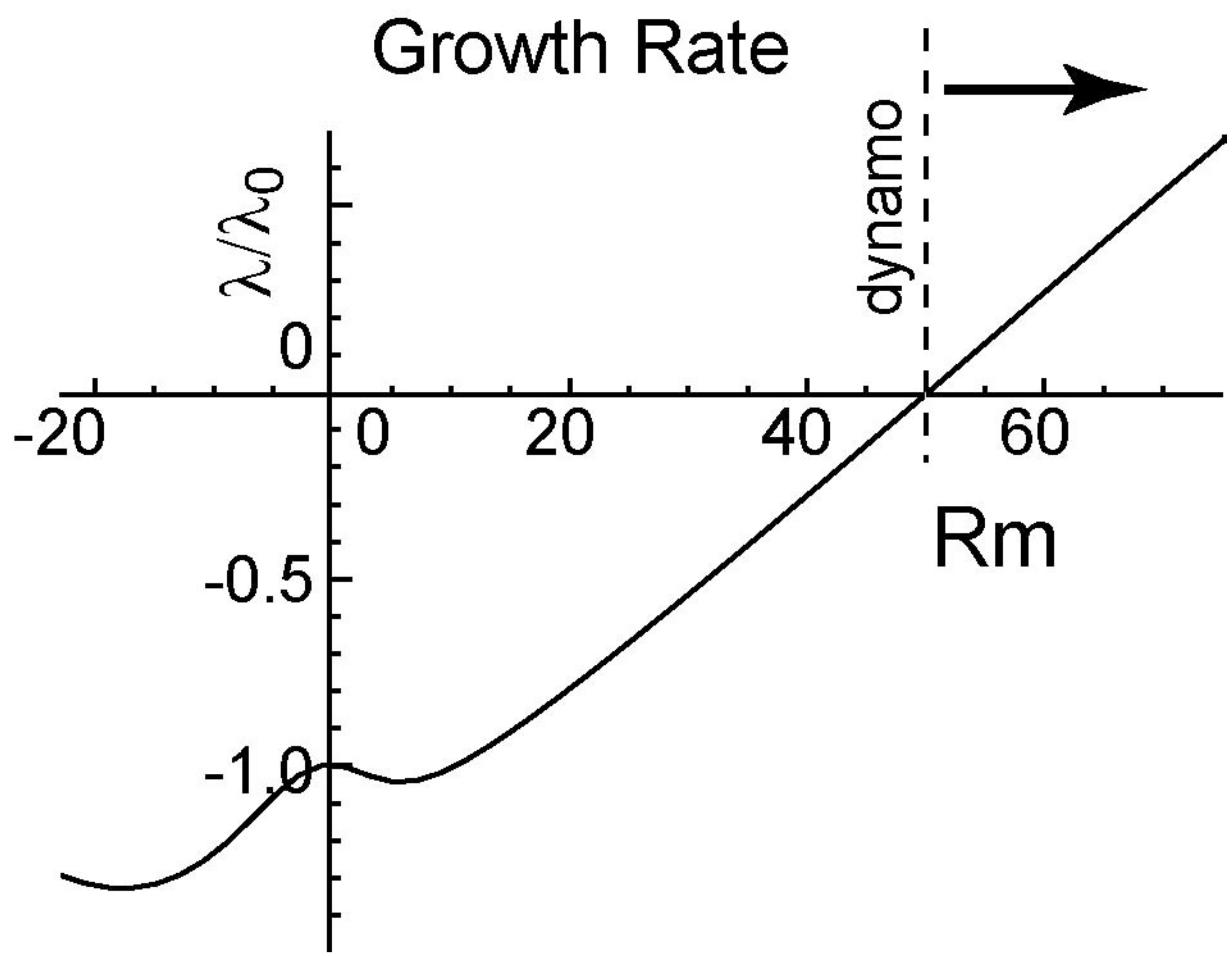
- Since its linear in B, use separation of variables:

$$\boldsymbol{B}(\boldsymbol{x}, t) = \sum_n e^{\lambda_n \tau} \boldsymbol{B}_n(\boldsymbol{x})$$

- Solve eigenvalue equation for given V(r) profile

$$\lambda_n \boldsymbol{B}_n = \mathrm{Rm} \nabla \times \hat{\mathbf{V}} \times \boldsymbol{B}_n + \nabla^2 \boldsymbol{B}_n$$

General solution shows growth/damping depends upon Rm

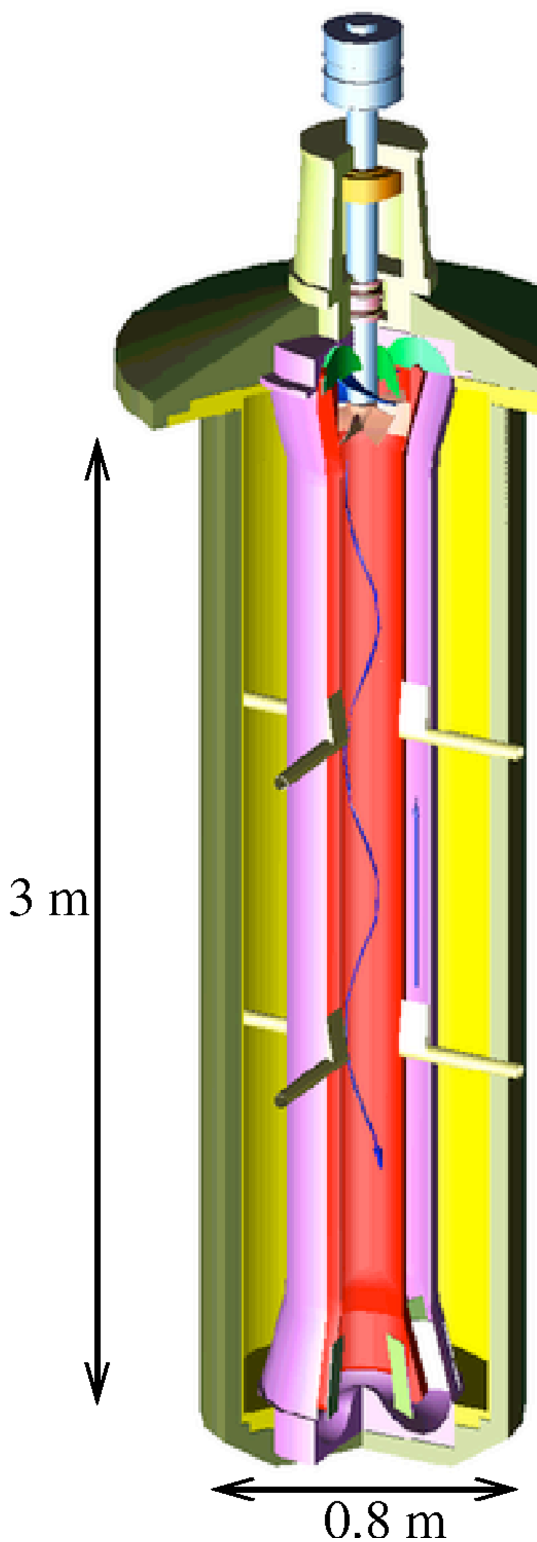


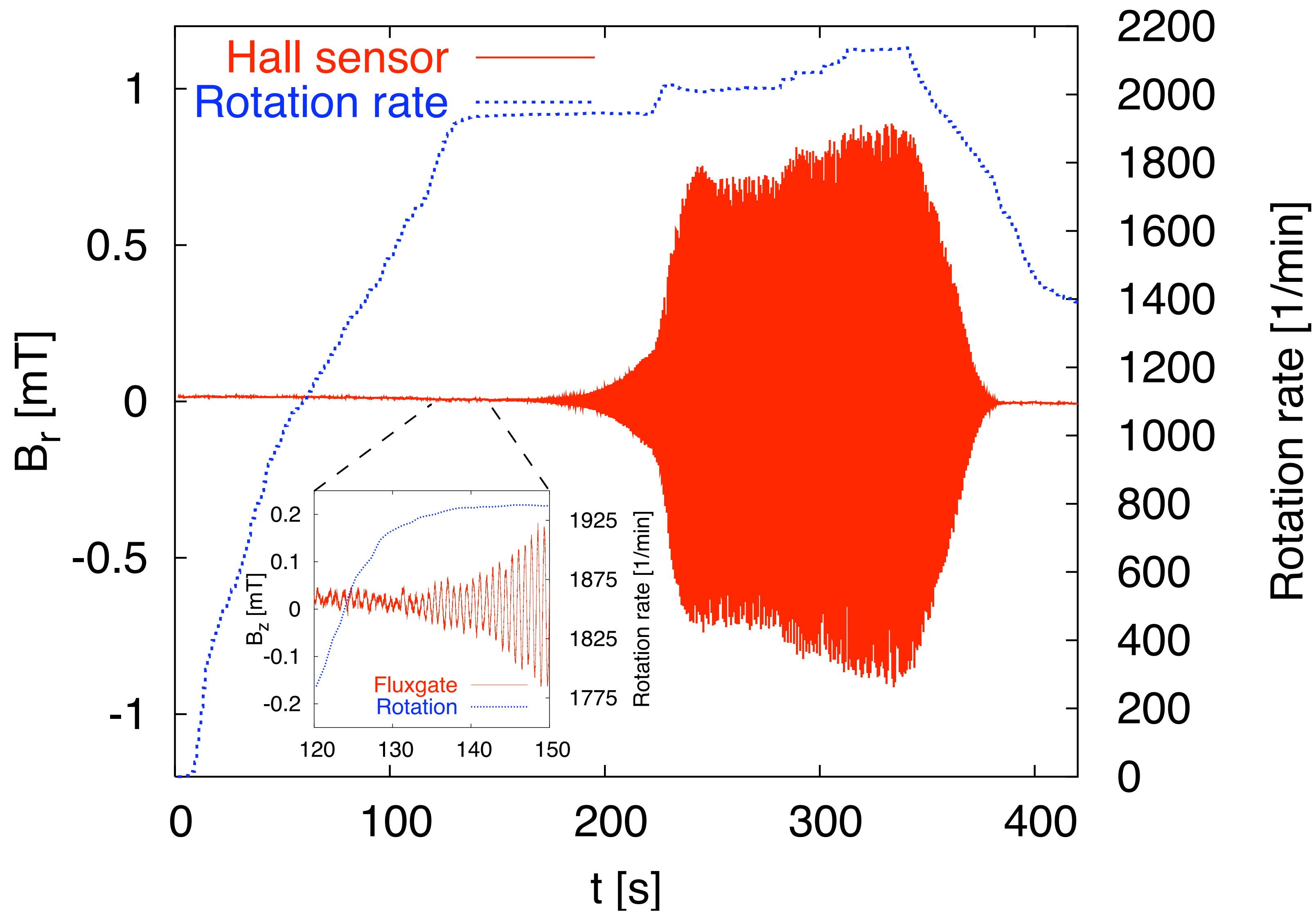
- Rm = $\mu_0 \sigma a V_{max}$ ~ conductivity X size X velocity
- must exceed critical value for system to self-excite (typically 50 to 100)

Flows of liquid sodium can achieve high Rm

- Why sodium?
  - Sodium is more conducting than any other liquid metal (melts at 100 C)
- How big must an experiment be to provide Rm=100?
  - Power $\geq$ 100 kW
  - $a \geq 0.5$ m (volume $\approx 1\ m^3$)
  - V_{max} ~15 m/s

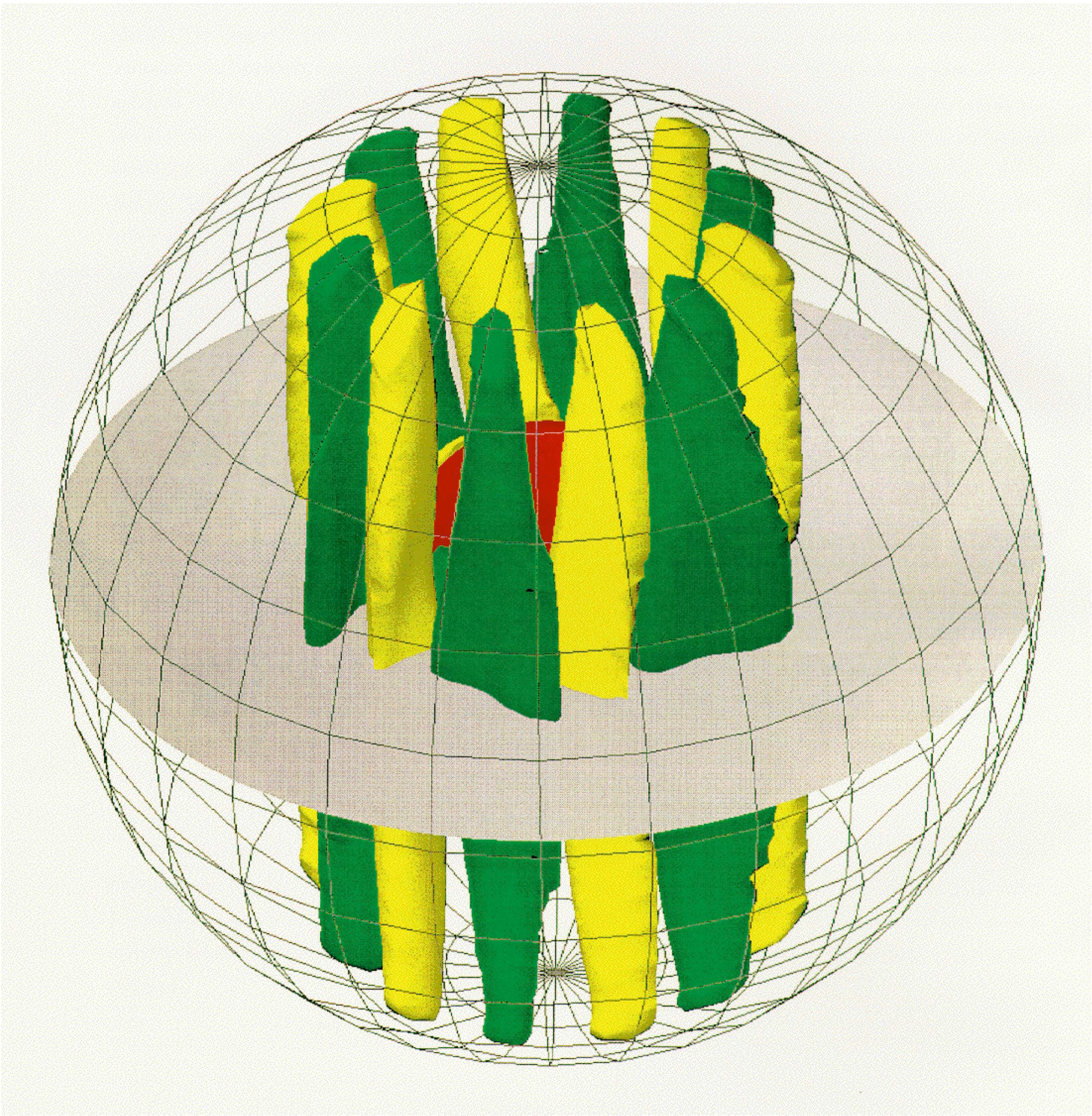
Riga experiment successfully self-generated a dynamo in 2001 (essentially single helical vortex)





A. Gailitis, et al.,*Magnetic Field Saturation in the Riga Dynamo Experiment*, Phys. Rev. Lett. **86** 3024 (2001)

The Karlsruhe experiment used channels to produce a small scale helical eddies to mimic the helical turbulence in the Earth's core



Muller and Stieglitz, *Experimental demonstration of a homogeneous dynamo*, Phys. Fluids **13** 561 (2001).

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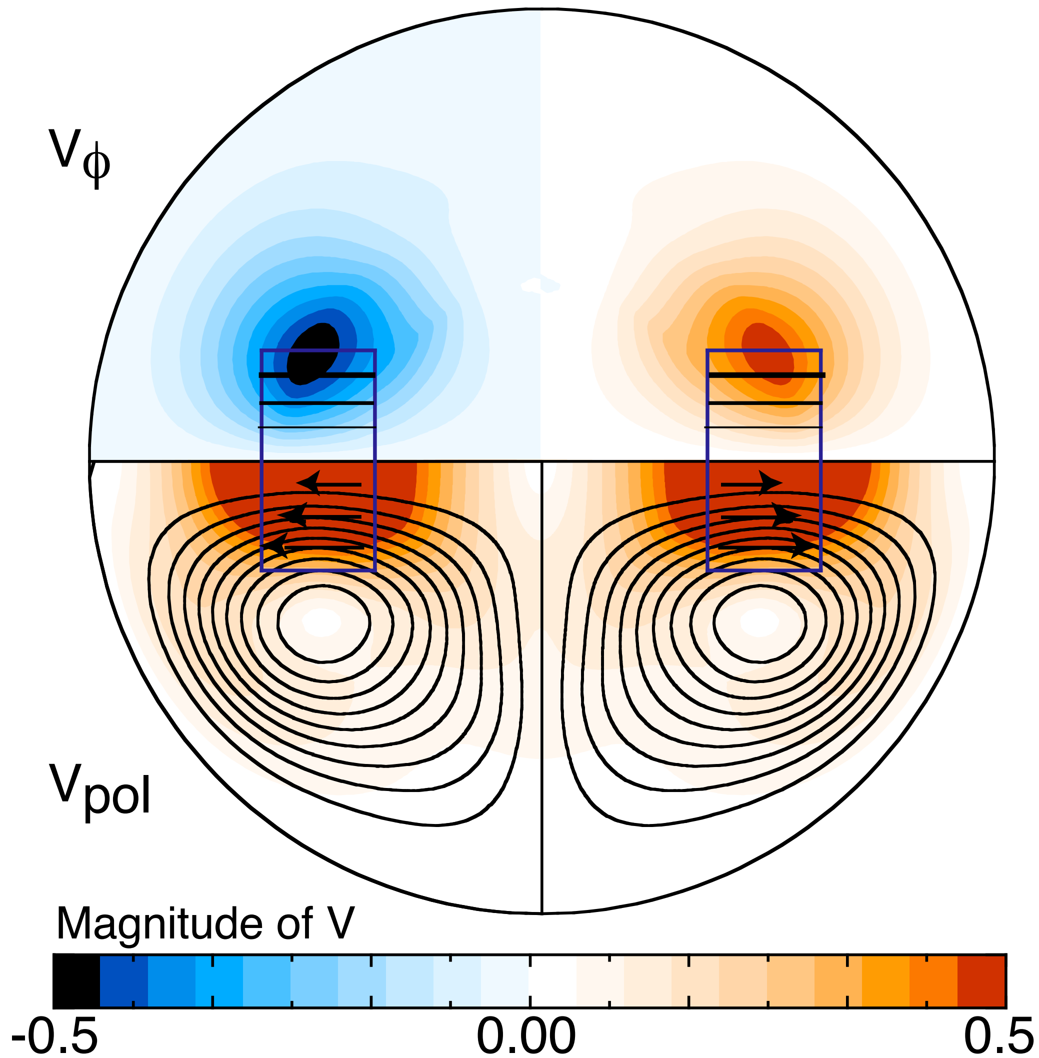
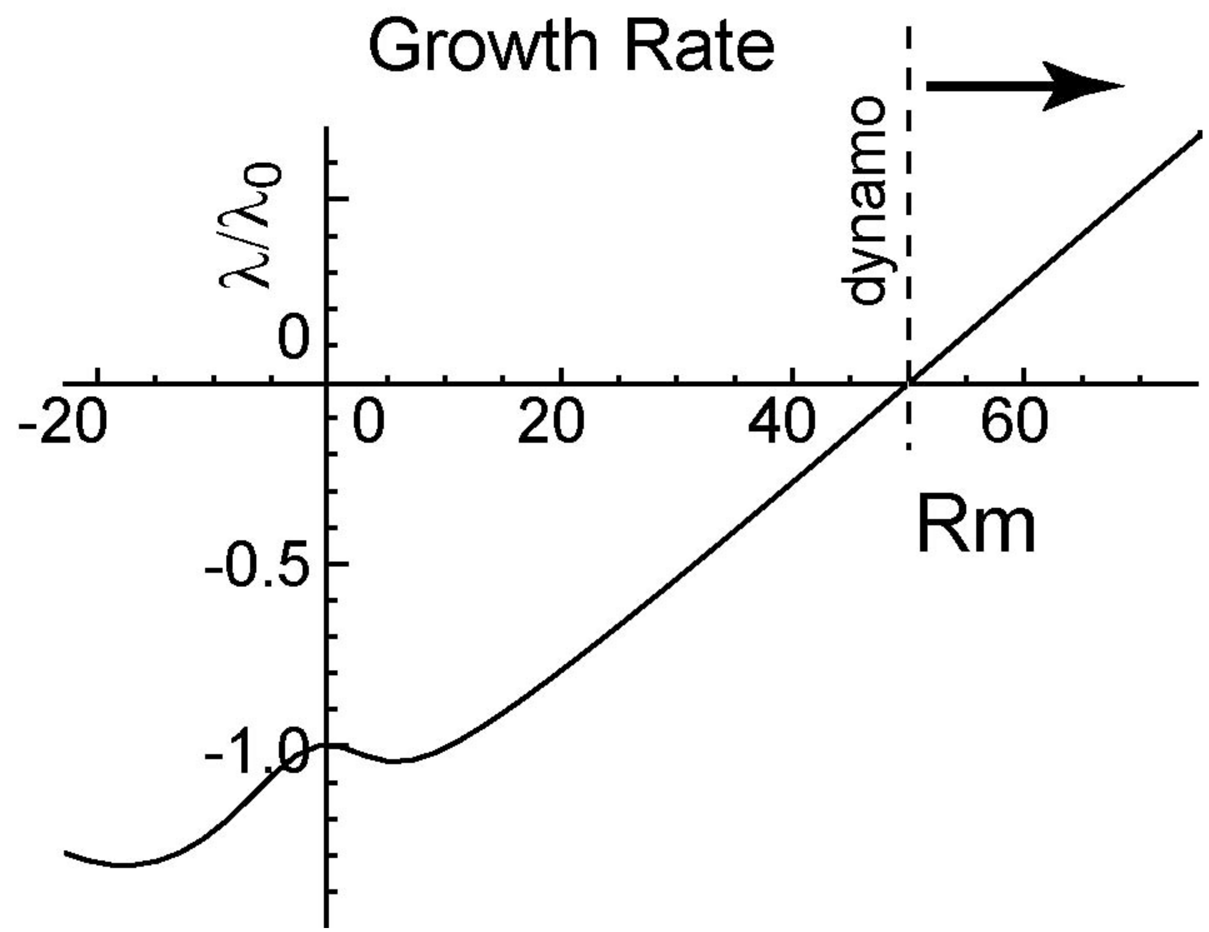
Big questions remain unanswered by experiments

- What happens if the velocity field is not constrained by pipes and baffles?

- What role does turbulence play in self-excitation?
  - Can turbulence generate current?
  - Does mean-field theory make sense?

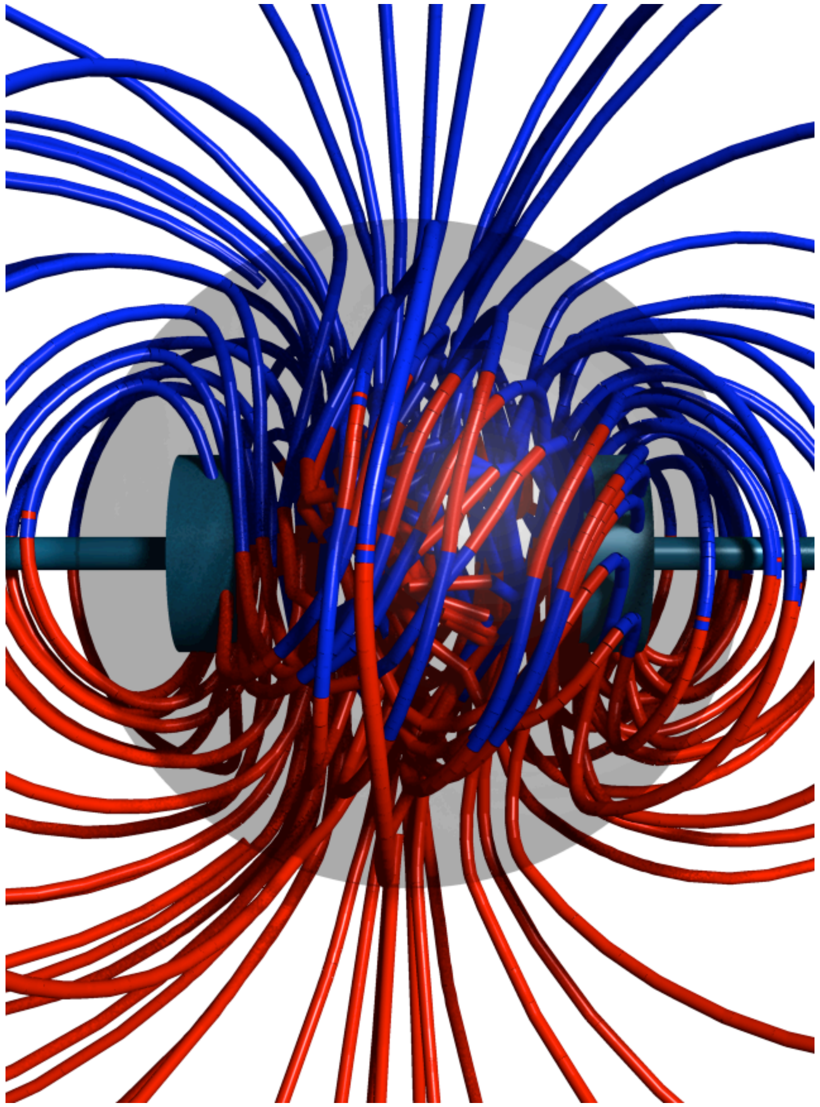
This simplest possible self-exciting flow: a two vortex flow with Rm_{crit}~50



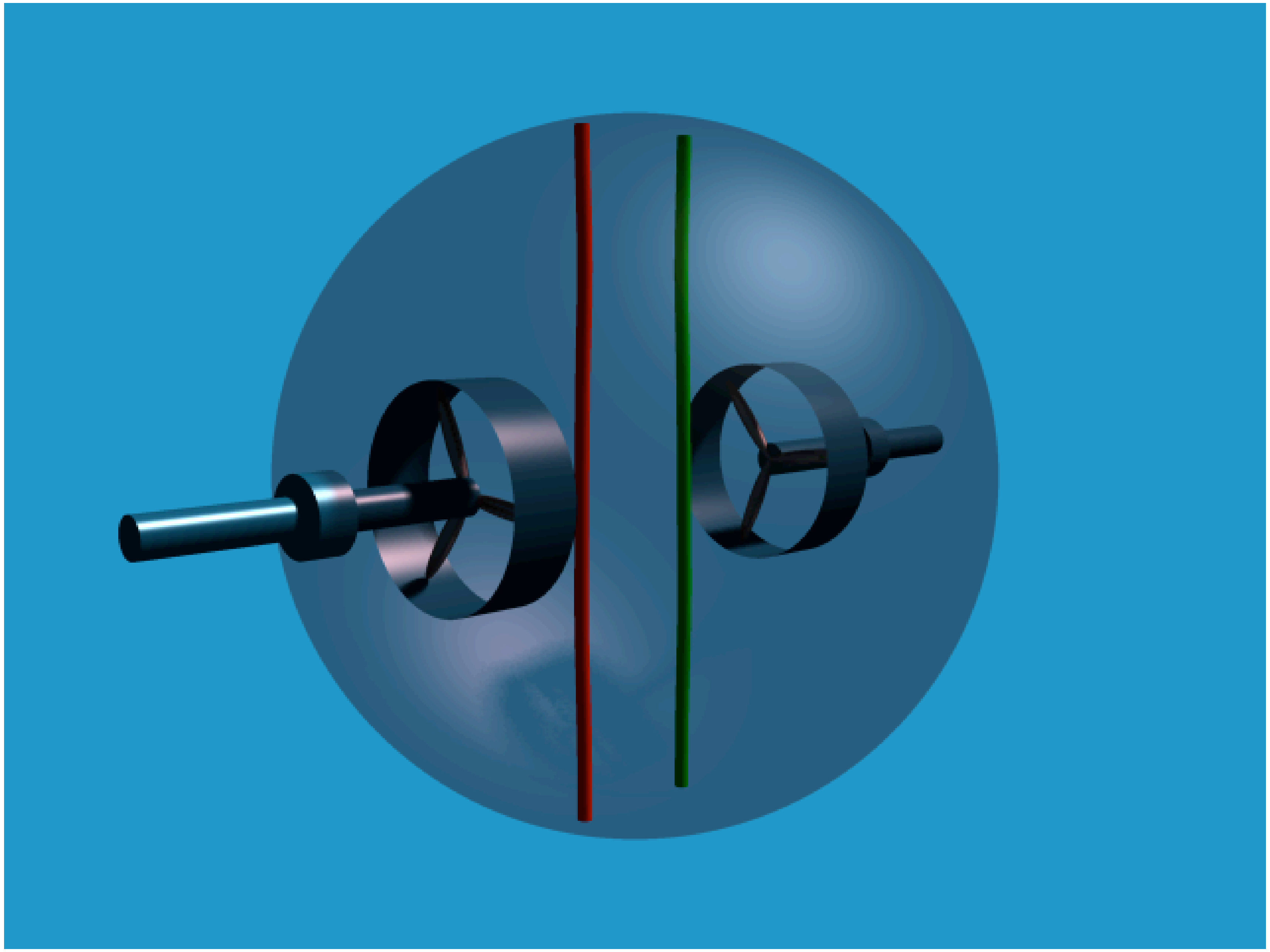


Dudley and James, *Time-dependent kinematic dynamos with stationary flows*, Proc. Roy. Soc. Lond. A. **425** 407 (1989).

Predicted eigenmode is an equatorial dipole



Dynamo is of the stretch-twist-fold type: field line stretching and reinforcement leads to dynamo

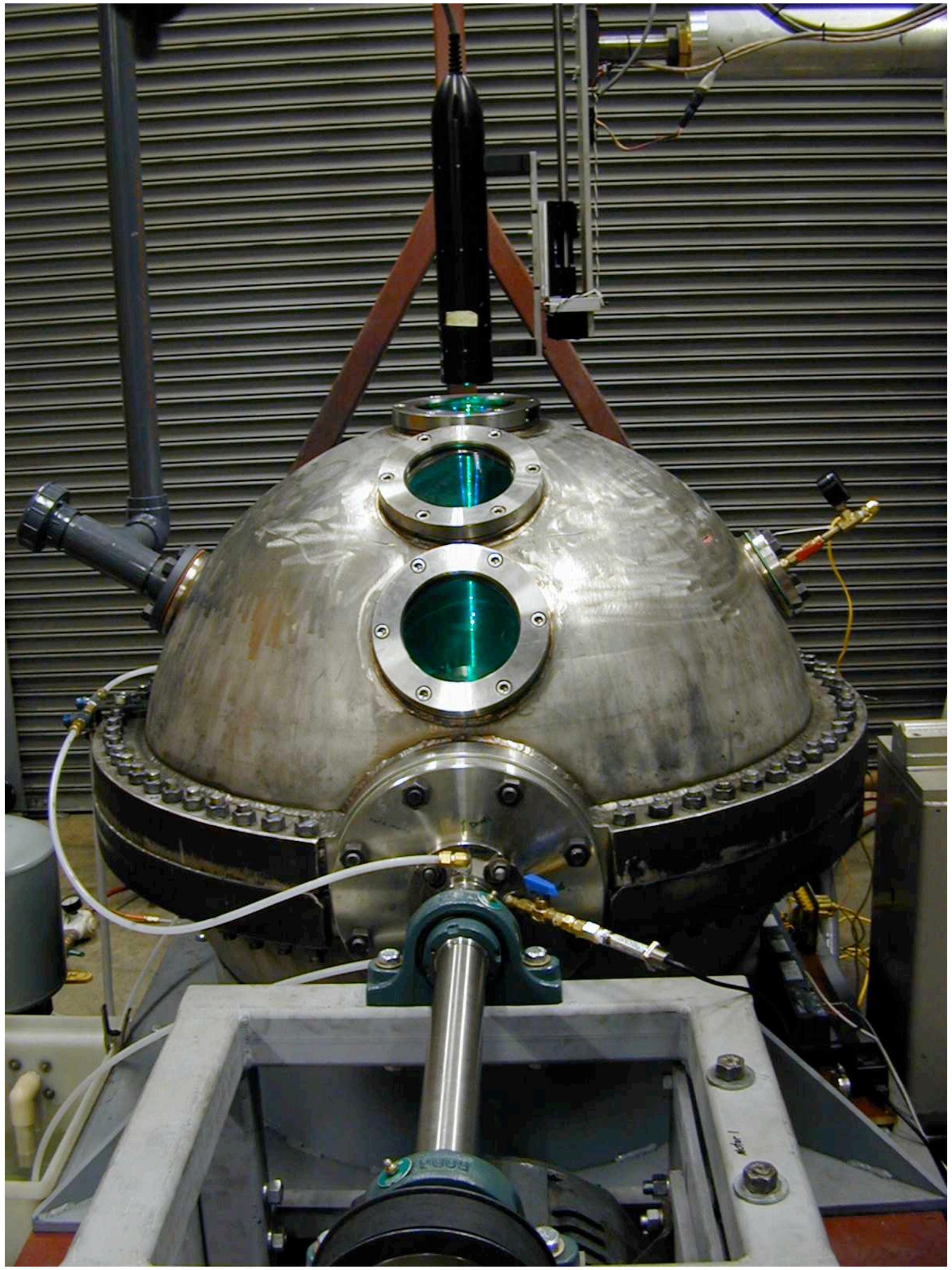


Dimensionally identical water experiment was used to demonstrate feasibility

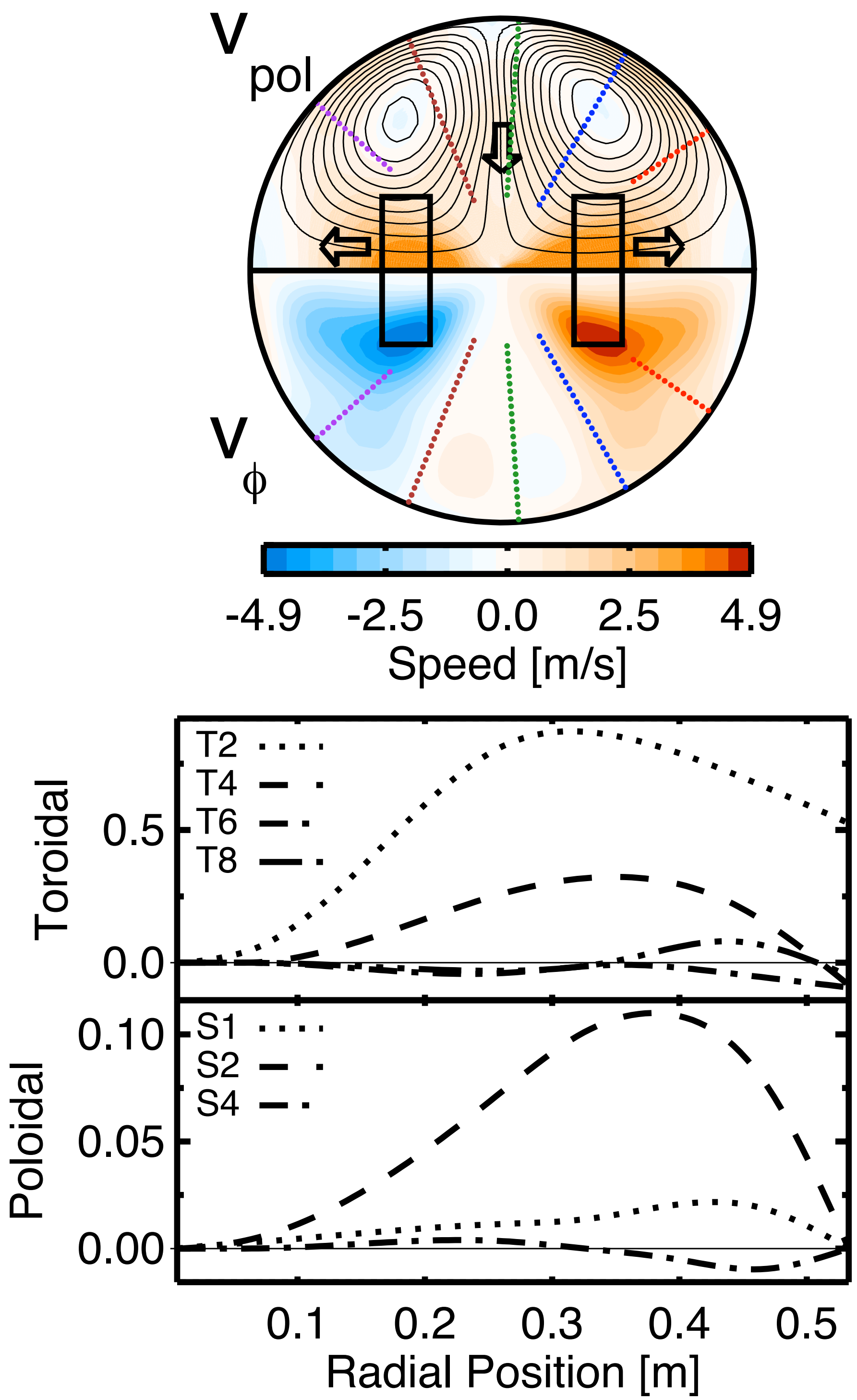
- Laser Doppler velocimetry is used to measure vector velocity field
- Measured flows are used as input to MHD calculation
- Full scale, half power

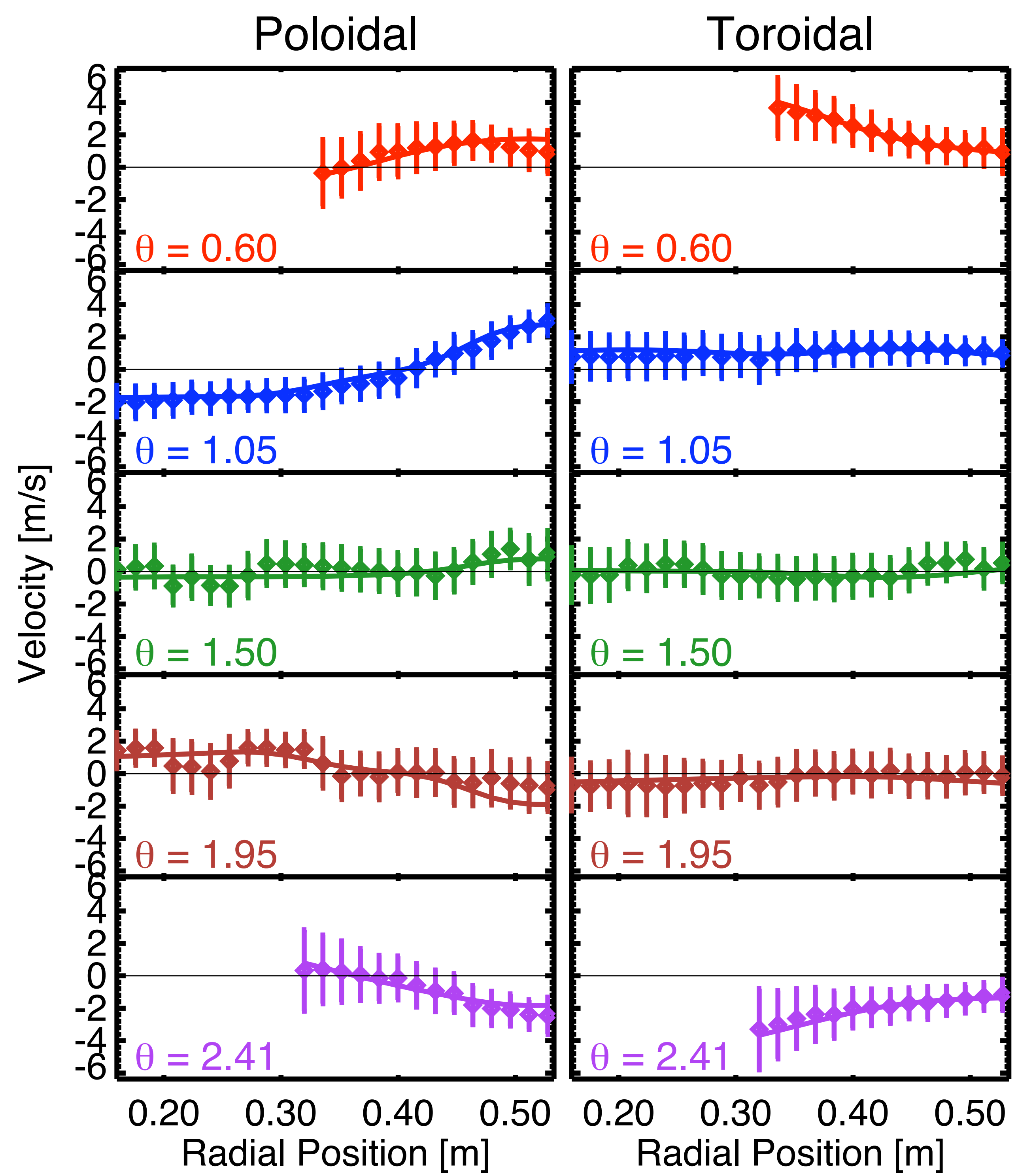
	Sodium	Water
Temperature	$110^\circ C$	$50^\circ C$
viscosity	$0.65\times10^{-6}\ \mathrm{m^2\,sec^{-1}}$	$0.65\times10^{-6}\ \mathrm{m^2\,sec^{-1}}$
mass density	$0.925\ \mathrm{gm\ cm^{-3}}$	$0.988\ \mathrm{gm\ cm^{-3}}$
resistivity	$10^{-7}\ \Omega\mathrm{m}$	

$$\longrightarrow \quad Rm = \frac{\mu_0 a V}{\eta} = 4\pi\, a(m)\, V(m/s)$$

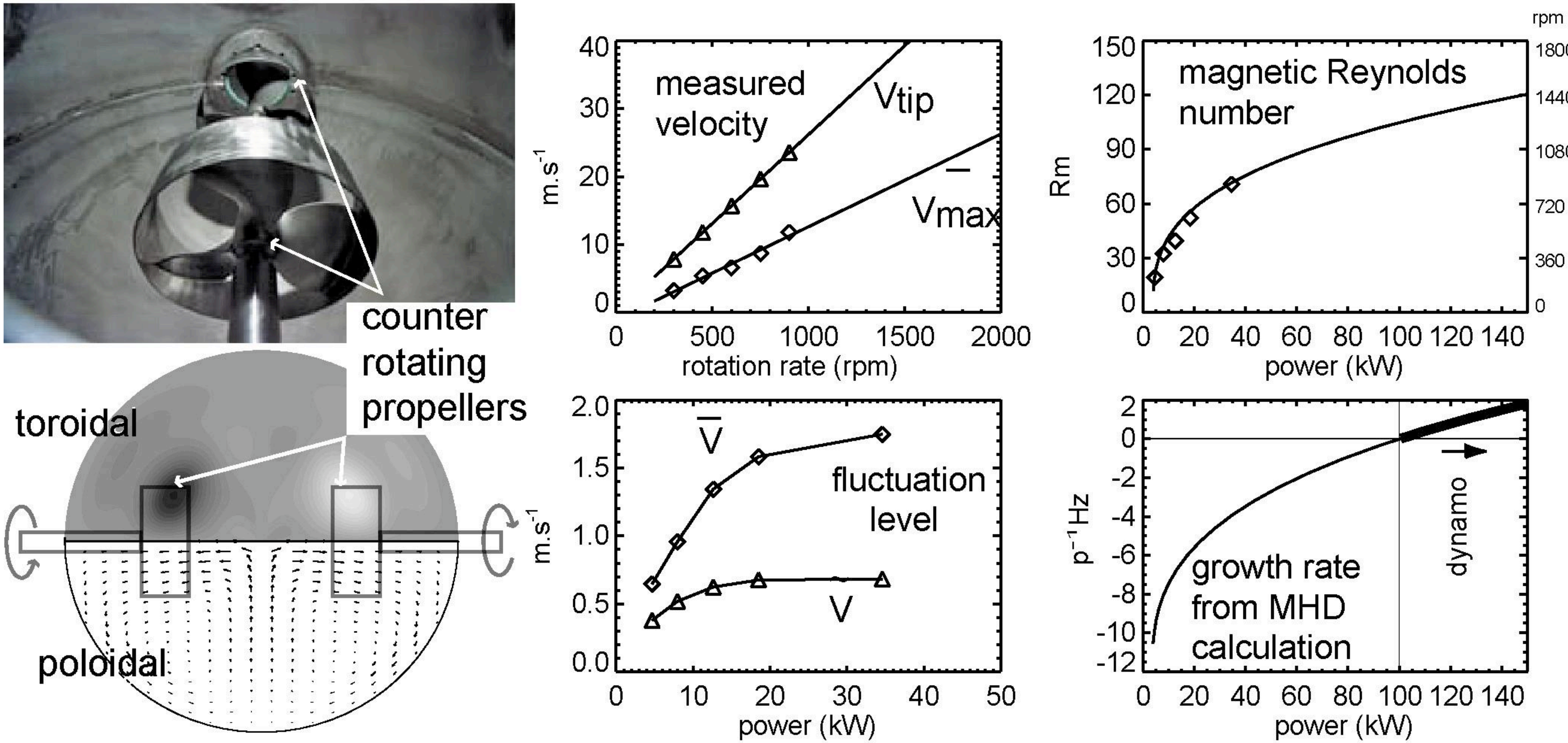


LDV measurements provide data for a reconstruction of the mean velocity field

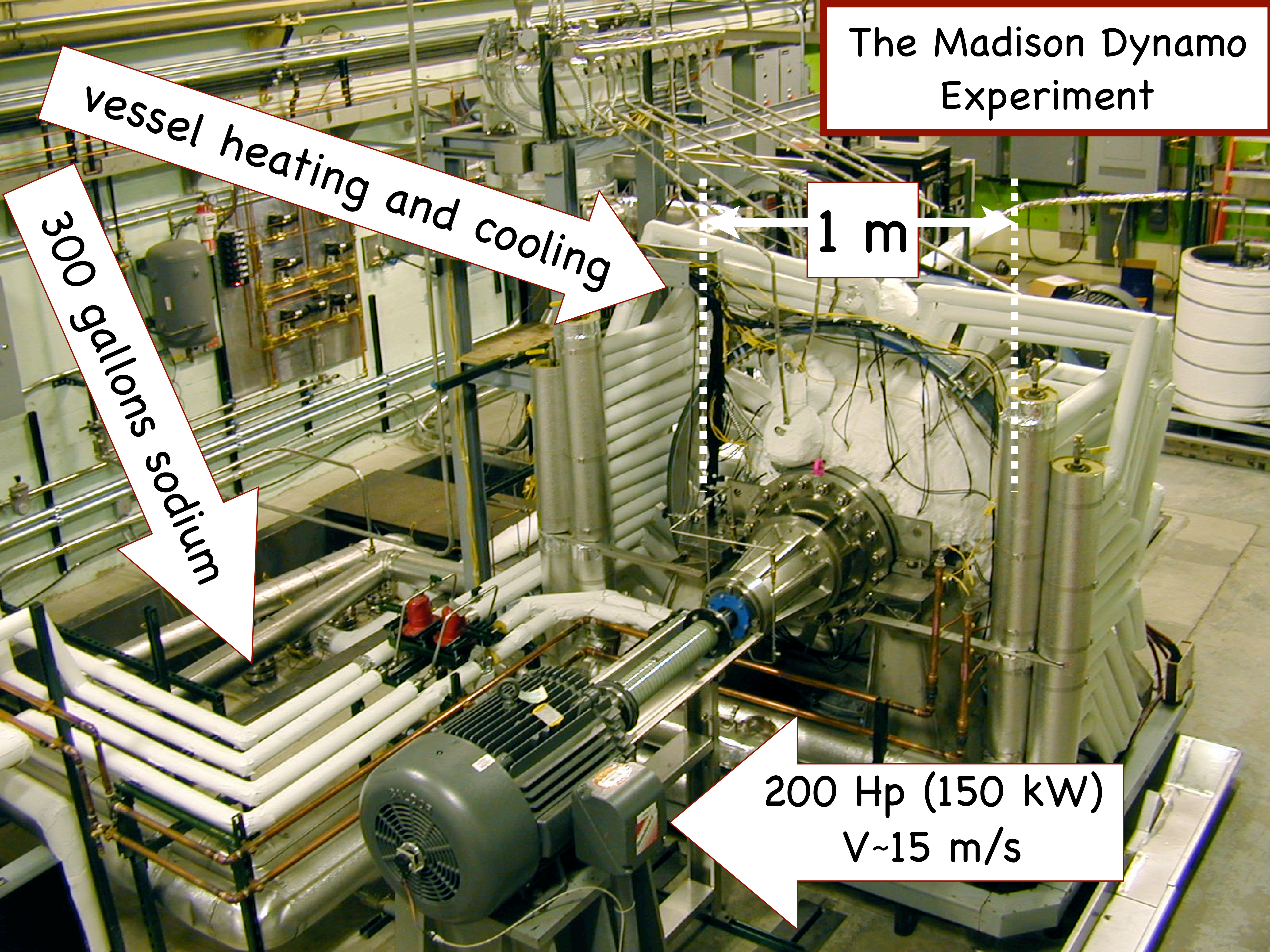




Velocity fields can be generated in water which lead to dynamo action

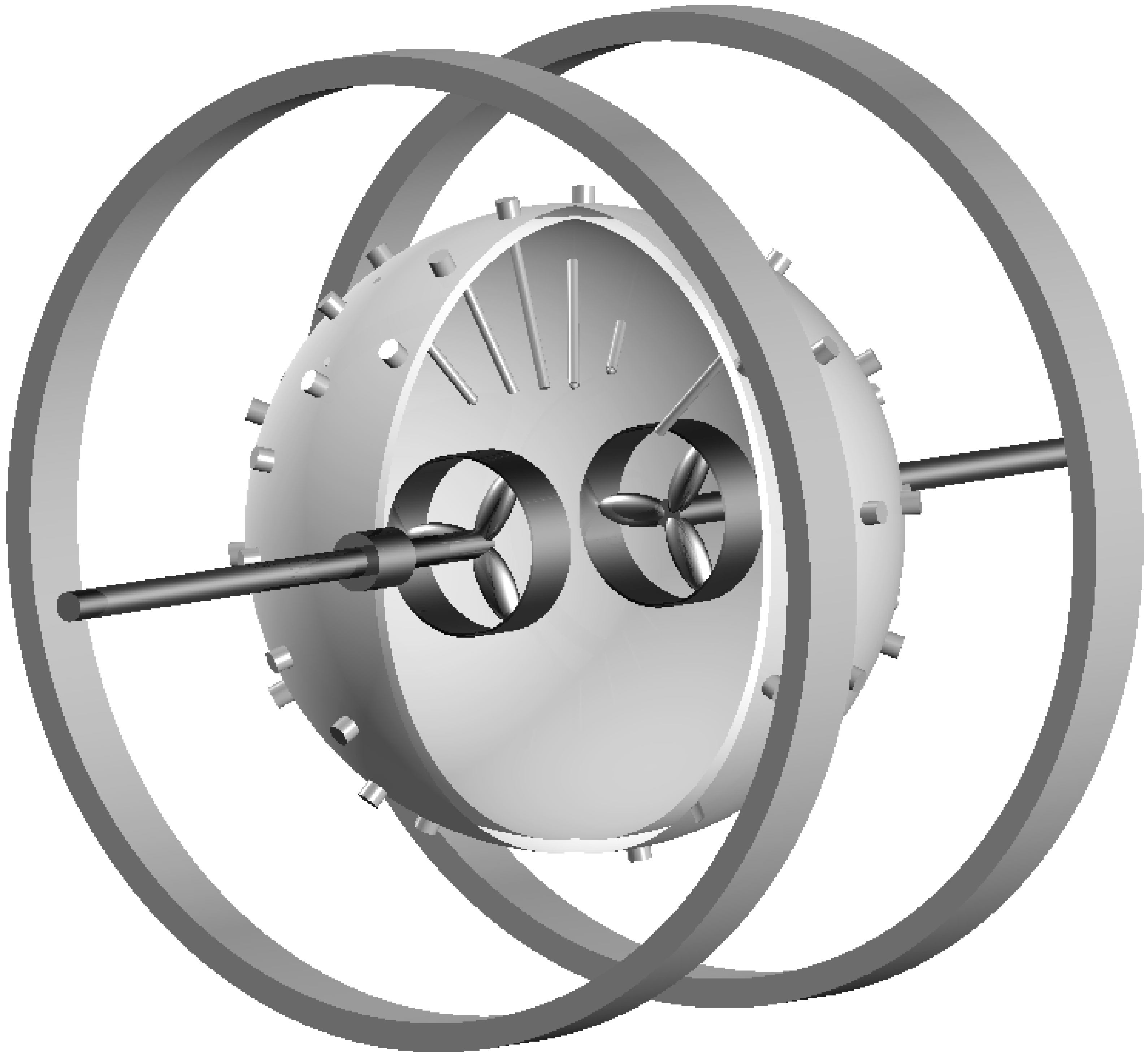


$a=0.5$ m, $\sigma=10^7$ mhos

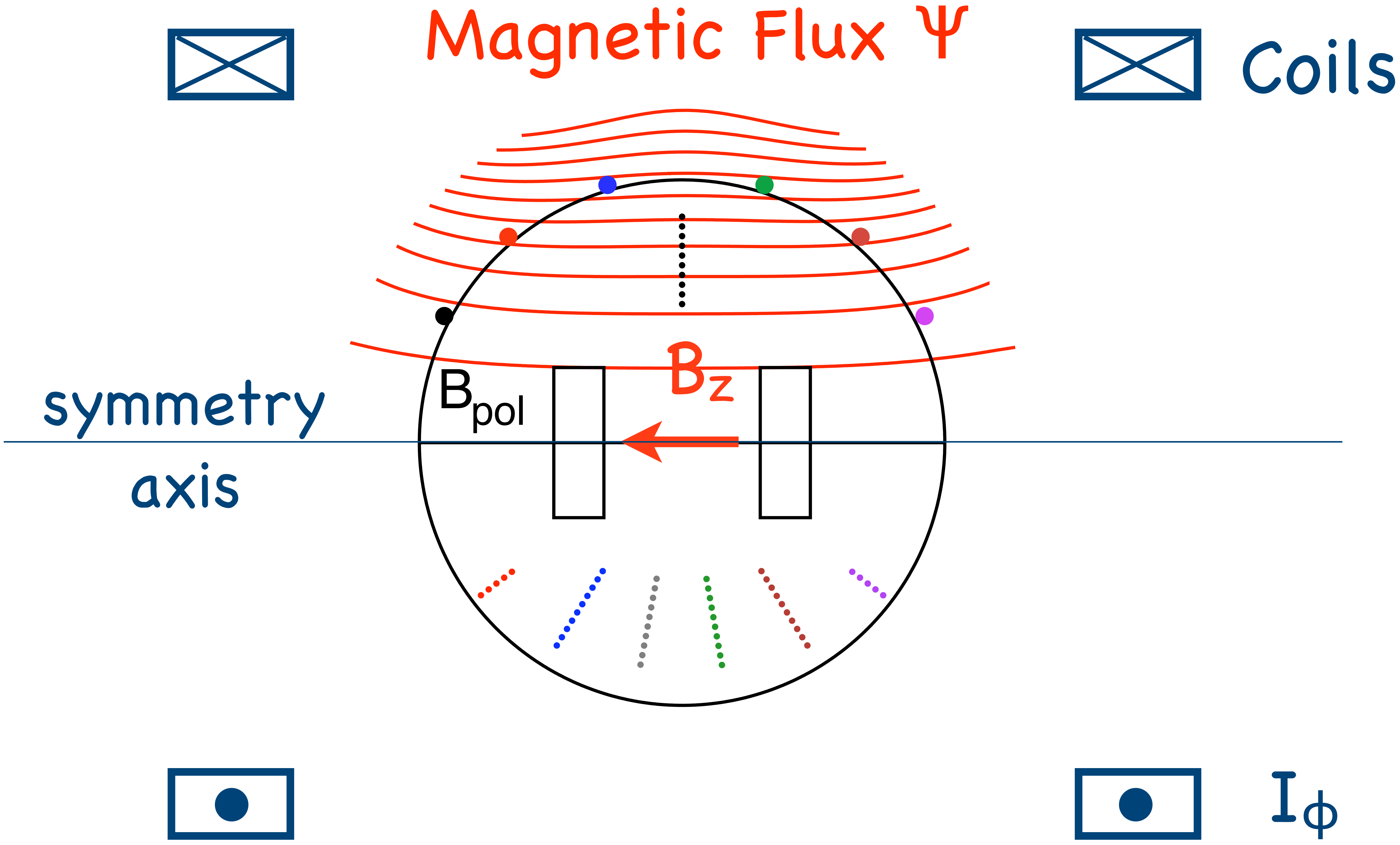

The Madison Dynamo Experiment
vessel heating and cooling
300 gallons sodium
1 m
200 Hp (150 kW)
V~15 m/s

Magnetic field is measured both internally and externally; external magnetic fields can be applied to probe experiment

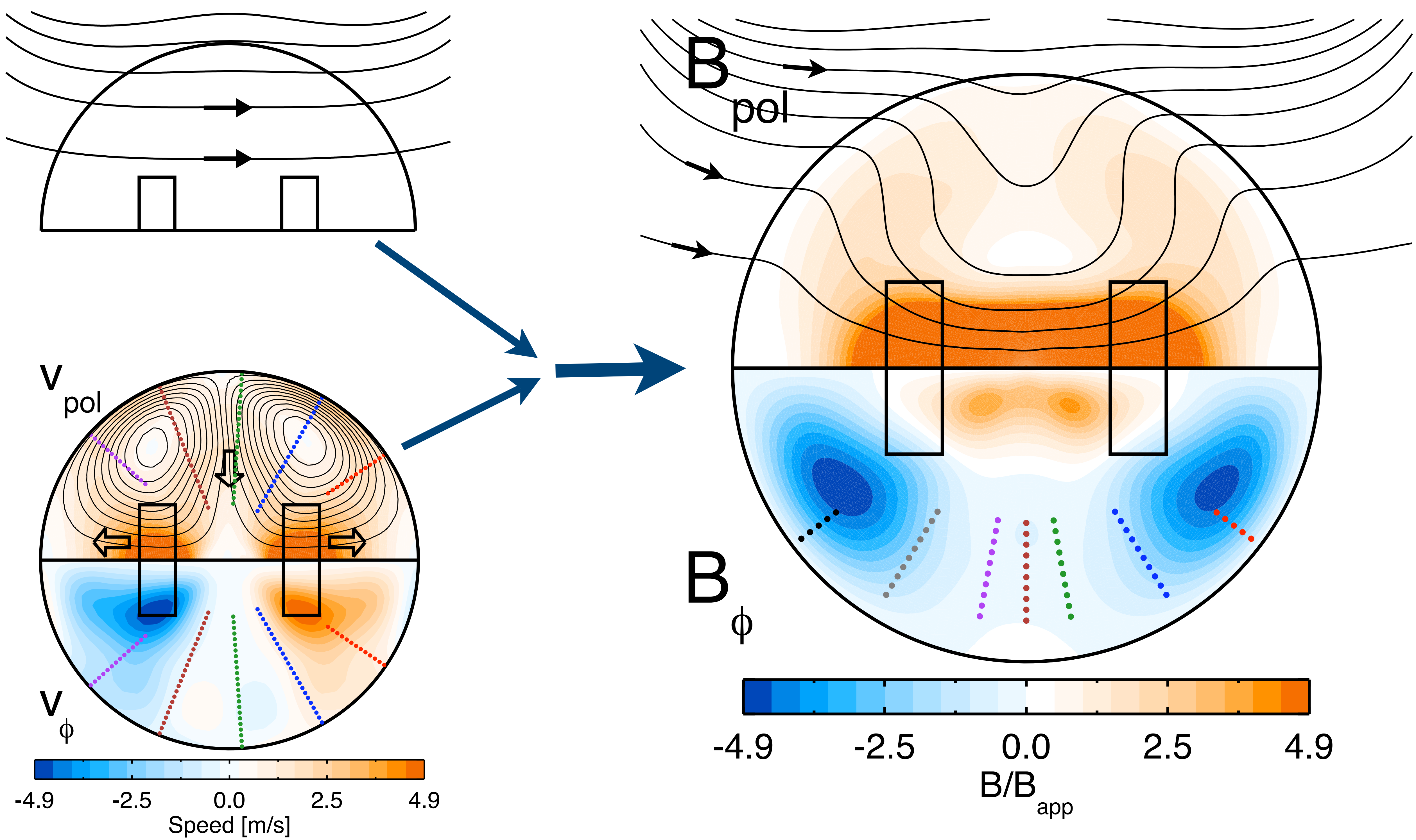
- $B_z \leq 100$ gauss
- Measure
  - surface probes
    - $B_r(a,\theta,\phi)$
    - Y_{lm} for $l \leq 6$, $|m| \leq 4$
  - Internal Probes
    - $B_\phi(r,\theta_p)$, 6 arrays
    - $B_z(r,\theta=\pi/2)$



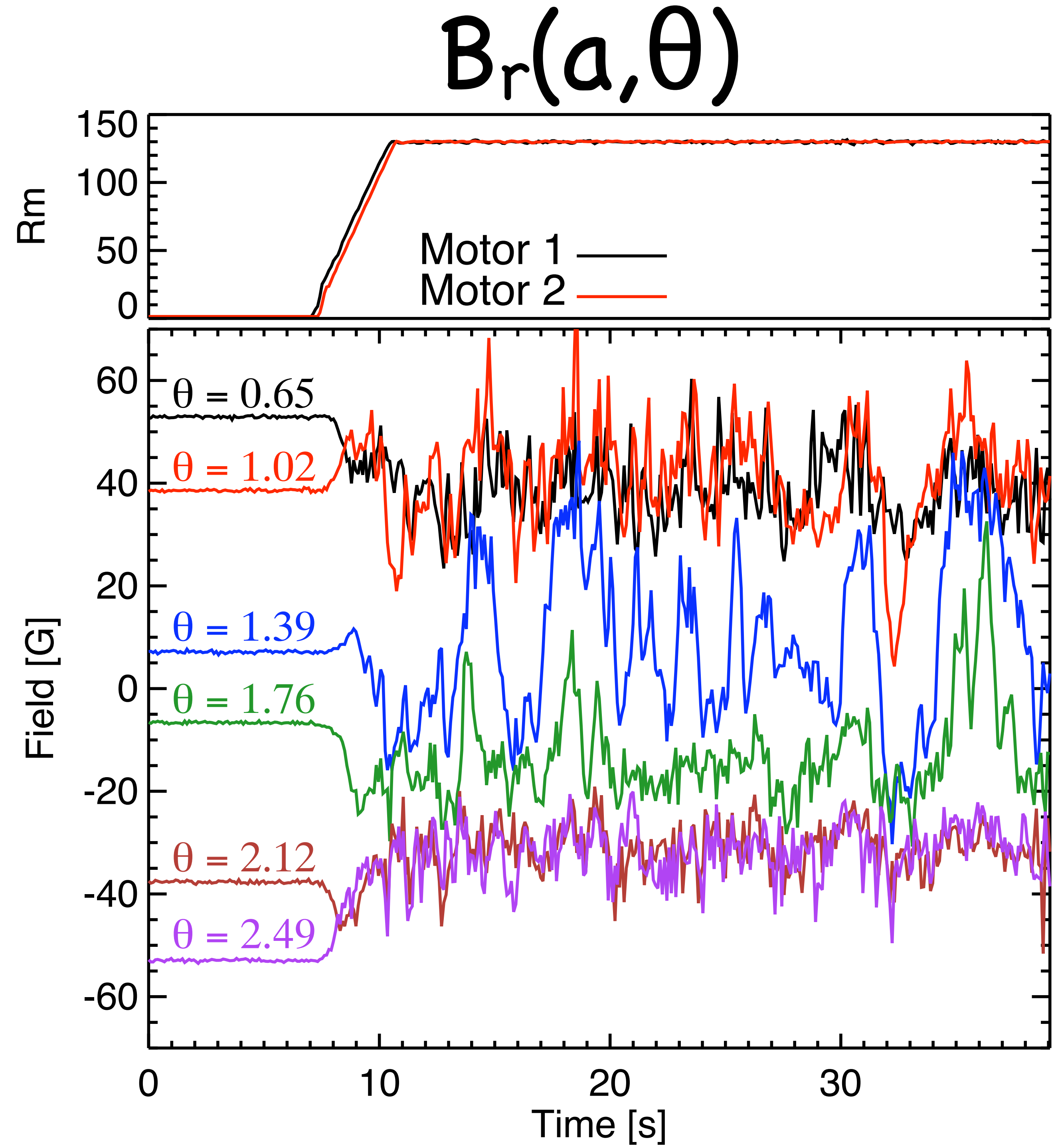
Experiment: apply axisymmetric poloidal seed field to sphere and measure induced magnetic fields

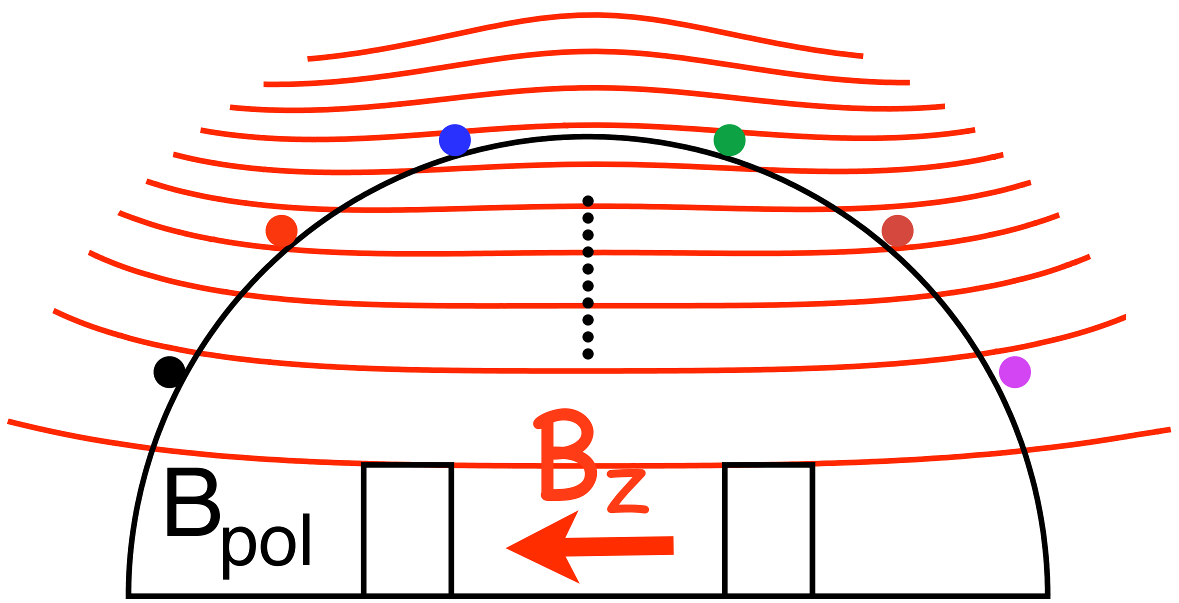


Predicted total magnetic fields

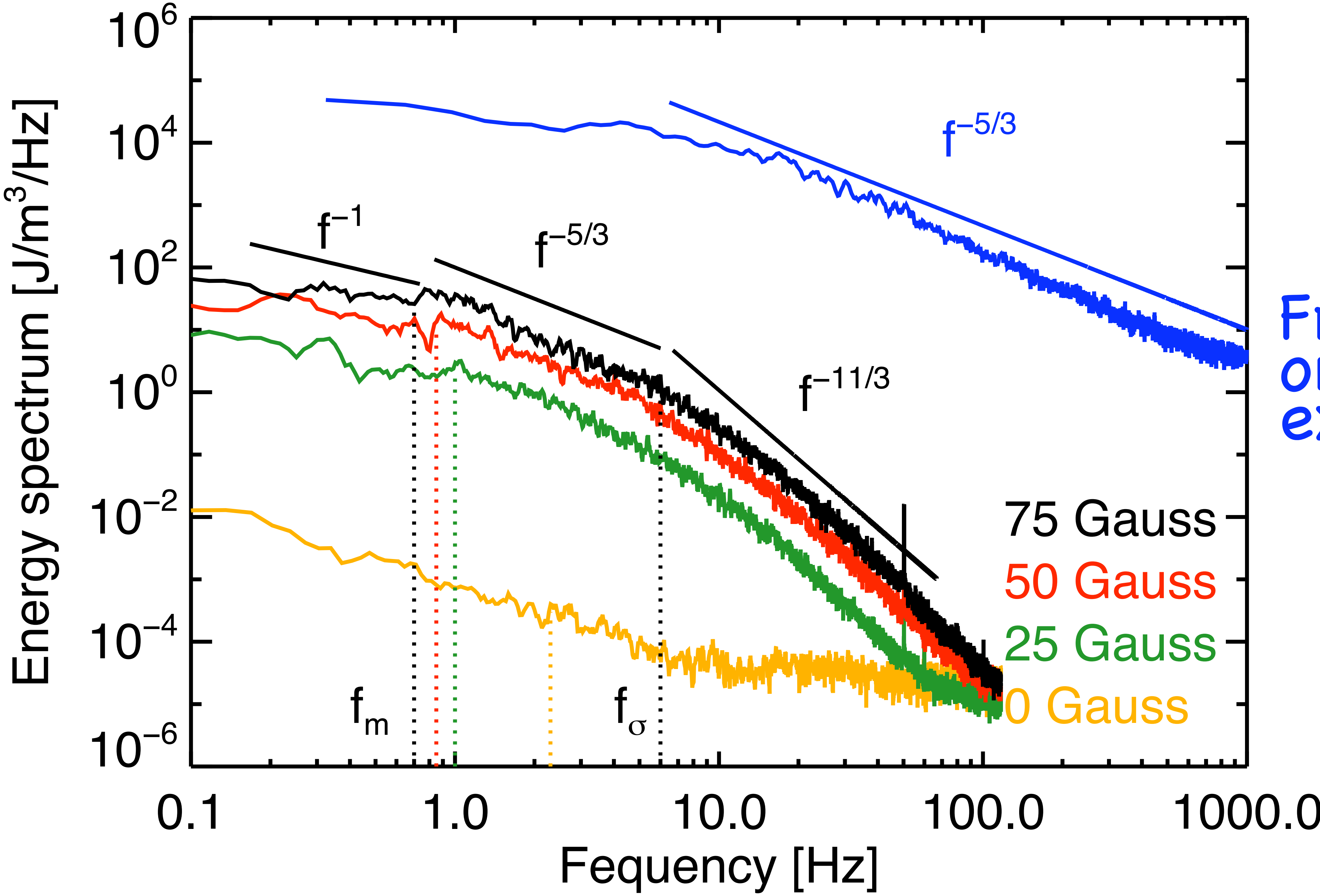


Large scale (mean) and small scale (turbulent) magnetic fields are generated by liquid sodium flows





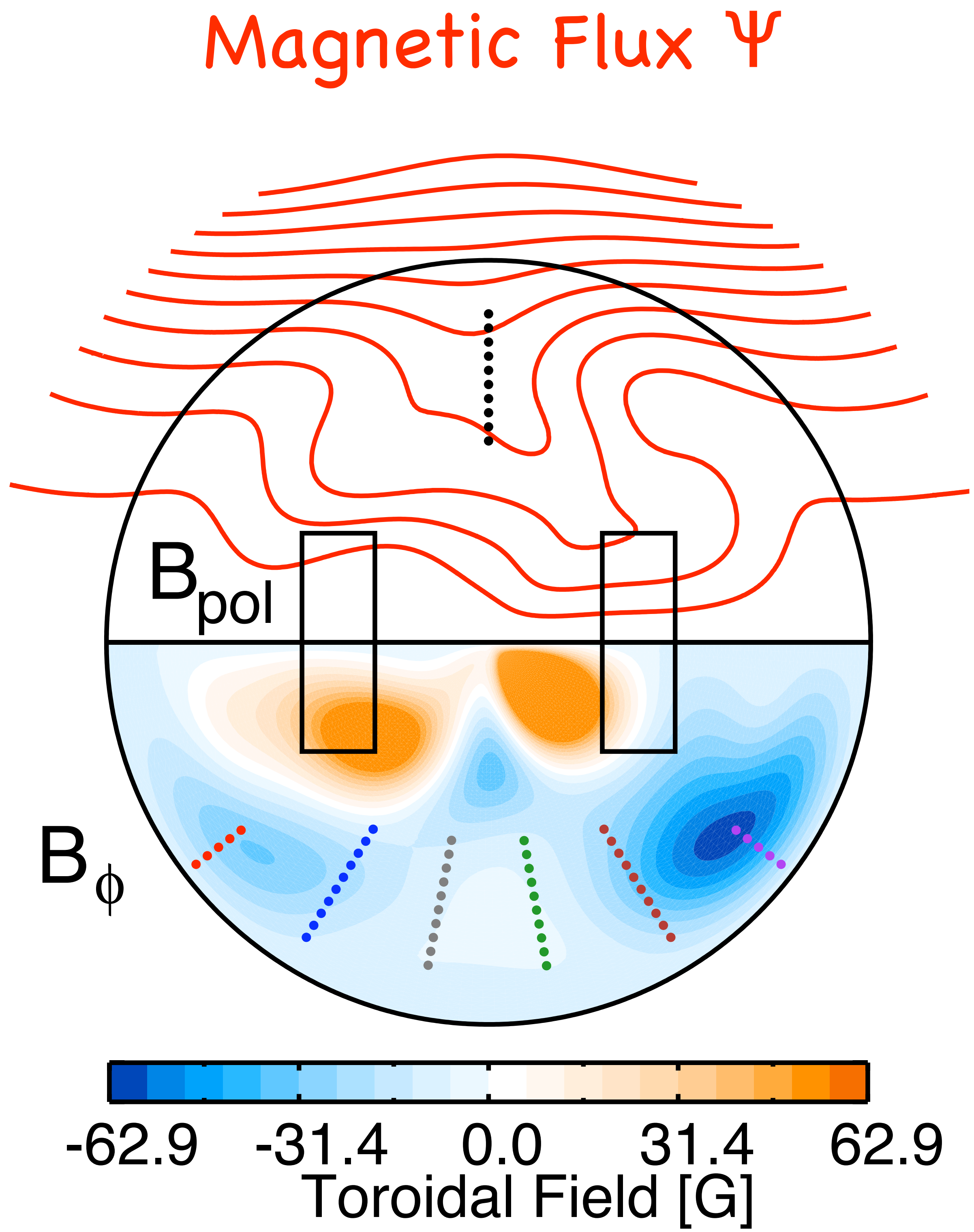
Spectra are turbulent: the turublent magnetic energy is much smaller than the kinetic energy



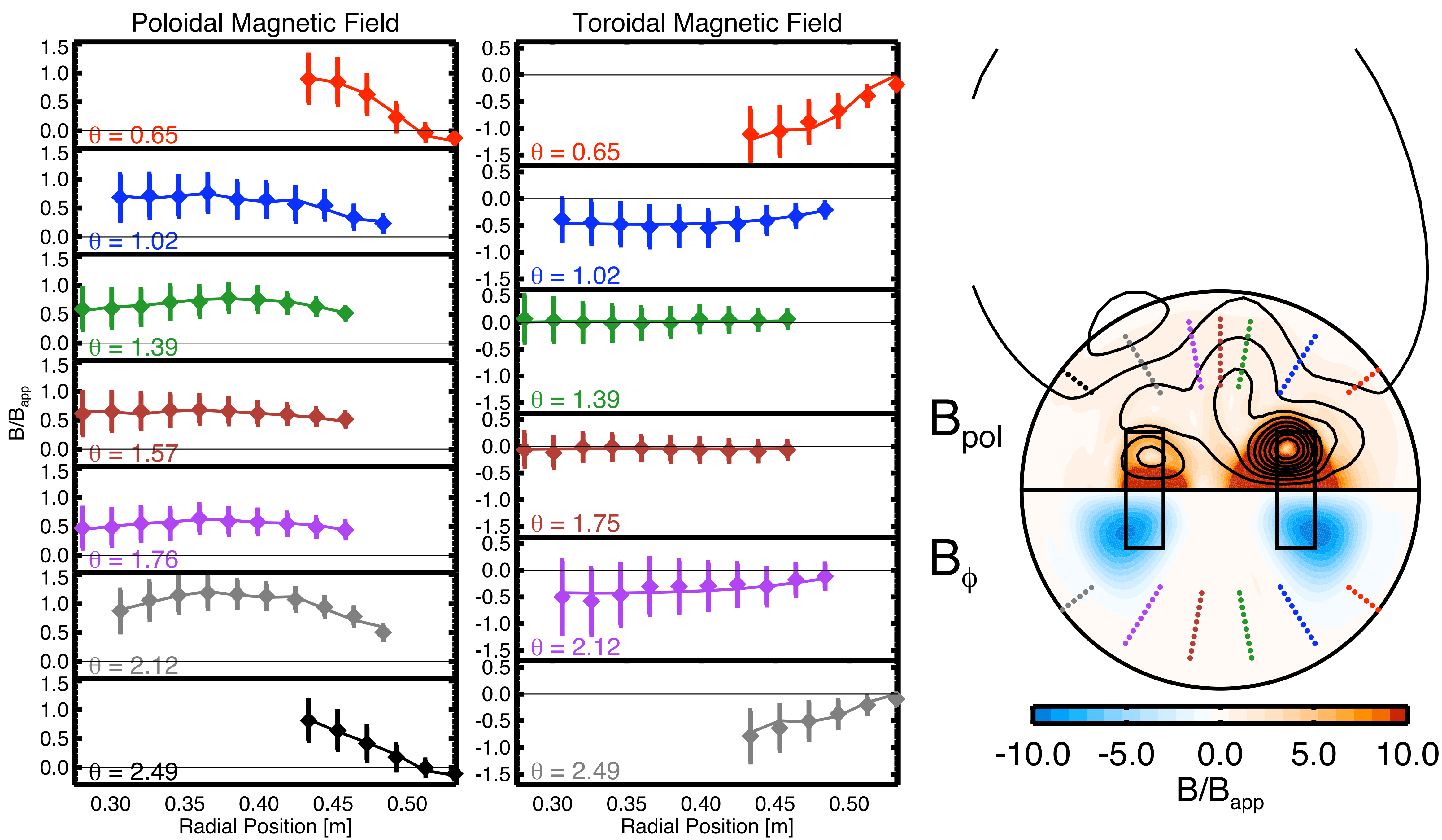
From LDV on water experiment

Nornberg, Spence, Bayliss, Kendrick, and Forest, *Measurements of the magnetic field induced by a turbulent flow of liquid metal,* Phys. Plasmas **13** 055901 (2006).

The time-averaged, axisymmetric part of the magnetic field shows poloidal flux expulsion and a strong Ω effect



Magnetic field is reconstructed from magnetic field measurements at discrete positions



Question: Does a simple Ohm's law make sense?

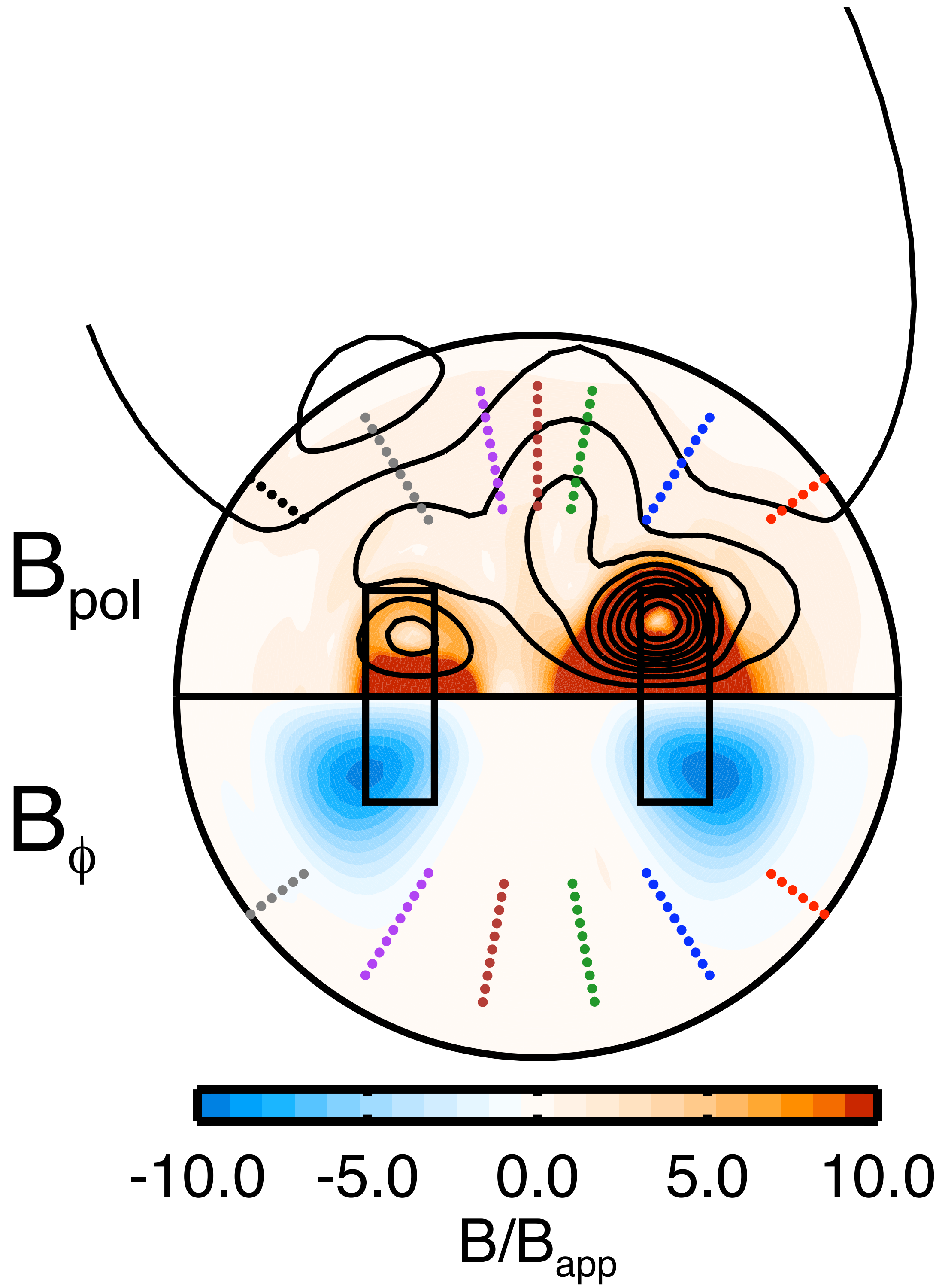
Measured by LDV

$$\langle \boldsymbol{J} \rangle = \sigma \left(\langle \boldsymbol{E} \rangle + \langle \boldsymbol{V} \rangle \times \langle \boldsymbol{B} \rangle + \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle \right)$$

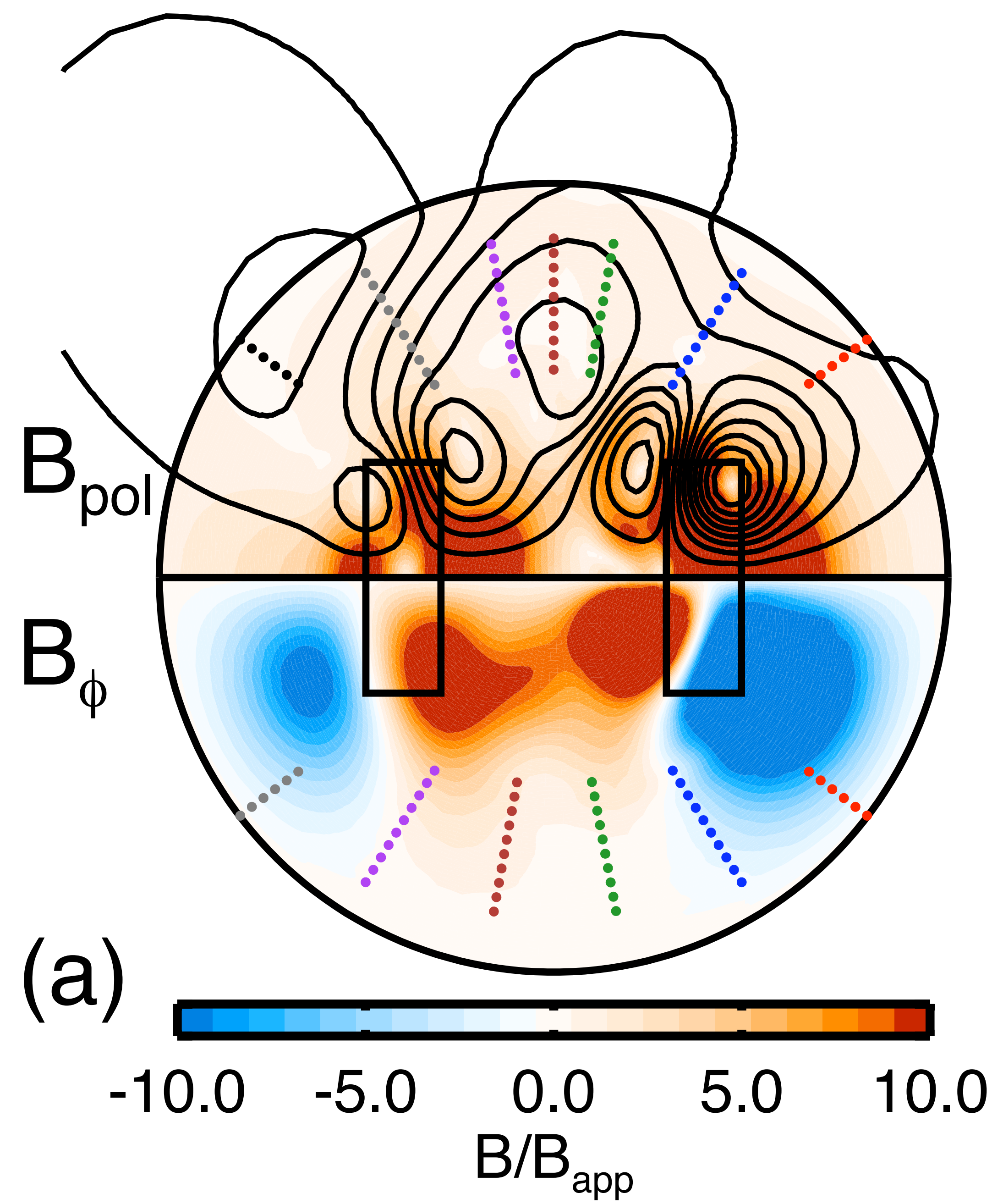
Time averaged current density generates measured <B>

Fluctuation driven

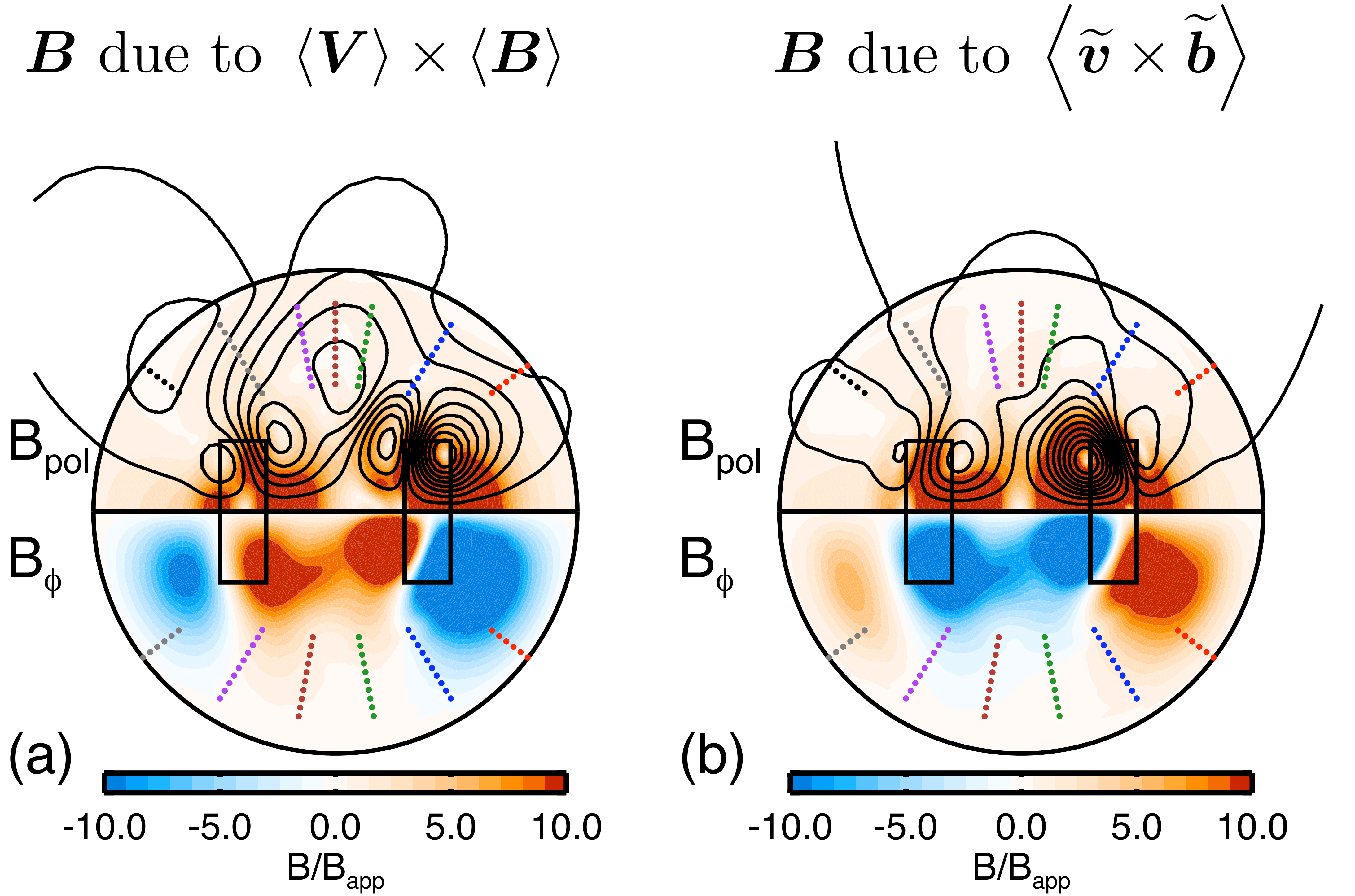
<V>x<B> does not account for measured field: turbulence must be generating current



$\boldsymbol{B}$ due to $\langle \boldsymbol{V} \rangle \times \langle \boldsymbol{B} \rangle$

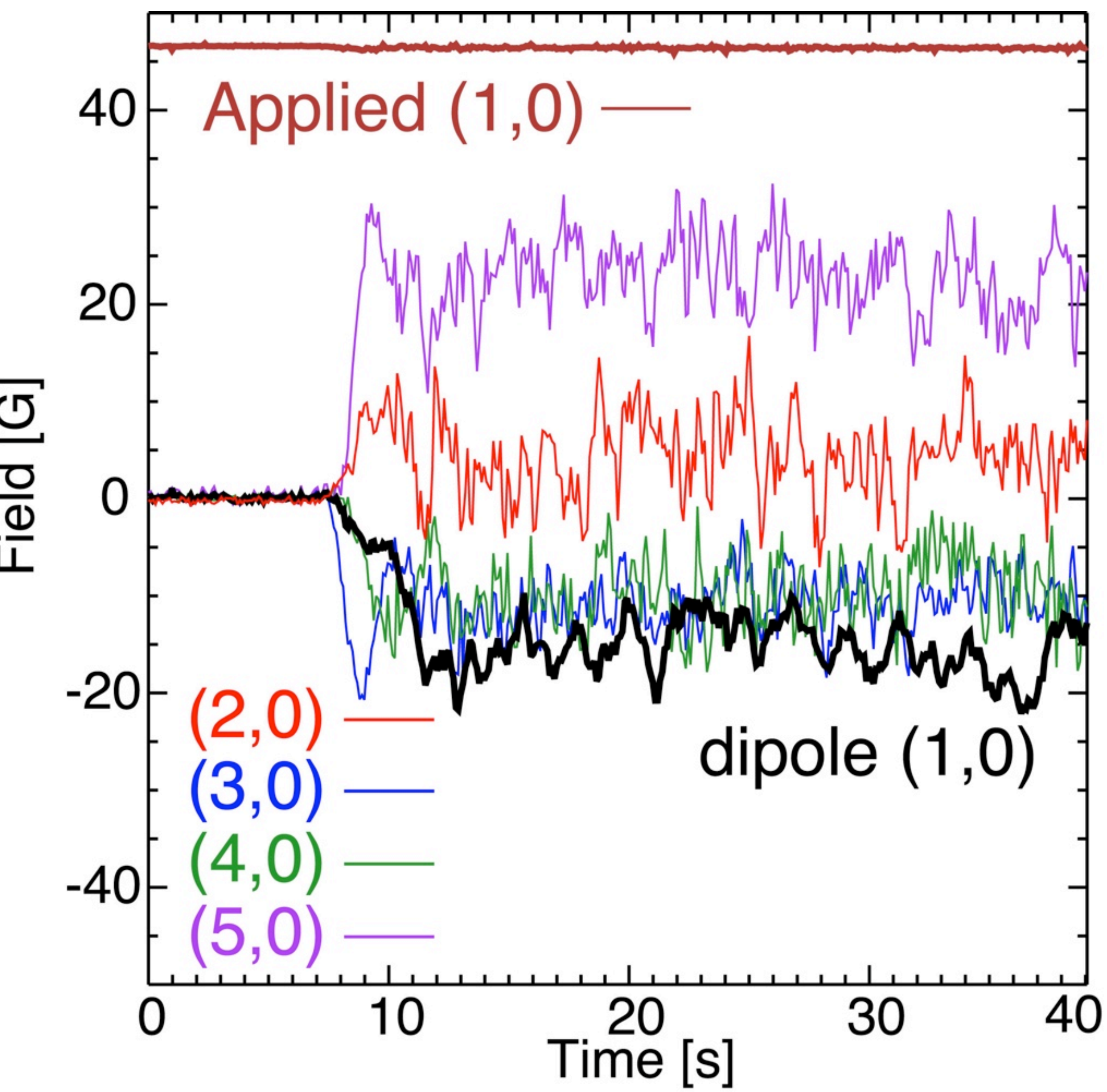


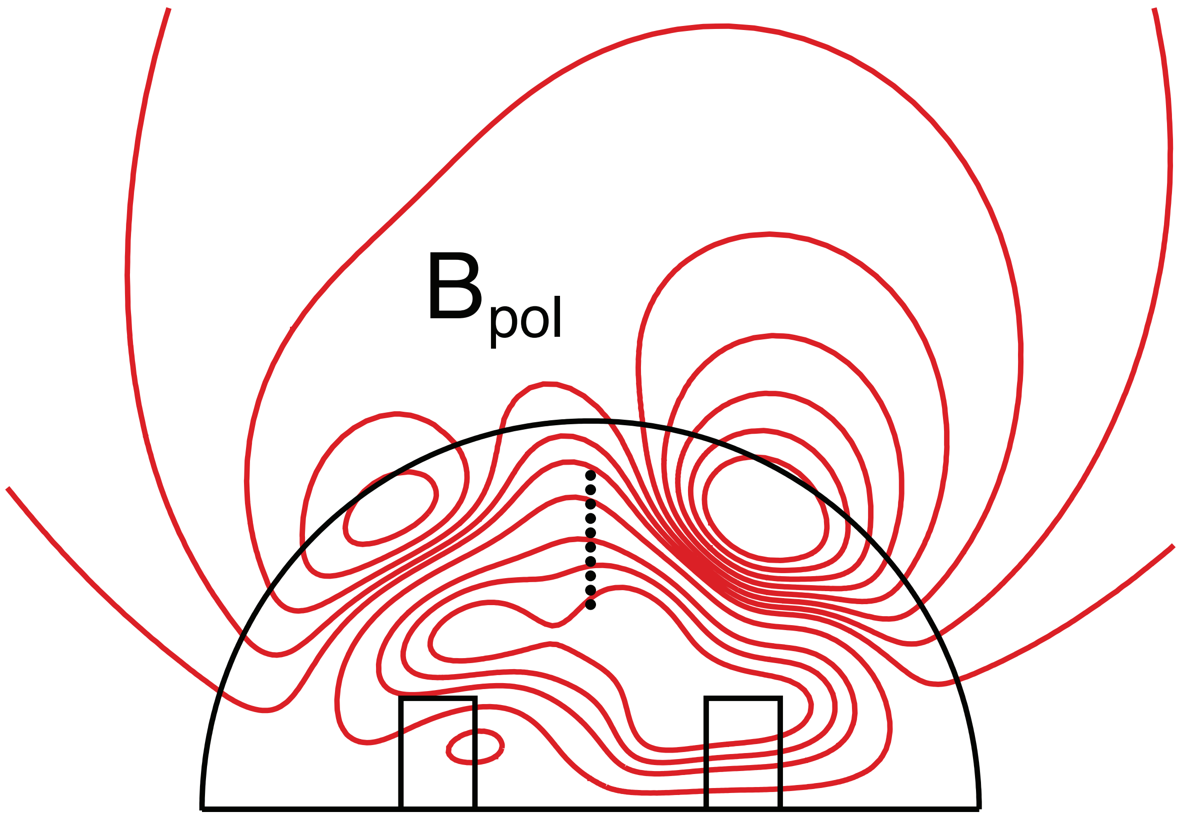
Field can be separated into mean-flow, mean-field driven currents and fluctuation generated currents



The mean induced magnetic field has a dipole moment

components of Y_{lm}

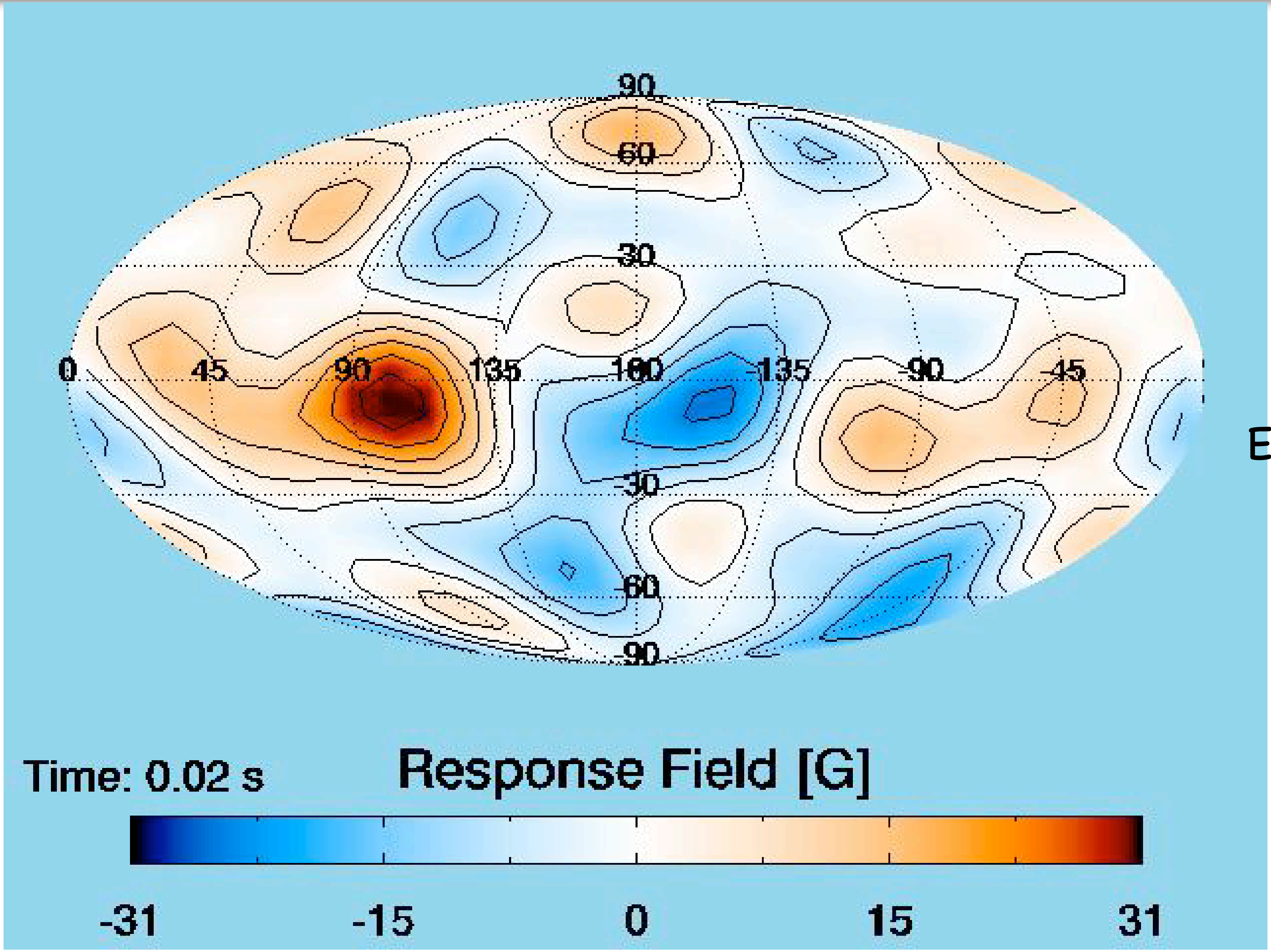




Impossible to reconstruct with axisymmetric flows!

Spence, Nornberg, Jacobson, Kendrick, and Forest, *Observation of a turbulence-induced large-scale magnetic field*, Phys. Rev. Lett. **96** 055002 (2006).

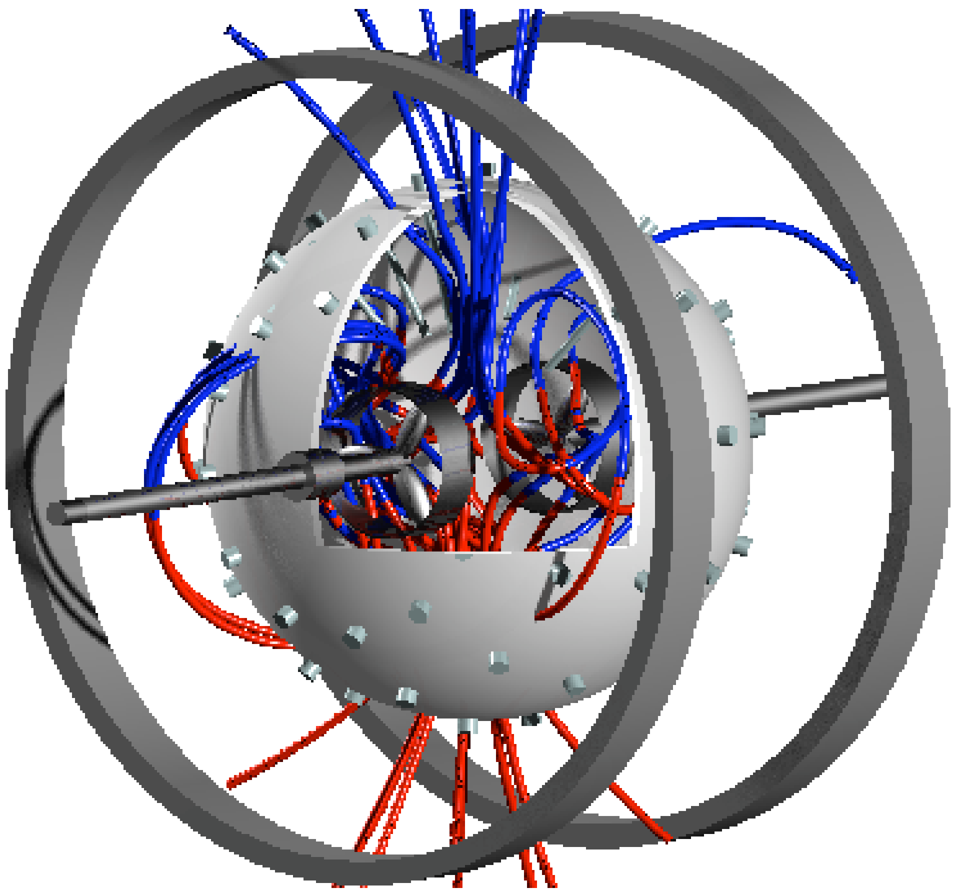
Intermittent equatorial dipole is observed on surface of sphere



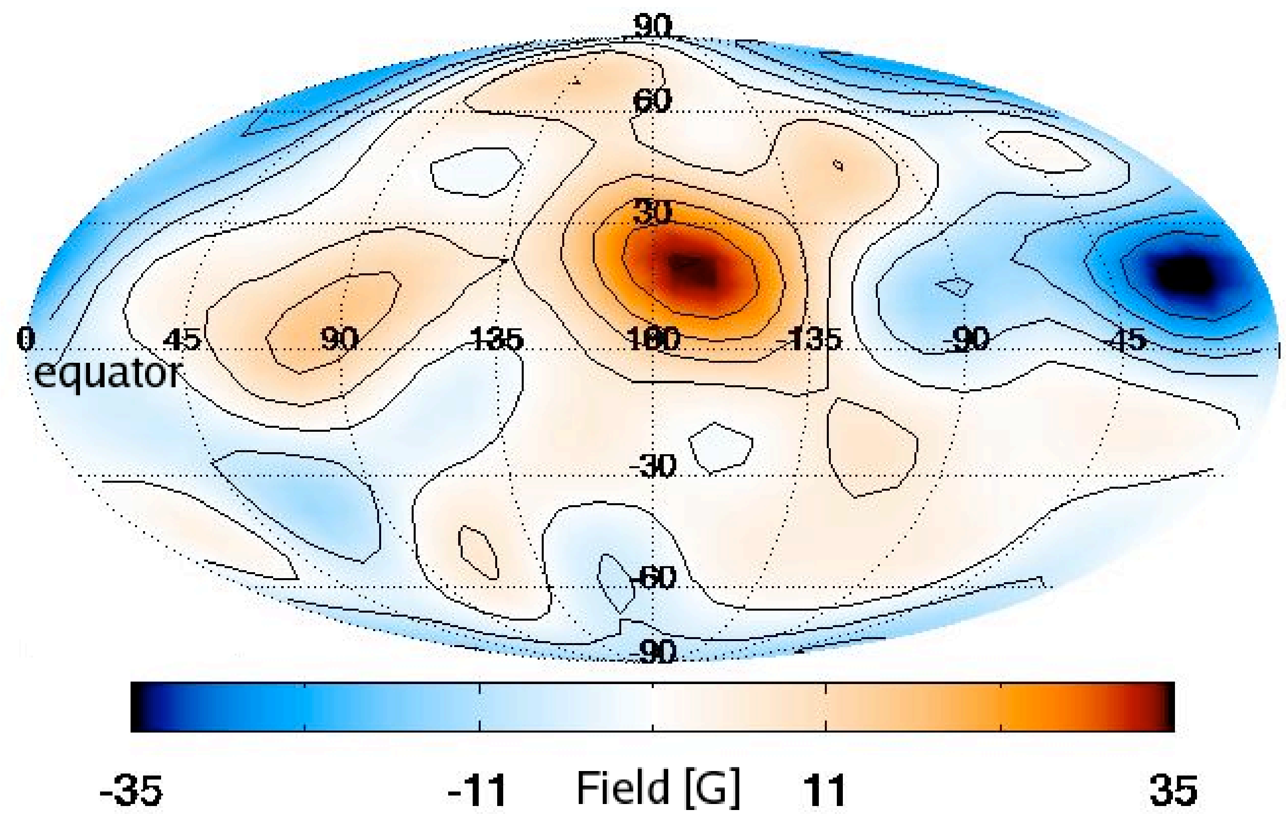
Equator

Excited eigenmode has structure similar to that predicted for the mean-flow, self-generated dynamo

Predicted



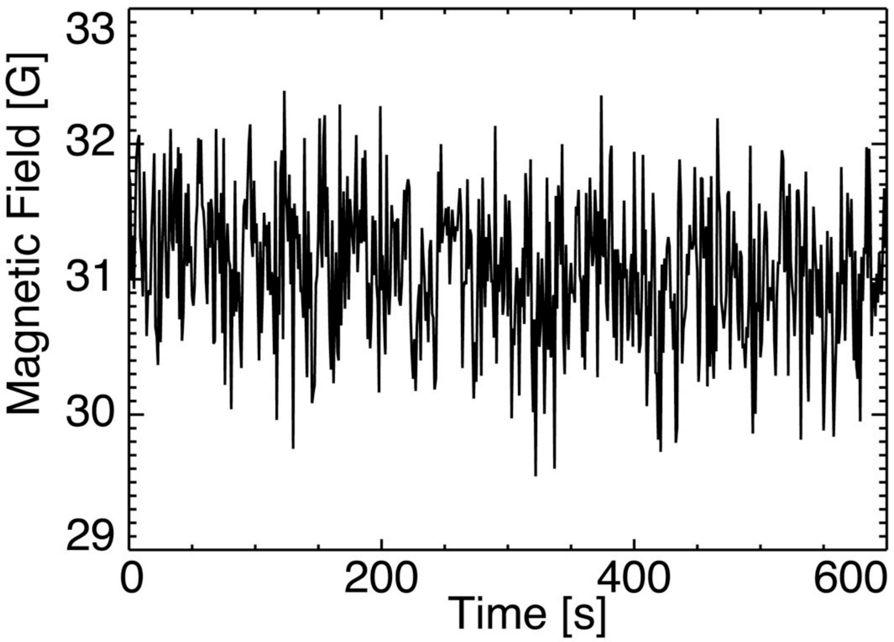
Observed

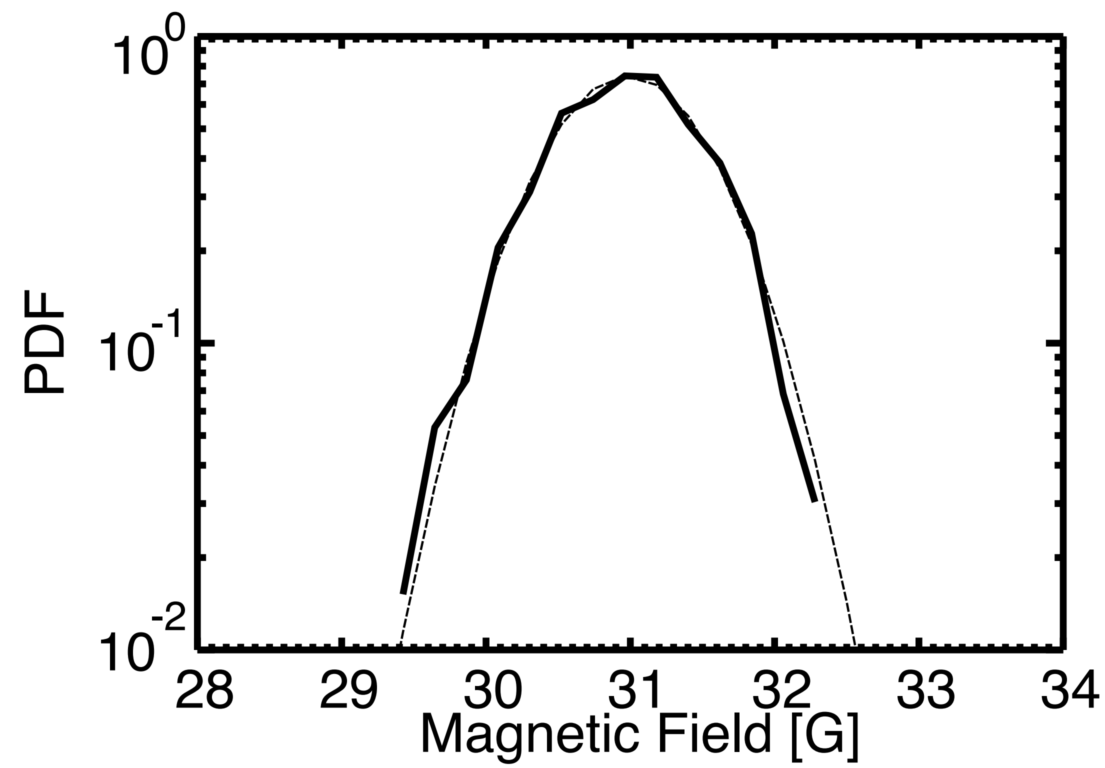


Nornberg, Spence, Jacobson, Kendrick, and Forest, *Intermittent magnetic field excitation by a turbulent flow of liquid sodium,* Phys. Rev. Lett. 97 044503 (2006).

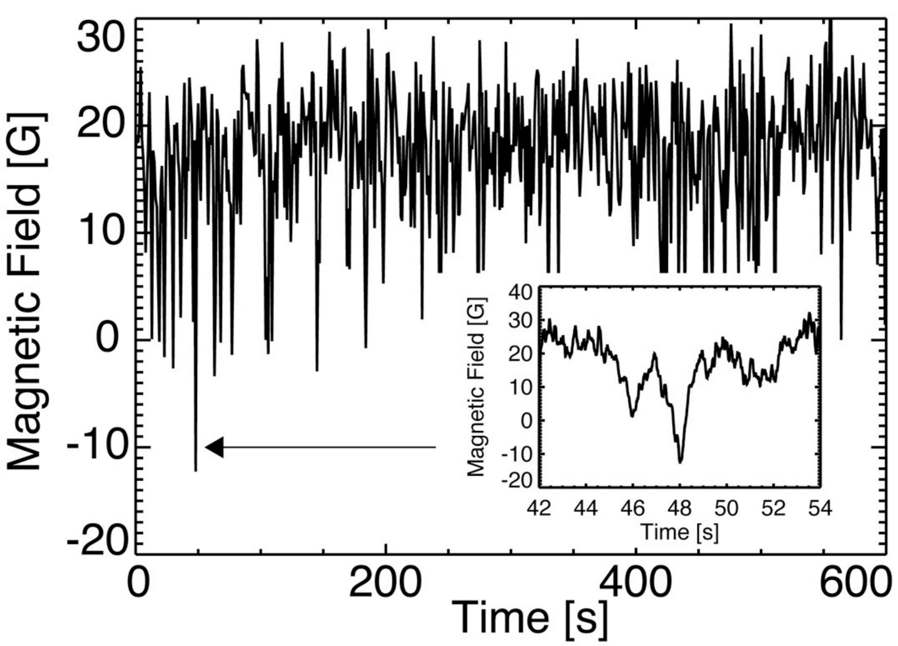
Surface magnetic field fluctuations have both normal distributions and intermittent characteristics

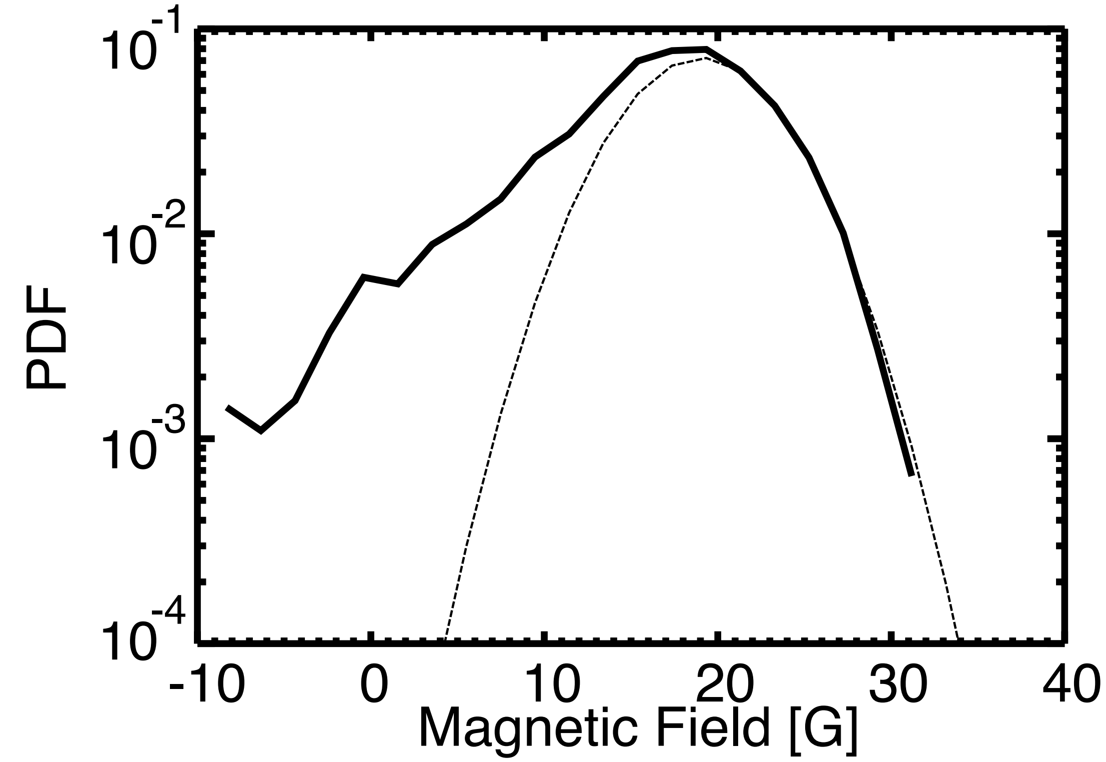
Near Poles



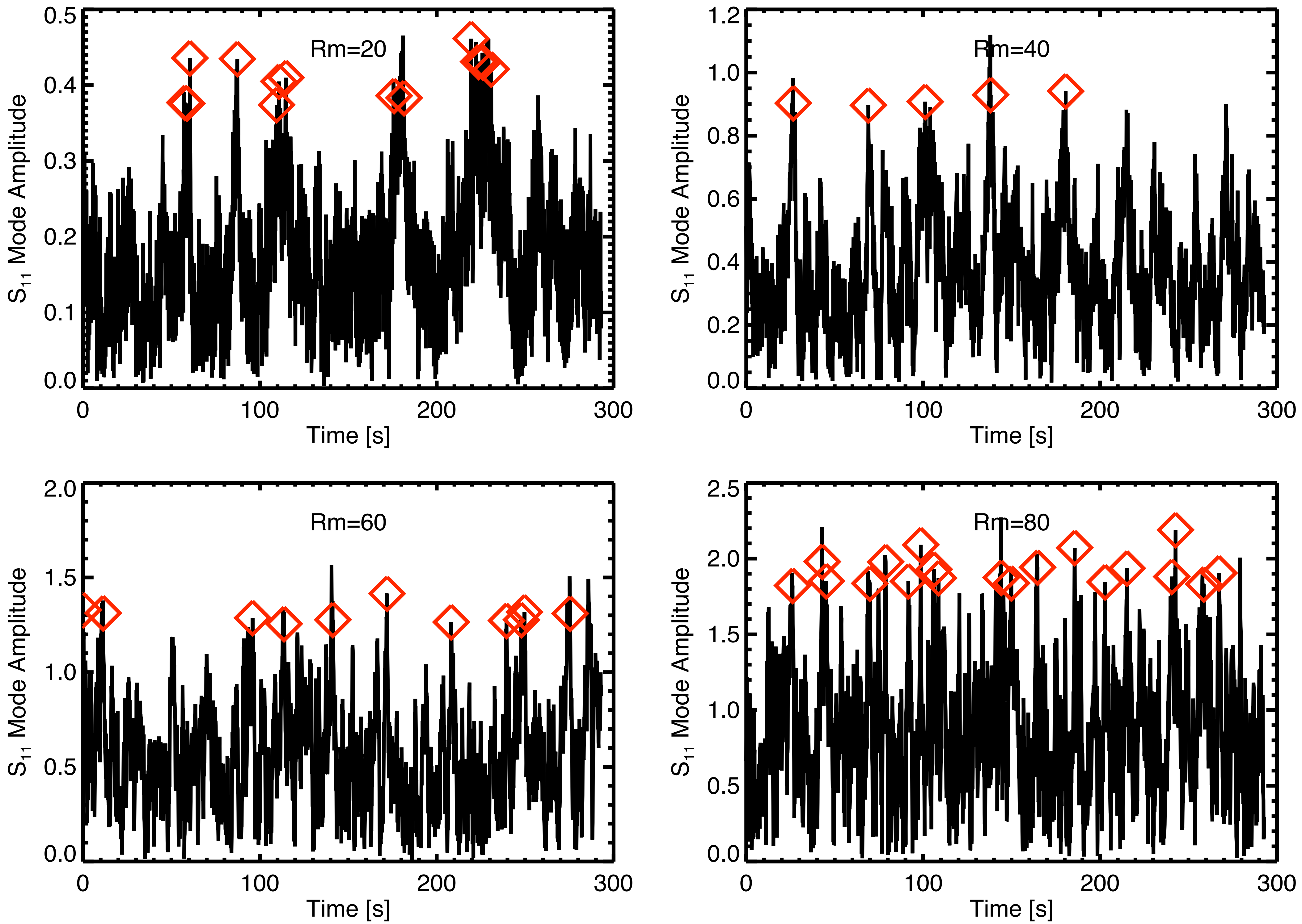


Near Equator

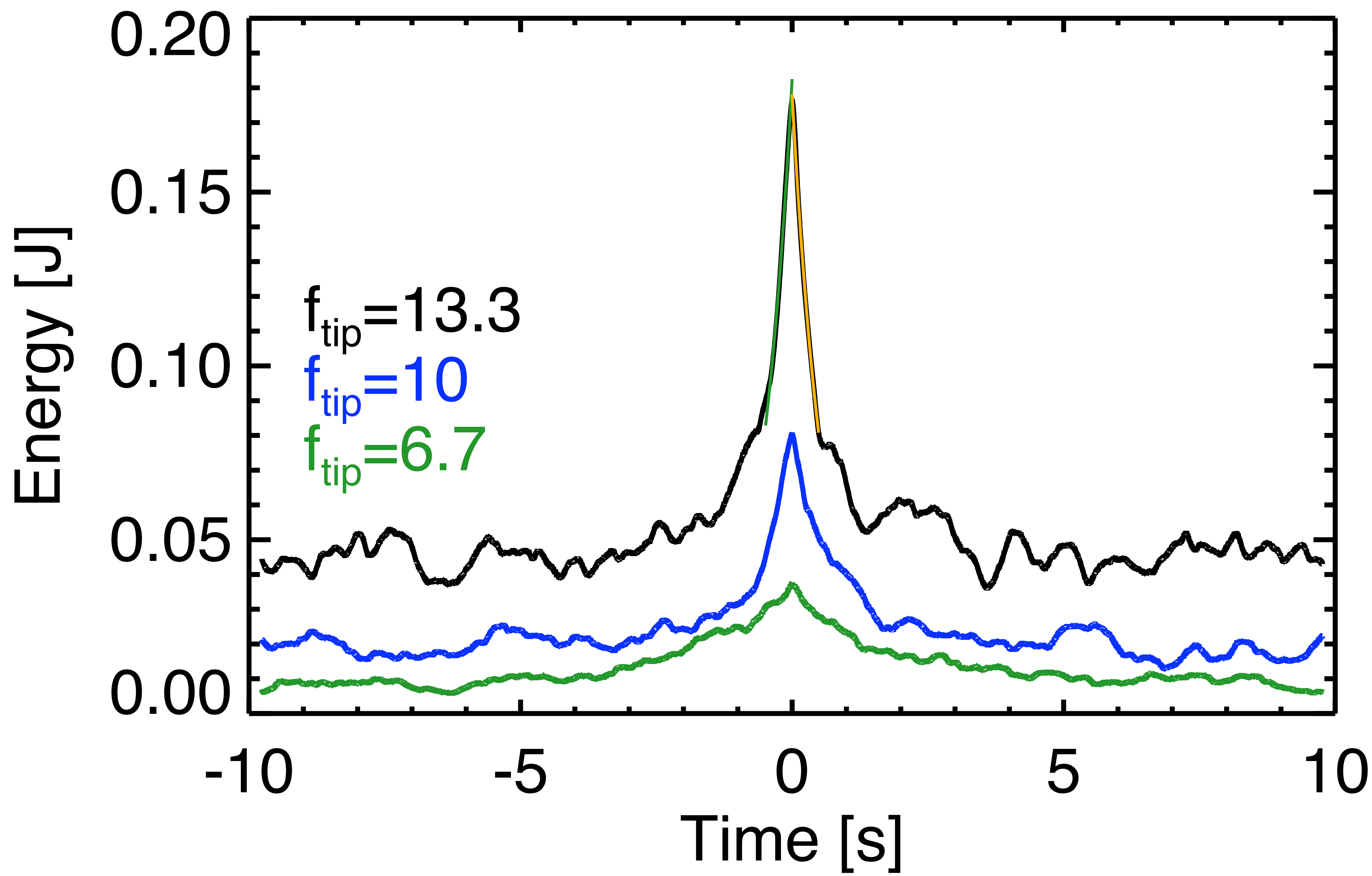




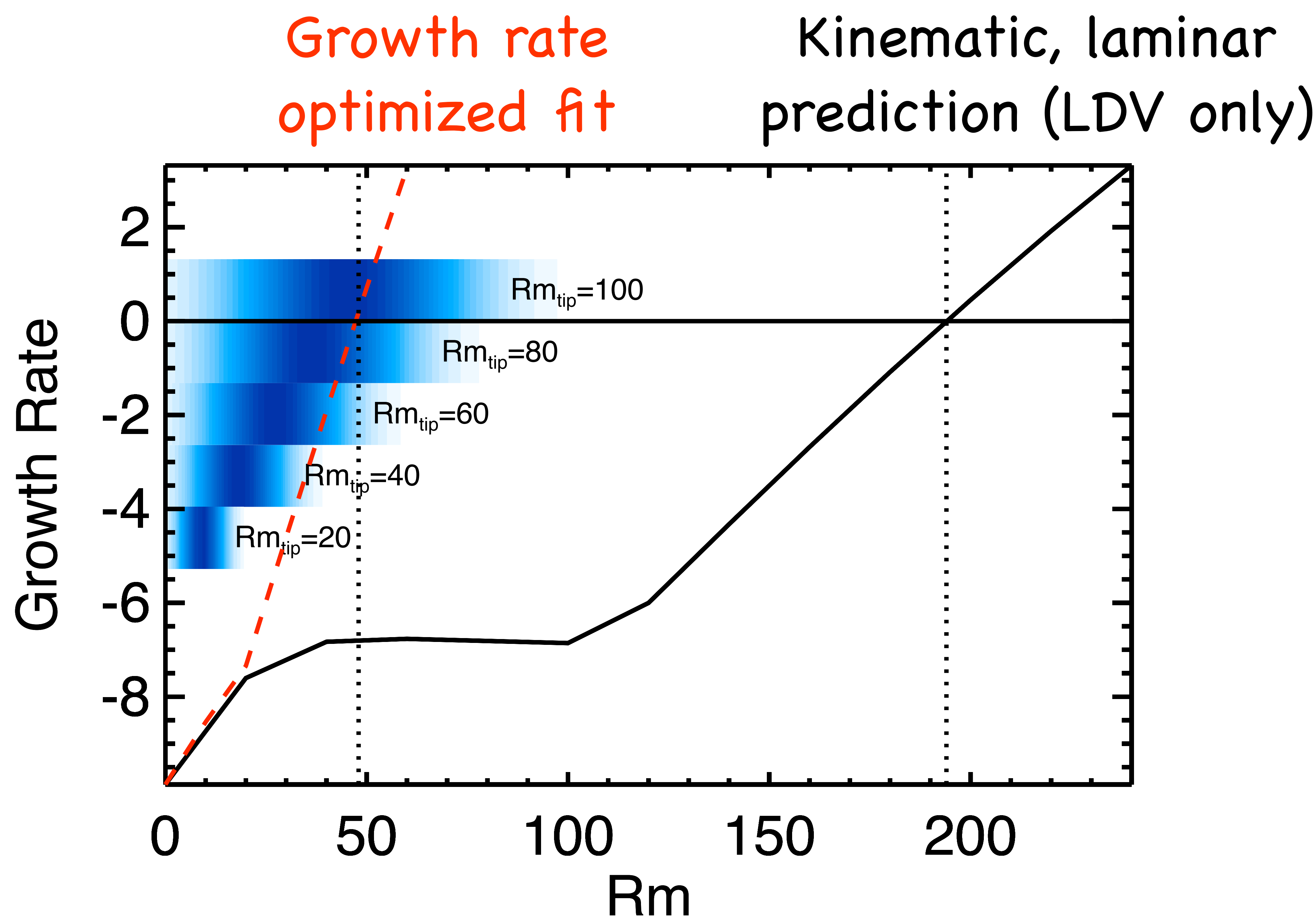
Intermittency varies with Rm



Conditional Averaging shows events get larger, and shorter with increasing Rm



Conjecture: Flow increasingly spends time as dynamo



Summary

- Liquid metal experiments are beginning to investigate self-exciting dynamos
  - constrained helical flows are dynamos
- Main Results from Madison Experiment
  - Dipole generation by turbulence
  - measurement of the magnetic field generated by fluctuations
  - Intermittent self-excitation

Theorem: For a stationary, axisymmetric flow and magnetic field, no dipole moment can exist for the current distribution inside the experiment (even with externally applied fields)

Use cylindrical coordinates (s, Z, ϕ) and stream functions for velocity and magnetic fields:

$$\vec{v} = \nabla\Phi \times \nabla\phi + v_\phi \hat{\phi} \quad (1)$$

$$\vec{B} = \nabla\Psi \times \nabla\phi + B_\phi \hat{\phi} \quad (2)$$

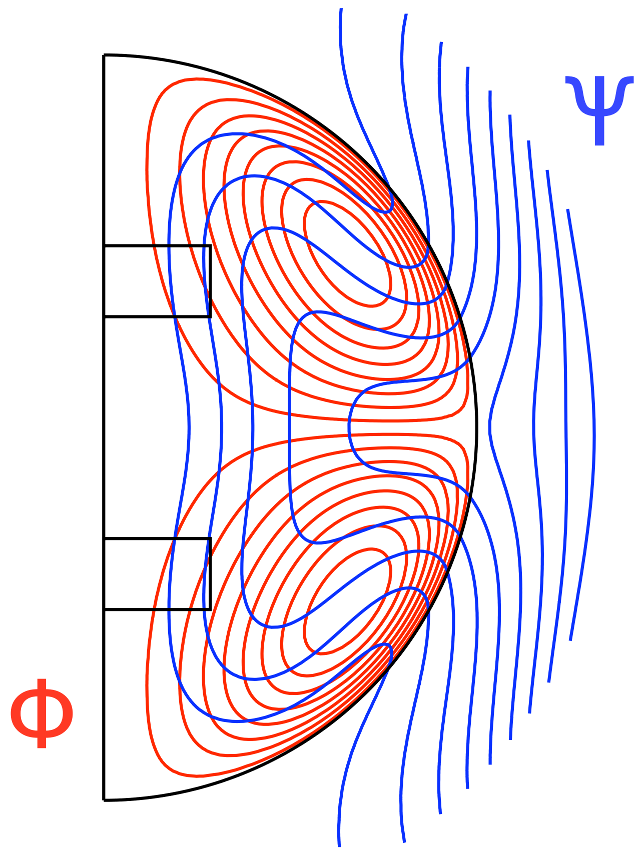
The dipole moment $\mu_z = \int sJ_\phi d^3x$ is generated by toroidal currents:

$$\begin{aligned} J_\phi &= \sigma \vec{v} \times \vec{B} \cdot \hat{\phi} && (3) \\ &= \sigma \frac{|\nabla\Phi \times \nabla\Psi|}{s^2} && (4) \end{aligned}$$

Switching to flux coordinates (Ψ, ℓ) where $d^3x = \frac{d\ell d\Psi}{B_p}$, the dipole becomes

$$\begin{aligned} \mu_z &= \sigma \int\int |\nabla\Phi \times \nabla\Psi| \frac{d\ell d\Psi}{sB_p} && (5) \\ &= \sigma \int d\Psi \int \frac{\partial\Phi}{\partial\ell} d\ell \equiv 0 && (6) \end{aligned}$$

Proof continued



Integrating Φ along open poloidal flux contours gives

$$\int_a^b \frac{\partial \Phi}{\partial \ell} d\ell = \Phi(b) - \Phi(a) = 0$$

since vessel boundary had $\Phi = const.$ Closed poloidal flux contours give

$$\oint \frac{\partial \Phi}{\partial \ell} d\ell \equiv 0$$

Therefore, $\mu_z = 0$ for axisymmetric flows. QED

- Conclusion: symmetry breaking fluctuations must be responsible for observed dipole
  - consistent with an α-effect and the self-generated toroidal field: $J_\varphi = \sigma \alpha B_\varphi$