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Direct observation of Anderson localization of matter waves in a controlled disorder

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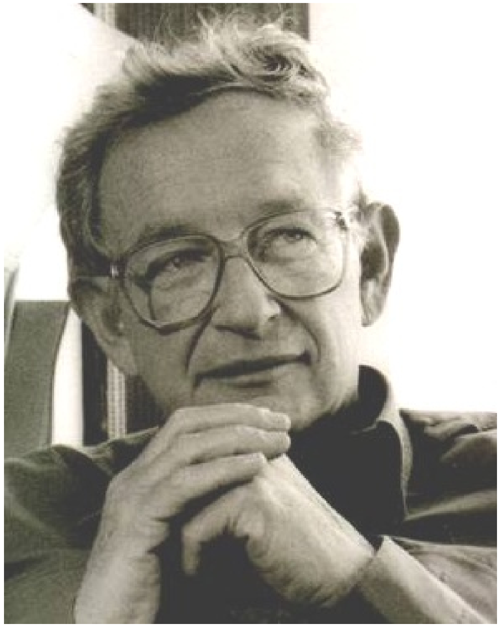

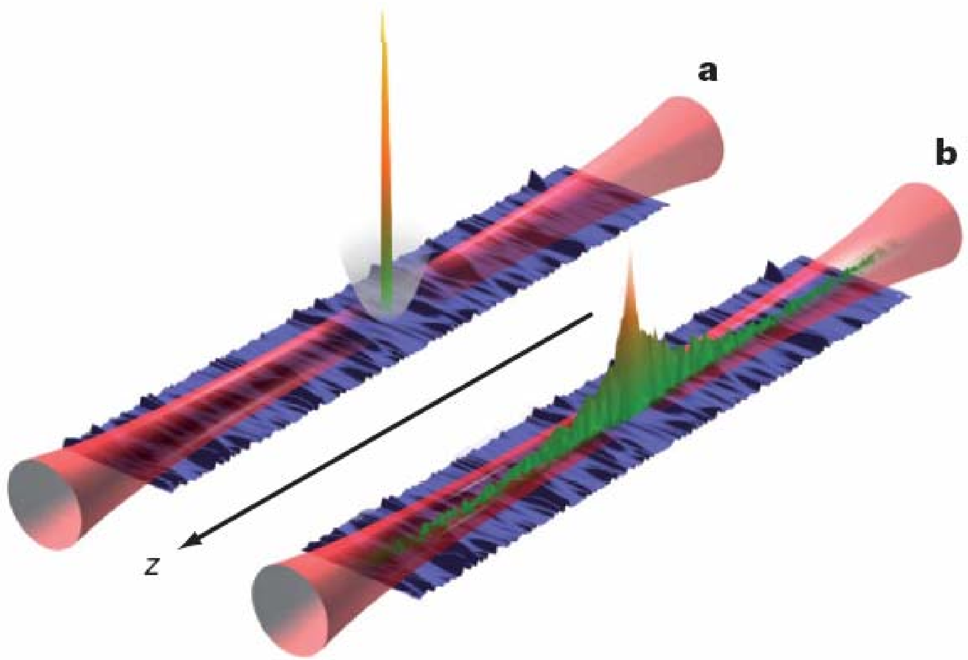

Rockson Chang Group meeting Jan 28th, 2009

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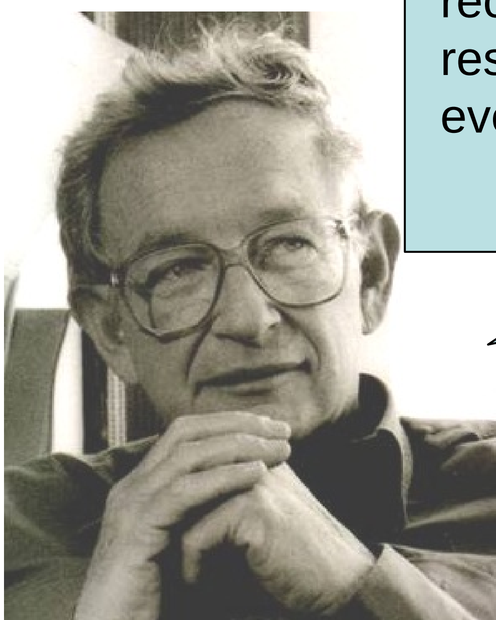
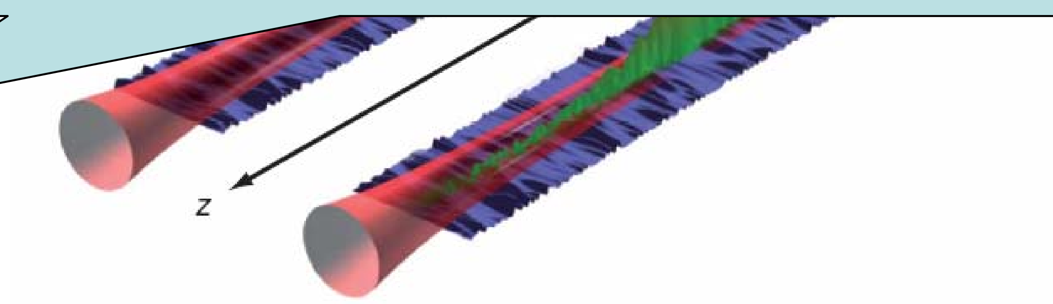
"Localization [..], very few believed it at the time, and even fewer saw its importance, among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it."

P.W. Anderson, Nobel Lecture, 1977





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Overview

1) Anderson localization
2) *J. Billy et. al.* experimental setup (basic)
3) Theory
    1) Description of speckle pattern
    2) Expansion of a BEC in a waveguide
    3) Localization in the presence of disorder
    4) Regimes of mobility
4) Experimental setup (detailed)
5) Observations

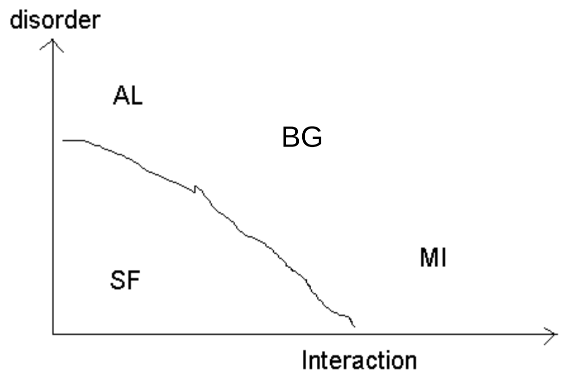
Anderson Localization

- Classical transport of particles through a material
  - scattering off impurities, mean free path, resistance
- But if meanfree path < de Broglie Wavelength, quantum effects can be important:
  - A wave phenomenon
  - Single particle interference between multiple scattering pathways
- Signature: suppression of diffusion and exponential localization

- Alternately: Bloch waves in a perfect lattice… disorder breaks translational symmetry leading to localization

Anderson Localization

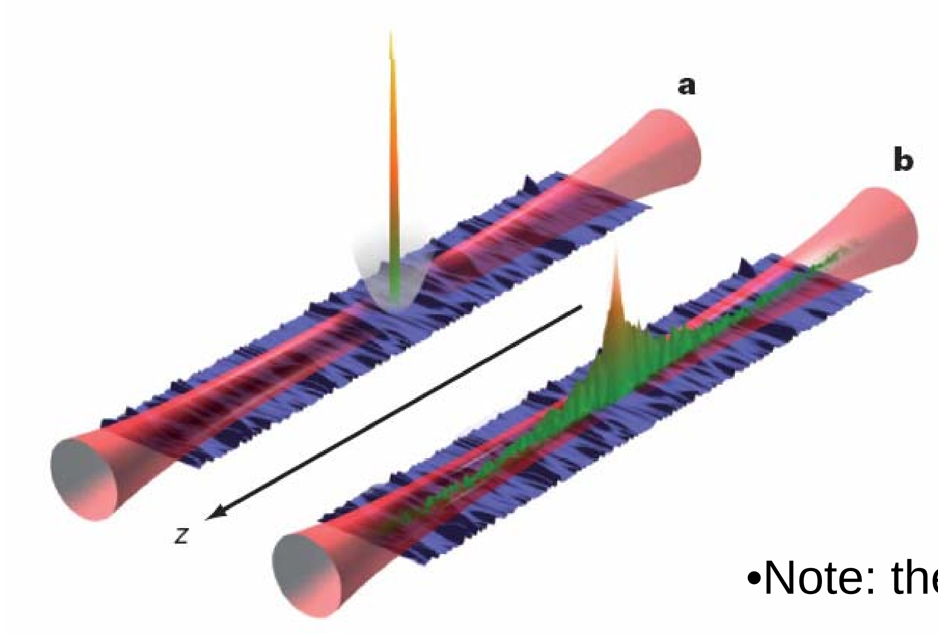
- Predicts a metal-to-insulator transition (delocalized to localized wavefunctions)
- Originally applied to non-interacting particles, but it is well known that interactions can play significant role
  - → disorder potential, non-linear interaction, hopping
    - → interesting phases: bose glass, anderson glass, Mott insulator …



- This work: avoid interactions to unambiguously see AL

Experimental setup (basic)

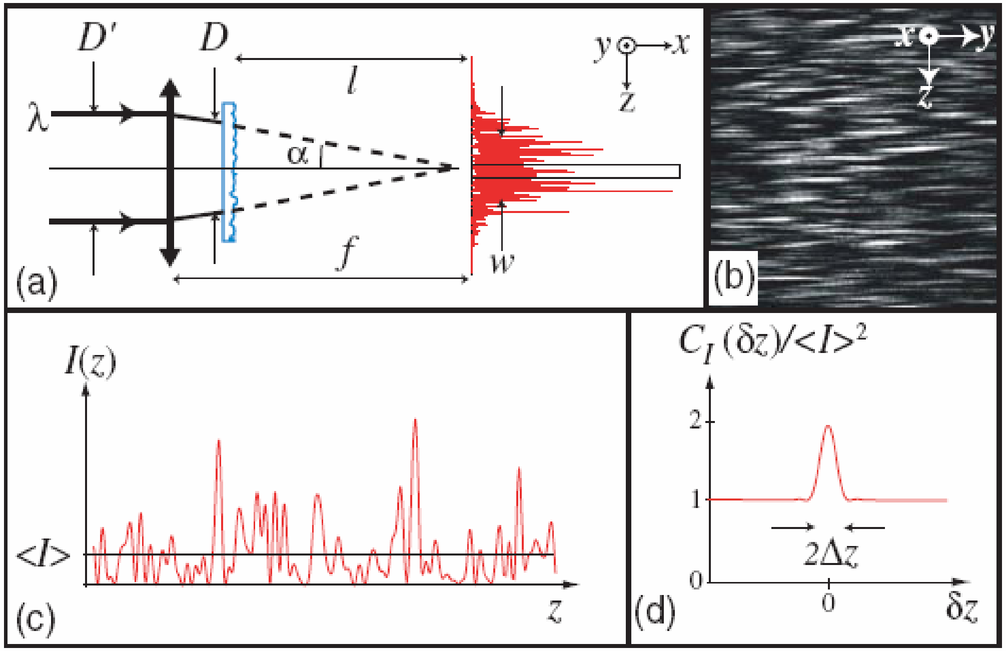
- Produce BEC in opto-magnetic hybrid trap
  - Optical waveguide + longitudinal magnetic confinement
- Apply weak disorder
  - Repulsive dipole force from optical speckle pattern generated by scattering light off a diffusing plate
- Remove longitudinal confinement
- Allow system to evolve for time, *t.*
- Turn off waveguide, image after TOF

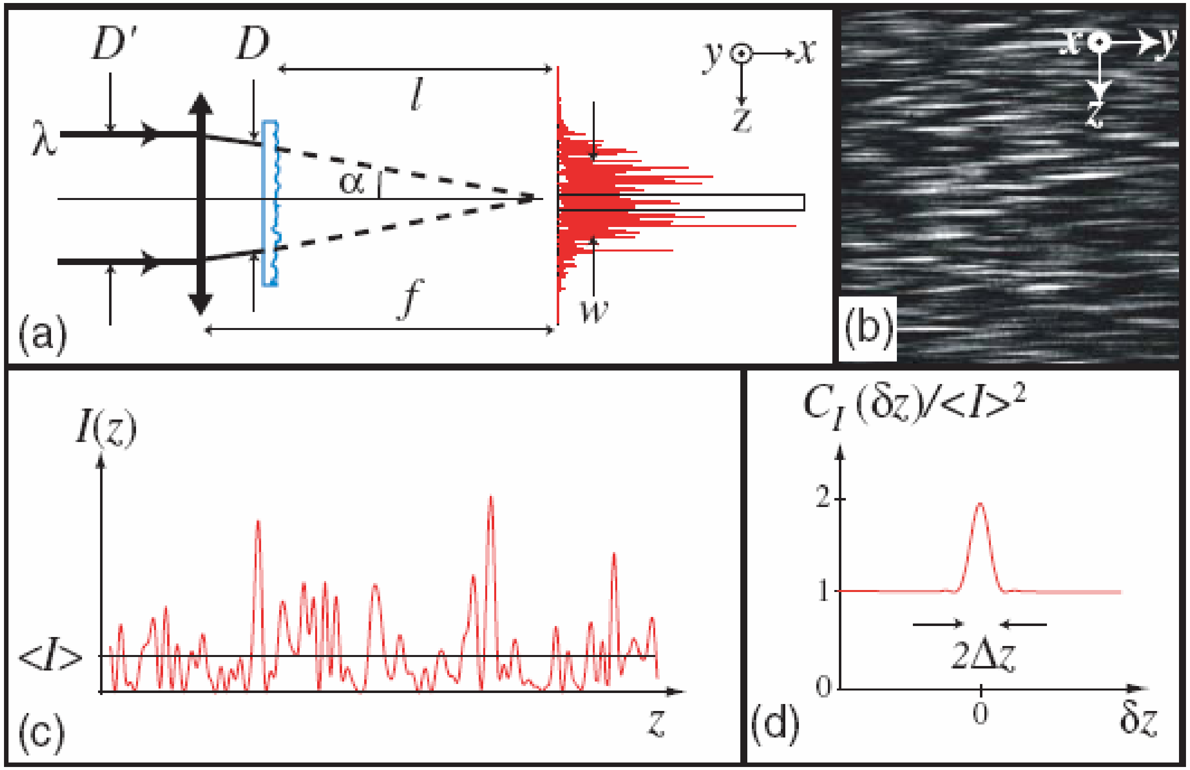


•Note: there is no lattice here.

Speckle pattern

- Produced by shining coherent laser light on a rough surface
- The scattered field from each scattering site has random amplitude and phase
- Characterized by intensity distribution and intensity correlation function





$$C_I(\delta\mathbf{r}) = \langle I(\mathbf{r}) I(\mathbf{r}+\delta\mathbf{r})\rangle$$

$$C(z) = V_R^2 c(z/\sigma_R),$$

$$c(u) = \sin^2(u)/u^2.$$

Grain size

$$\Delta z = \lambda \frac{l}{D_Z},$$

$$= \pi\sigma_R$$

$$\hat{C}(k) = V_R^2 \sigma_R \hat{c}(k\sigma_R),$$

$$\hat{c}(\kappa) = \sqrt{\pi/2}(1-\kappa/2)\Theta(1-\kappa/2),$$

$$\hat{C}(k) = 0 \text{ for } k > 2/\sigma_R.$$

D Clement et. al., New Journal of Physics **8** (2006) 165

Expansion of a BEC (Thomas-Fermi regime)

$$\mu\psi(z) = \left[\frac{-\hbar^2\partial_z^2}{2m} + \frac{m\omega^2 z^2}{2} + V(z) + g|\psi(z)|^2\right]\psi(z).$$

1D GPE

$$\psi_0(z) = \sqrt{n_0(z)} \qquad n_0(z) = \frac{\mu - m\omega^2 z^2/2}{g}$$

BEC wavefunction
in TF approximation

At long times (t>1/omega), interaction energy has been fully converted into kinetic energy

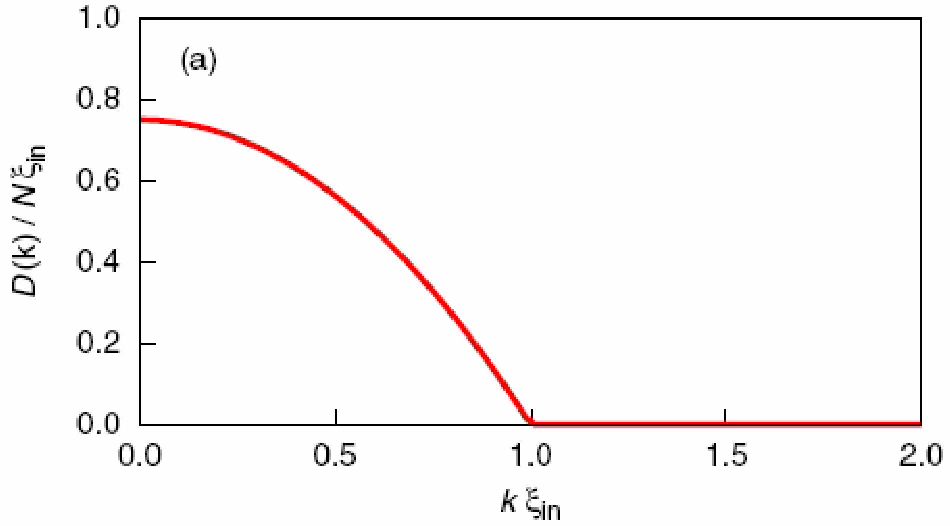
$$\psi(z,t) = \int \frac{\mathrm{d}k}{\sqrt{2\pi}}\hat{\psi}(k,t)\mathrm{e}^{\mathrm{i}kz} \qquad \mathcal{D}(k) = |\hat{\psi}(k,t)|^2$$

Assymptotic momentum distribution calculated from scaling solution of spatial wavefunction under free expansion

$$\mathcal{D}(k) \simeq \frac{3N\xi_{\mathrm{in}}}{4} \times \left[1 - (k\xi_{\mathrm{in}})^2\right] \times \Theta\left[1 - (k\xi_{\mathrm{in}})^2\right]$$

L Sanchez-Palencia, New Journal of Physics **10** (2008) 045019

$$\mathcal{D}(k) \simeq \frac{3N\xi_{\rm in}}{4} \times \left[1-(k\xi_{\rm in})^2\right] \times \Theta\left[1-(k\xi_{\rm in})^2\right]$$



Distribution exhibits a high-momentum cut-off

$$k_{\rm c} = 1/\xi_{\rm in}$$

$$\xi_{\rm in} = \hbar/\sqrt{4m\mu}$$

At long times (t>1/omega) almost all interaction energy has been converted to kinetic energy. Momentum distribution is stationary, and we c*an treat the interaction of each k-wave with the disorder independently.*

$$\psi(z,t) = \int \frac{{\rm d}k}{\sqrt{2\pi}} \hat{\psi}(k,t){\rm e}^{{\rm i}kz}$$

L Sanchez-Palencia, New Journal of Physics **10** (2008) 045019

Exponential localization

In Anderson theory, each *k* vector plane wave becomes exponentially localized

$$e^{ikz} \longrightarrow \phi_k(z)$$

with localization length L_{loc}

$$\ln |\phi_k(z)| \simeq -\gamma(k)|z| \qquad \gamma(k) = 1/L_{\mathrm{loc}}(k)$$

Density distribution is then the sum of all localized states weighted by the momentum distribution

$$n_0(z) \simeq 2\int_0^\infty \mathrm{d}k\mathcal{D}(k)\langle|\phi_k(z)|^2\rangle$$

L Sanchez-Palencia, New Journal of Physics **10** (2008) 045019

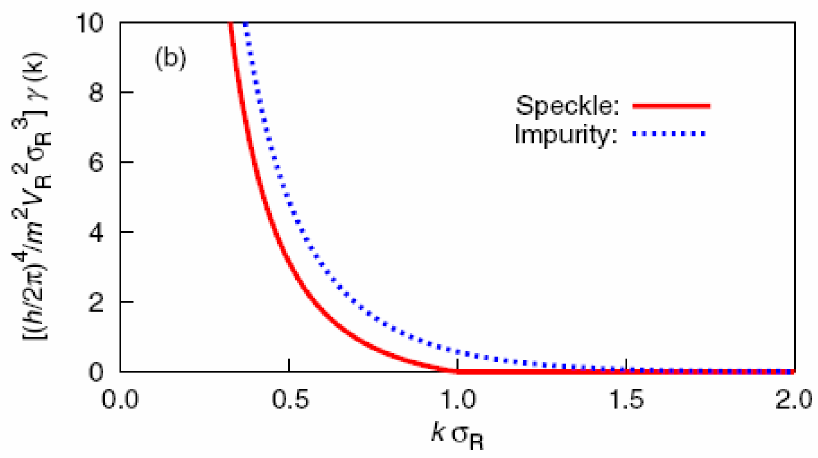
Localization length

The localization length is connected to the correlation function of the disorder, $\hat{C}(k) = V_R^2 \sigma_R \hat{c}(k\sigma_R)$,

$$\gamma(k) = 1/L_{\text{loc}}(k)$$

$$\gamma(k) = \frac{\pi m^2 V_{\text{R}}^2 \sigma_{\text{R}}}{2\hbar^4 k^2}(1 - k\sigma_{\text{R}})\Theta(1 - k\sigma_{\text{R}}).$$

$\gamma(k) > 0$ only for $k\sigma_R < 1$



Mobility edge at $k = 1/\sigma_R$, above which k-states are not exponentially localized

Localization of a BEC by a speckle pattern

$$n_0(z) \simeq 2 \int_0^\infty \mathrm{d}k \mathcal{D}(k) \langle |\phi_k(z)|^2 \rangle$$

$$\mathcal{D}(k) \simeq \frac{3N\xi_{\mathrm{in}}}{4} \times \left[1 - (k\xi_{\mathrm{in}})^2\right] \times \Theta\left[1 - (k\xi_{\mathrm{in}})^2\right]$$

$$\gamma(k) = \frac{\pi m^2 V_{\mathrm{R}}^2 \sigma_{\mathrm{R}}}{2\hbar^4 k^2} (1 - k\sigma_{\mathrm{R}}) \Theta(1 - k\sigma_{\mathrm{R}}).$$

$$k_{\mathrm{c}} = \min\{1/\xi_{\mathrm{in}}, 1/\sigma_{\mathrm{R}}\}$$

Localization of a BEC by a speckle pattern

$$n_0(z) \simeq 2\int_0^\infty \mathrm{d}k\mathcal{D}(k)\langle|\phi_k(z)|^2\rangle$$

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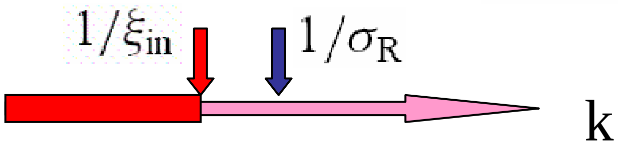

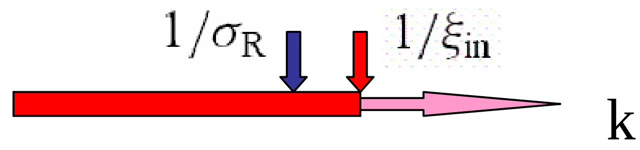

Localization of a BEC by a speckle pattern

$$n_0(z) \simeq 2\int_0^\infty \mathrm{d}k\mathcal{D}(k)\langle|\phi_k(z)|^2\rangle$$

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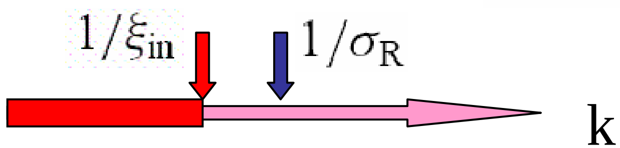
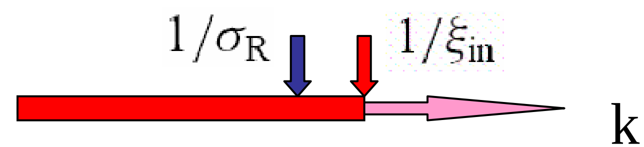
$$\gamma(k) = \frac{\pi m^2 V_{\mathrm{R}}^2 \sigma_{\mathrm{R}}}{2\hbar^4 k^2}(1-k\sigma_{\mathrm{R}})\Theta(1-k\sigma_{\mathrm{R}}).$$

$$k_{\mathrm{c}} = \min\{1/\xi_{\mathrm{in}}, 1/\sigma_{\mathrm{R}}\}$$





if $k_{\mathrm{c}} = 1/\xi_{\mathrm{in}}$

Exponential tails $k_{\mathrm{c}}\sigma_{\mathrm{R}} < 1$

$$n_0(z) \propto \frac{\exp\{-2\gamma_{\mathrm{eff}}|z|\}}{|z|^{7/2}},$$

where $\gamma_{\mathrm{eff}} = \gamma(k = 1/\xi_{\mathrm{in}})$.

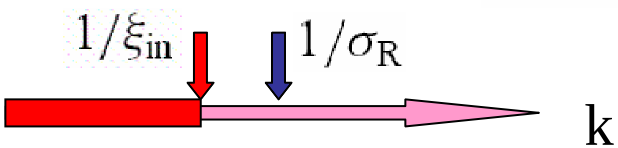
Localization of a BEC by a speckle pattern

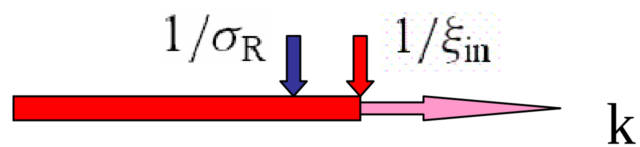
$$n_0(z) \simeq 2 \int_0^\infty \mathrm{d}k \mathcal{D}(k) \langle |\phi_k(z)|^2 \rangle$$

$$\mathcal{D}(k) \simeq \frac{3N\xi_{\mathrm{in}}}{4} \times \left[1 - (k\xi_{\mathrm{in}})^2\right] \times \Theta\left[1 - (k\xi_{\mathrm{in}})^2\right]$$

$$\gamma(k) = \frac{\pi m^2 V_{\mathrm{R}}^2 \sigma_{\mathrm{R}}}{2\hbar^4 k^2} (1 - k\sigma_{\mathrm{R}}) \Theta(1 - k\sigma_{\mathrm{R}}).$$

$$k_{\mathrm{c}} = \min\{1/\xi_{\mathrm{in}}, 1/\sigma_{\mathrm{R}}\}$$





if $k_{\mathrm{c}} = 1/\xi_{\mathrm{in}}$

Exponential tails $k_{\mathrm{c}}\sigma_{\mathrm{R}} < 1$

$$n_0(z) \propto \frac{\exp\{-2\gamma_{\mathrm{eff}}|z|\}}{|z|^{7/2}},$$

where $\gamma_{\mathrm{eff}} = \gamma(k = 1/\xi_{\mathrm{in}})$.

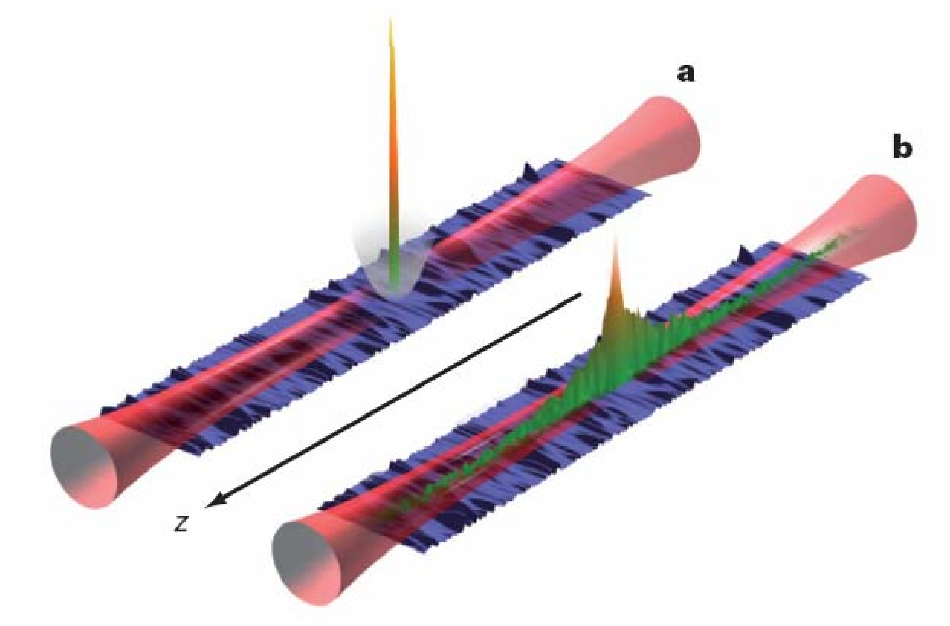
if $k_{\mathrm{c}} = 1/\sigma_{\mathrm{R}}$

Algebraic tails $k_{\mathrm{c}}\sigma_{\mathrm{R}} > 1$

$$n_0(z) \propto \frac{1}{|z|^2}.$$

Experimental setup

- 87-Rb BEC, 1.7x10^4 atoms.
- opto-magnetic hybrid trap
  - transverse confinement (70 Hz) [optical waveguide] 3 μm
  - longitudinal confinement (5.4 Hz) [magnetic trap] 35 μm
- Turn on disorder potential
- Switch off magnetic confinement
- Weakly anti-trapping magnetic field compensates the residual longitudinal trapping of the optical waveguide, atoms expand freely over several millimetres.
- Wait for time *t*, turn off trap, image after TOF.

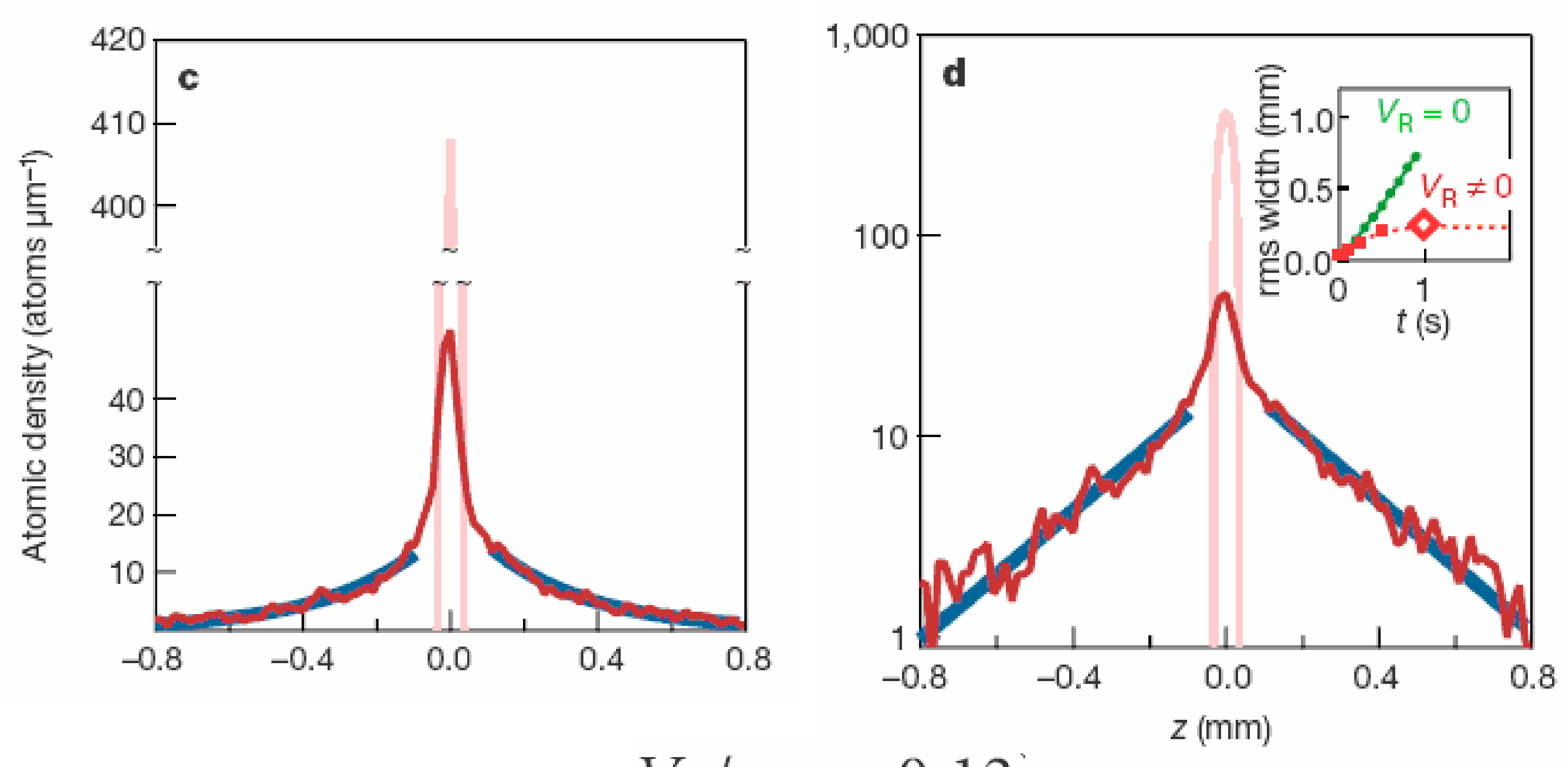


Experimental setup

Speckle pattern

- Lambda 0.513 um
- Transverse correlation function: 97 and 10 um (>> 3 um, atomic matter wave)
- Longitudinal: $\sigma_R = 0.26 \pm 0.03\ \mu m\ (\pm 1\ s.e.m.)$ $\pi\sigma_R = 0.82\ \mu m.$
- Can treat as 1D: $L_{TF} \gg \Delta z$ $R_{TF} \ll \Delta y, \Delta x,$
- Amplitude of disorder – intensity of laser
- Operate in regime of weak disorder: $V_R/\mu_{in} = 0.15.$

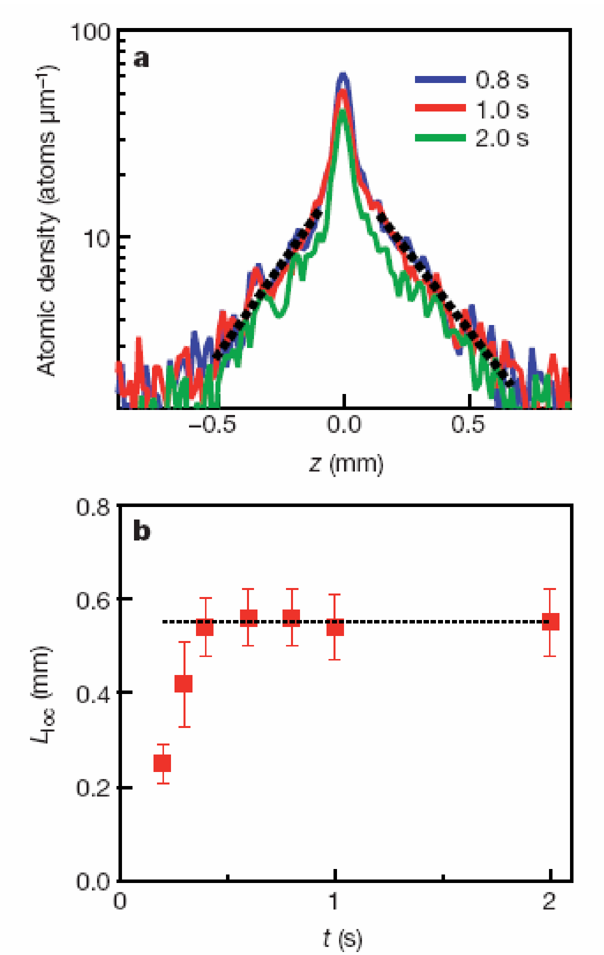
Exponential localization

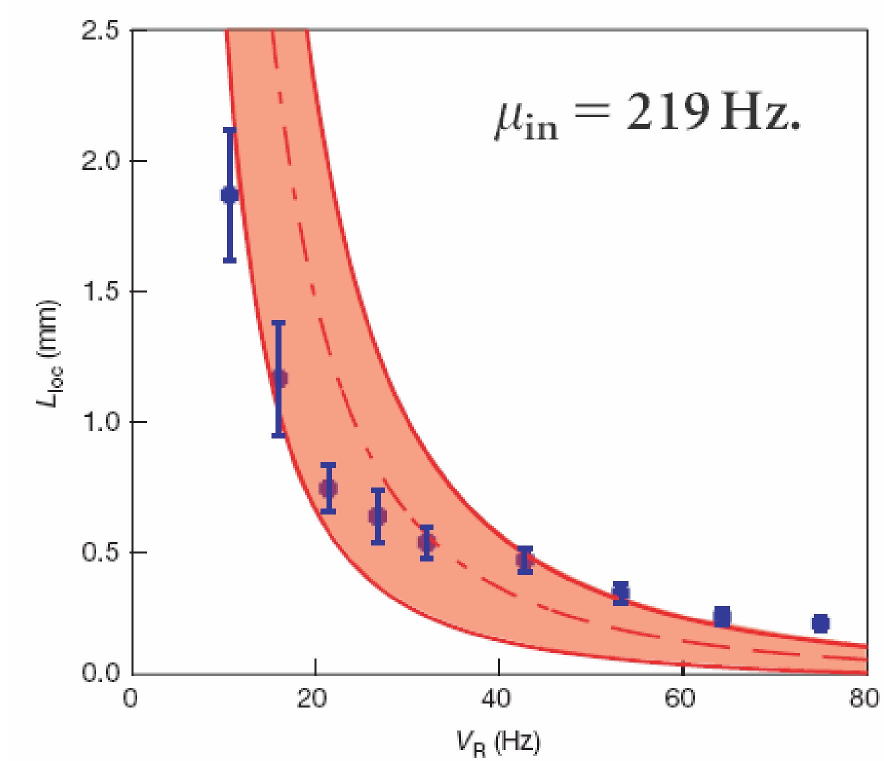


$V_{\rm R}/\mu_{\rm in} = 0.12$

Localization length

$V_R/\mu_{in} = 0.12$





$$L_{loc} = \frac{2\hbar^4 k_{max}^2}{\pi m^2 V_R^2 \sigma_R (1 - k_{max}\sigma_R)}$$

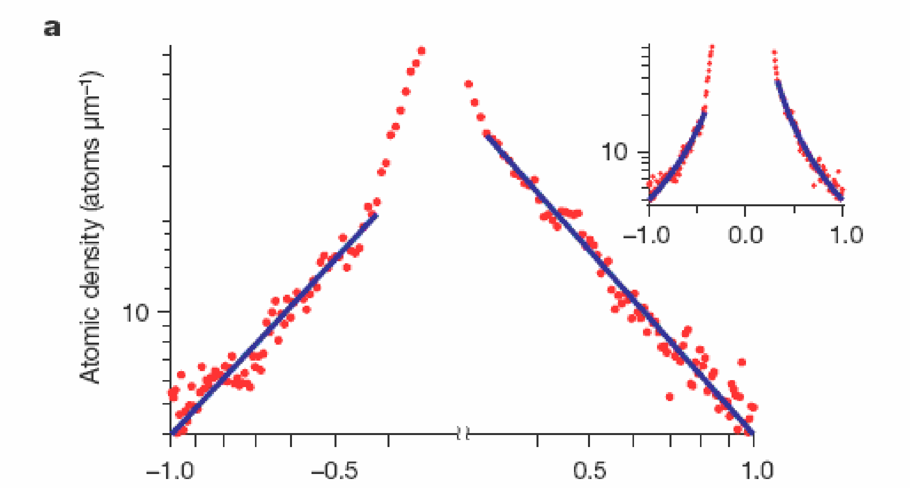
valid only for $k_{max}\sigma_R < 1$

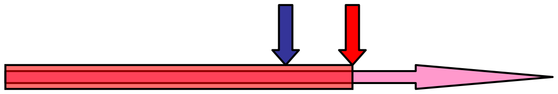
Beyond the mobility edge

1.7×10^5 atoms, $\mu_{in}/h = 519$ Hz

$$k_{max}\sigma_R = 1.16 \pm 0.14 \ (\pm 2 \text{ s.e.m.}).$$

$$V_R/\mu_{in} = 0.12$$





$1/|z|^{\beta}$

$\beta = 1.92 \pm 0.06 \ (\pm 2 \text{ s.e.m.})$ Left tail

$\beta = 2.01 \pm 0.03 \ (\pm 2 \text{ s.e.m.})$ Right tail

Beyond the mobility edge

1.7×10^5 atoms, $\mu_{in}/h = 519$ Hz

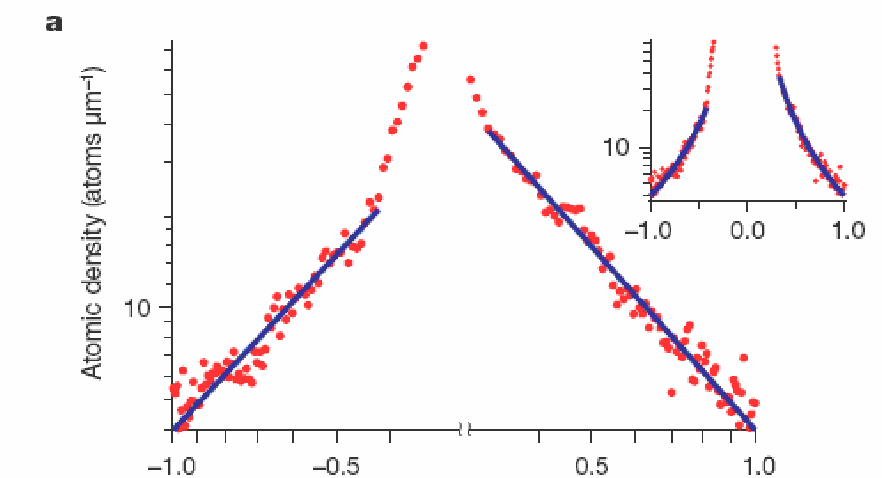
$k_{max}\sigma_R = 1.16 \pm 0.14$ (± 2 s.e.m.).

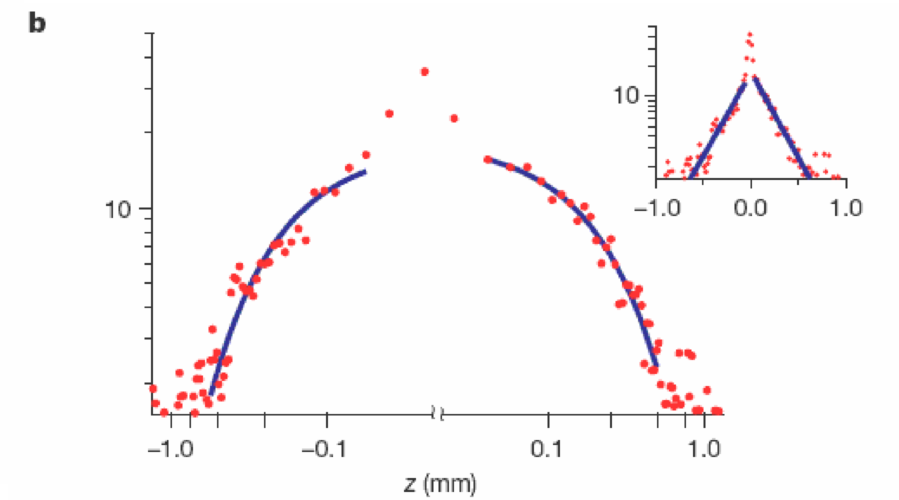
$V_R/\mu_{in} = 0.12$

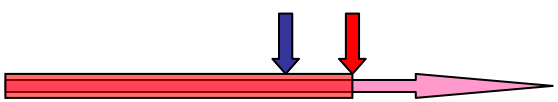
1.7×10^4 atoms $\mu_{in}/h = 219$ Hz,

$k_{max}\sigma_R = 0.65 \pm 0.09$ (± 2 s.e.m.)

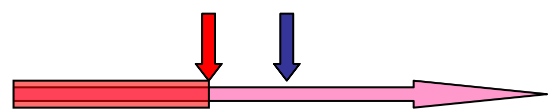
$V_R/\mu_{in} = 0.15.$







k



k

$1/|z|^\beta$

$\beta = 1.92 \pm 0.06$ (± 2 s.e.m.) Left tail

$\beta = 2.01 \pm 0.03$ (± 2 s.e.m.) Right tail

Summary

- First direct observation of exponentially localized matter waves in space, algebraic tails above the mobility edge.

Outlook

- Quantum simulation of Anderson localization in higher dimensions
- Add interactions: competition between interactions, disorder, and kinetic energy poorly understood