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Simplifying quantum logic using higher-dimensional Hilbert spaces

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Quantum computation promises to solve fundamental, yet otherwise intractable, problems across a range of active fields of research. Recently, universal quantum logic-gate sets—the elemental building blocks for a quantum computer—have been demonstrated in several physical architectures. A serious obstacle to a full-scale implementation is the large number of these gates required to build even small quantum circuits. Here, we present and demonstrate a general technique that harnesses multi-level information carriers to significantly reduce this number, enabling the construction of key quantum circuits with existing technology. We present implementations of two key quantum circuits: the three-qubit Toffoli gate and the general two-qubit controlled-unitary gate. Although our experiment is carried out in a photonic architecture, the technique is independent of the particular physical encoding of quantum information, and has the potential for wider application.

Wed Group Meeting

Hoda Hossein-Nejad

7 Jan 2009

OUTLINE

- What I was going to talk about
- This paper;

  - The Idea in a nutshell

  - Linear Optics Implementation
- Schlafen

WHAT I AM NOT GOING TO TALK ABOUT

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Generating entangled photons from the vacuum by accelerated measurements: Quantum-information theory and the Unruh-Davies effect

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Building on the well-known Unruh-Davies effect, we examine the effects of projective measurements and quantum communications between accelerated and stationary observers. We find that the projective measurement by a uniformly accelerated observer can excite real particles from the vacuum in the inertial frame, even if no additional particles are created by the measurement process in the accelerating frame. Furthermore, we show that the particles created by this accelerating measurement can be highly entangled in the inertial frame, and it is also possible to use this process to generate even maximally entangled two-qubit states by a certain arrangement of measurements. As a by-product of our analysis, we also show that a single qubit of information can be perfectly transmitted from the accelerating observer to the inertial one. In principle, such an effect could be exploited in designing an entangled-state generator for quantum communication.

THIS PAPER…

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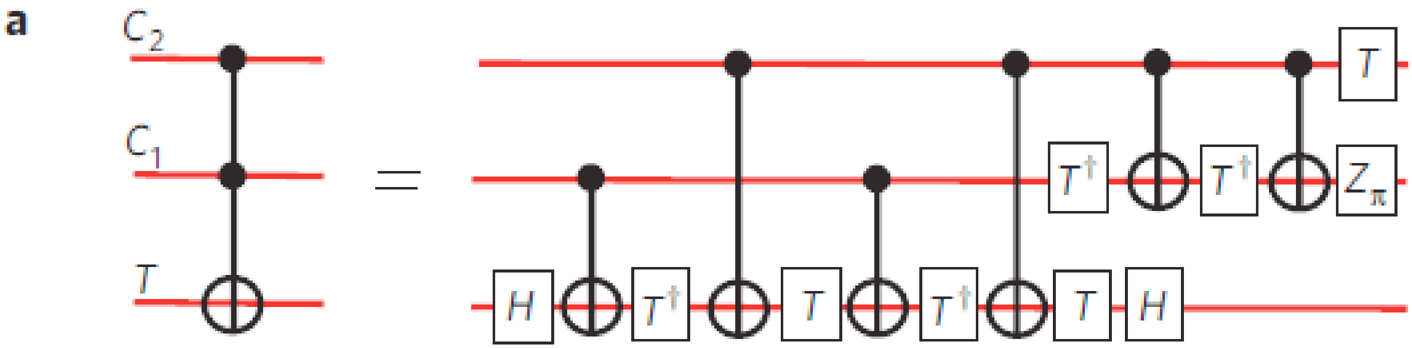
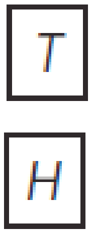
Simplifying quantum logic using higher-dimensional Hilbert spaces

- Motivation: The huge number of gates is the most serious obstacle to scalability of quantum computers.
- Multi-level info. carriers can significantly reduce this number.

EXAMPLE; TOFFOLI GATE IMPLEMENTATION



$$|1\,1\,1\rangle \mapsto |1\,1\,0\rangle$$
$$|1\,1\,0\rangle \mapsto |1\,1\,1\rangle$$



$T \; \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

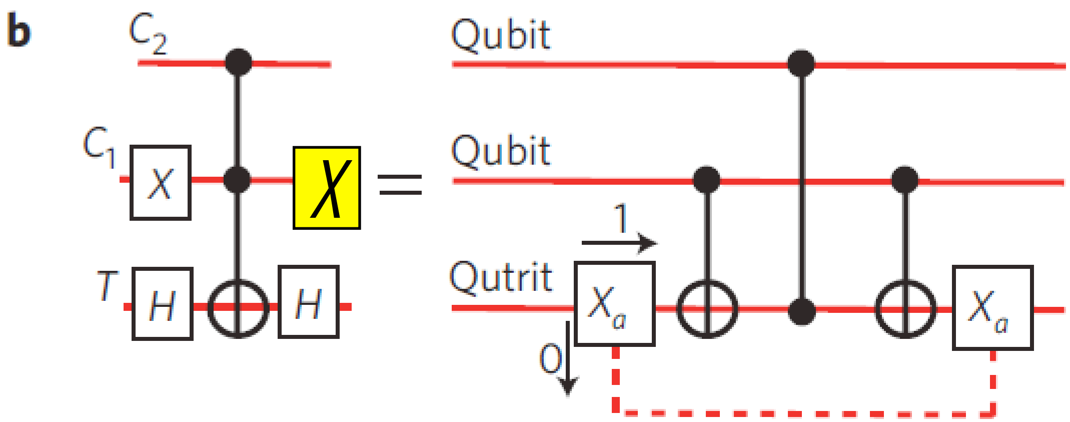
$H \; \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

→ 6 two qubit gates – (restrict to CNOT and controlled-Z)

$${}^{n}T \propto n^{2}$$

(Nicht gut …☹)

‘SUPER’ TOFFOLI IMPLEMENTATION



$$C_1, C_2 = |0\rangle, |1\rangle$$

$$T = |0\rangle, |1\rangle, |2\rangle$$

Qutrit operator

$$X_a = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Qubit operators

Controlled-Z (two filled dots): $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

CNOT (filled dot, ⊕ target): $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

T $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

Z_θ $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$

H $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

X $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$|1,1,0\rangle \mapsto |1,1,2\rangle$$
$$\mapsto |1,1,0\rangle$$

$$|1,1,0\rangle \mapsto |1,1,0\rangle$$
$$|1,1,1\rangle \mapsto |1,1,1\rangle$$
$$|1,0,1\rangle \mapsto -|1,0,1\rangle$$
$$|1,0,0\rangle \mapsto |1,0,0\rangle$$

GENERALIZED 'SUPER' TOFFOLI

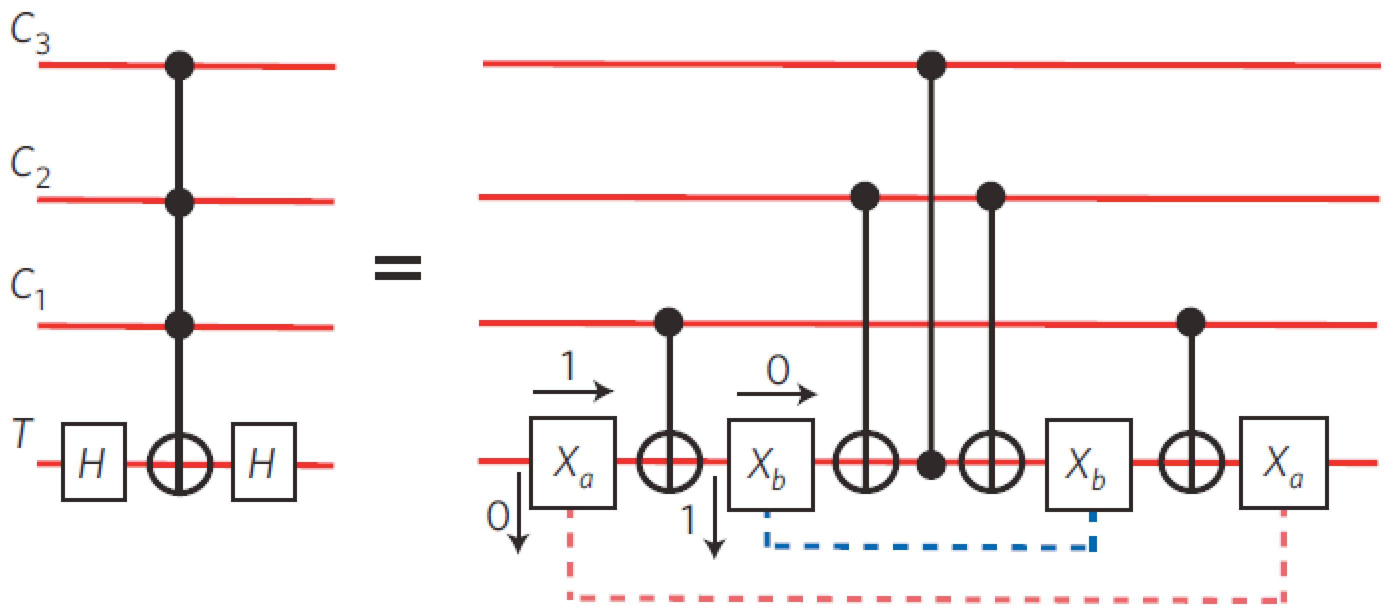


Figure 2 | Simplifying higher-order Toffoli gates. Three-control-qubit Toffoli[11]. The X_a gate swaps information between the logical $|\mathbf{0}\rangle$ and $|\mathbf{2}\rangle$ states of the target. The X_b gate flips information between the logical $|\mathbf{1}\rangle$ and $|\mathbf{3}\rangle$ state of the target. Thus, we require access to a four-level target information carrier: two levels in the original rail and one in each of the dashed rails. The target undergoes a sign shift only for the input term $|C_3, C_2, C_1, T\rangle = |\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}\rangle$. This operation is equivalent to the Toffoli under the action of only two one-qubit gates, as shown. See Fig. 1 for gate operations.

SUMMERY

<u>Qubit imp. of Toffoli</u>

12n – 11 gates , n-1 ancillas

OR ~ n^2 gates, no ancilla

<u>Multi level-bit imp. of Toffoli</u>

2n – 1 gates

one bit with (n+1) levels

CONTROLLED UNITARY

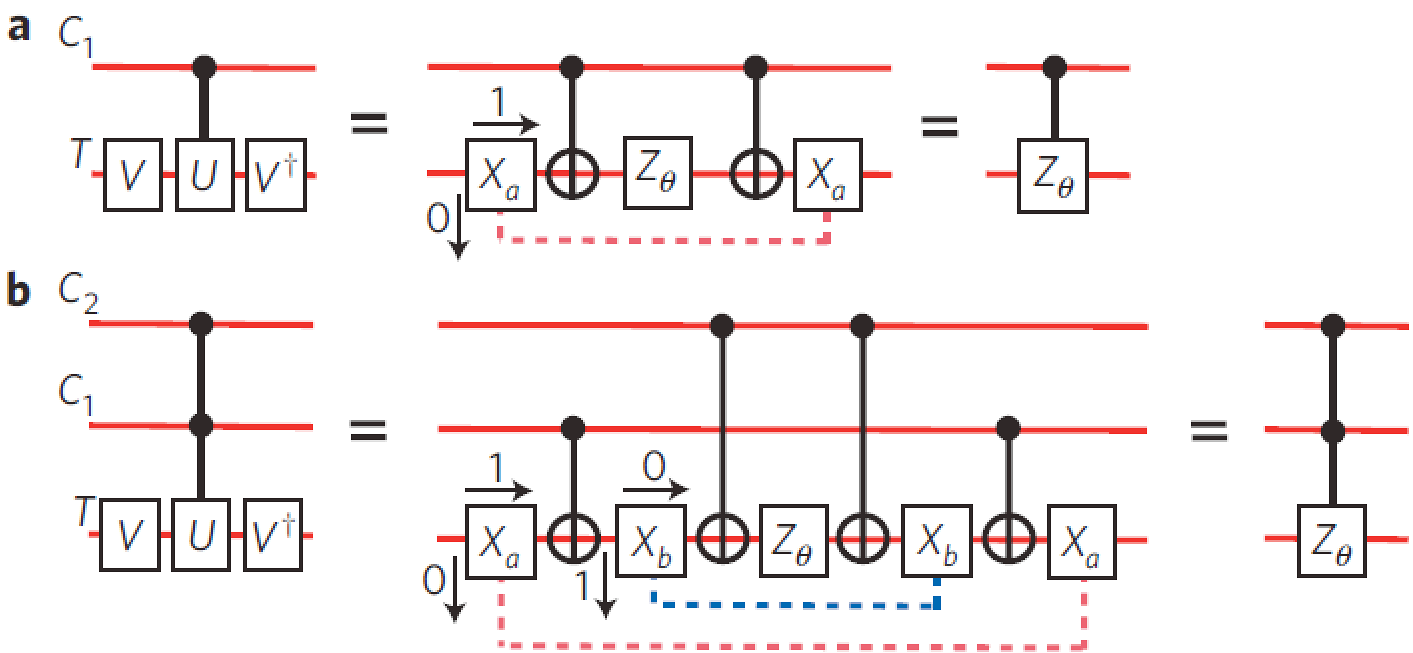


Figure 3 | Simplifying controlled-unitary gates. a, One control qubit (we implement a simplified version, see Fig. 5): the control operation occurs if $|C_1\rangle = |\mathbf{0}\rangle$. **b**, Two control qubits: the control operation occurs if $|C_2, C_1\rangle = |\mathbf{1}, \mathbf{1}\rangle$. $VZ_\theta V^\dagger$ is the spectral decomposition of U, up to a global phase factor. See Fig. 1 for gate operations.

GENERALIZED UNITARY, $C^N U$

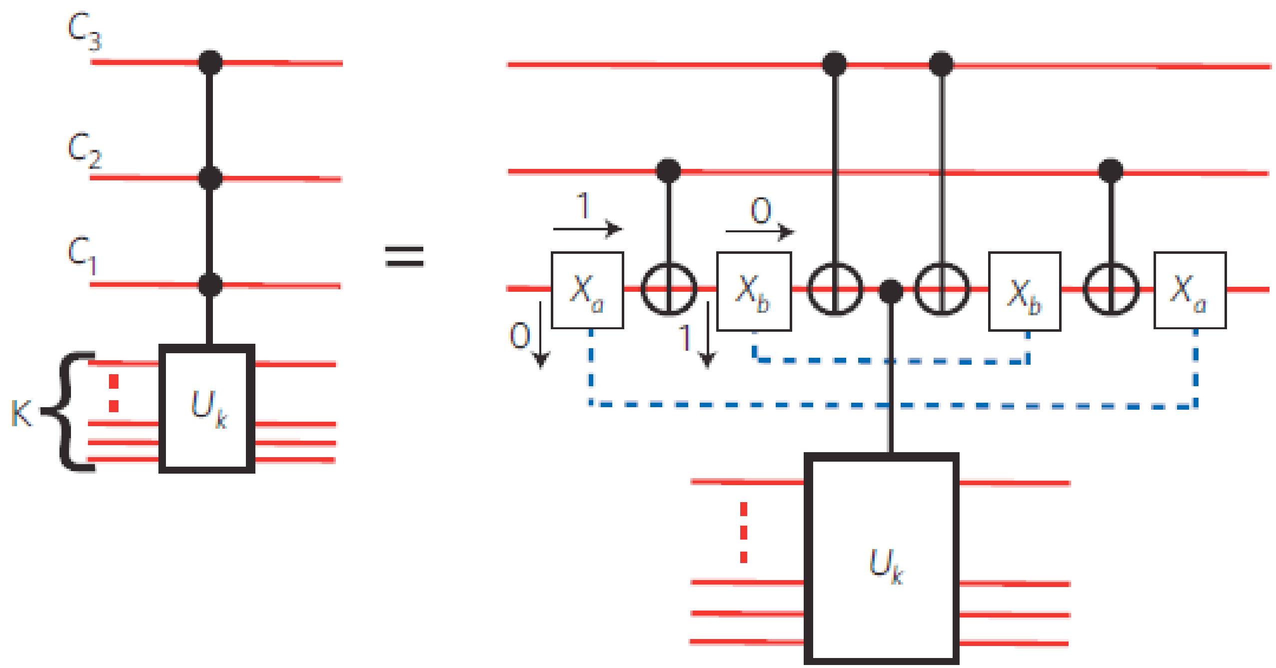


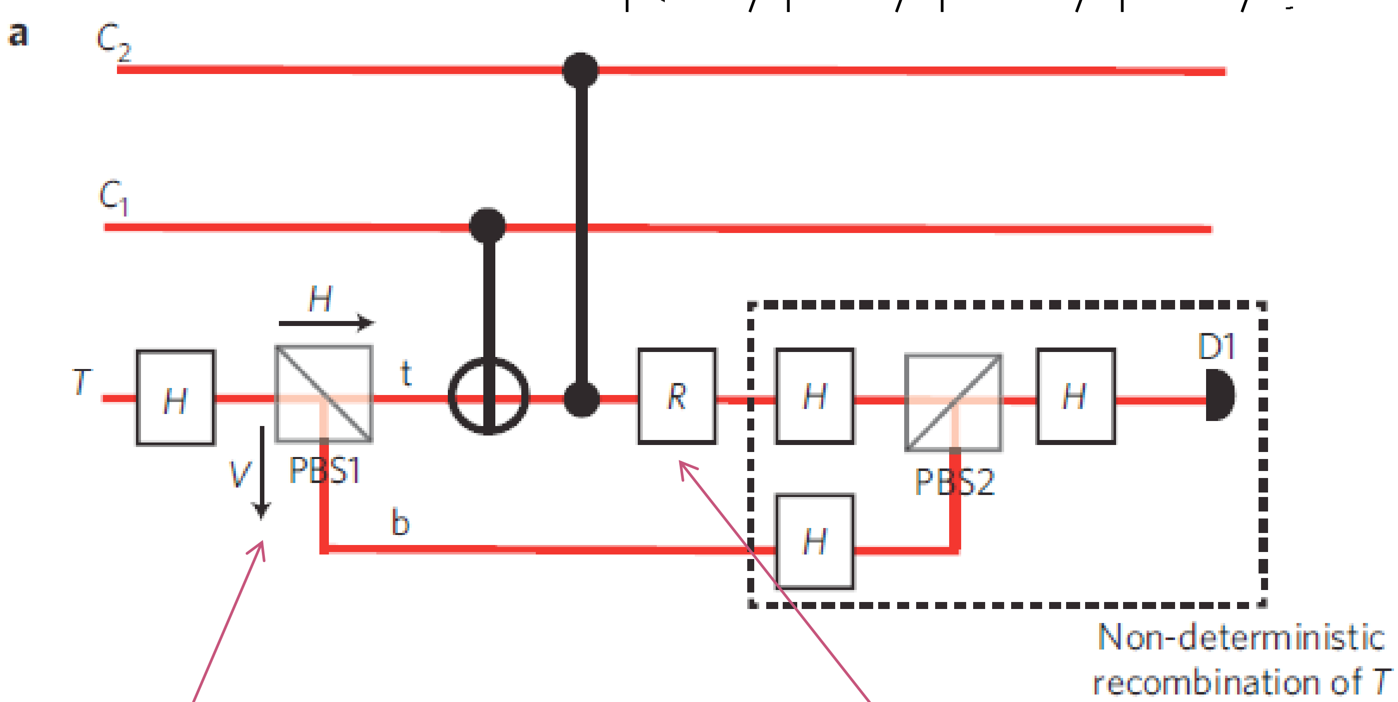
Figure 4 | Efficiently adding control qubits to an arbitrary controlled circuit. Circuit for a three-control-qubit unitary acting on k qubits, c^3u_k. Given the ability to carry out a single instance of a c^1u_k, n extra control qubits can be added at a cost of an extra $2n$ two-qubit gates and an extra n levels in C_1. The X_j perform as described in the caption of Fig. 2. The control operation occurs if $|C_3, C_2, C_1\rangle = |\mathbf{1}, \mathbf{1}, \mathbf{1}\rangle$.

HOW TO IMPLEMENT?

- Any multi-level system with the ability to coherently swap info. between the levels.
- Trapped ions
- Linear optics

DEMONSTRATION

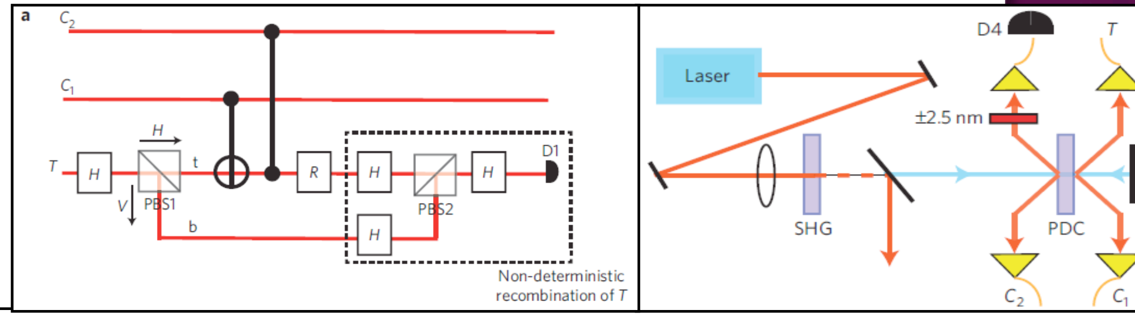
$$T = \{|H,t\rangle, |V,t\rangle, |H,b\rangle, |V,b\rangle\}$$



Expansion of the Hilbert space of the target photon

R= I Toffoli
R=Z(θ) C^1U
between C1 & T

EXPERIMENT



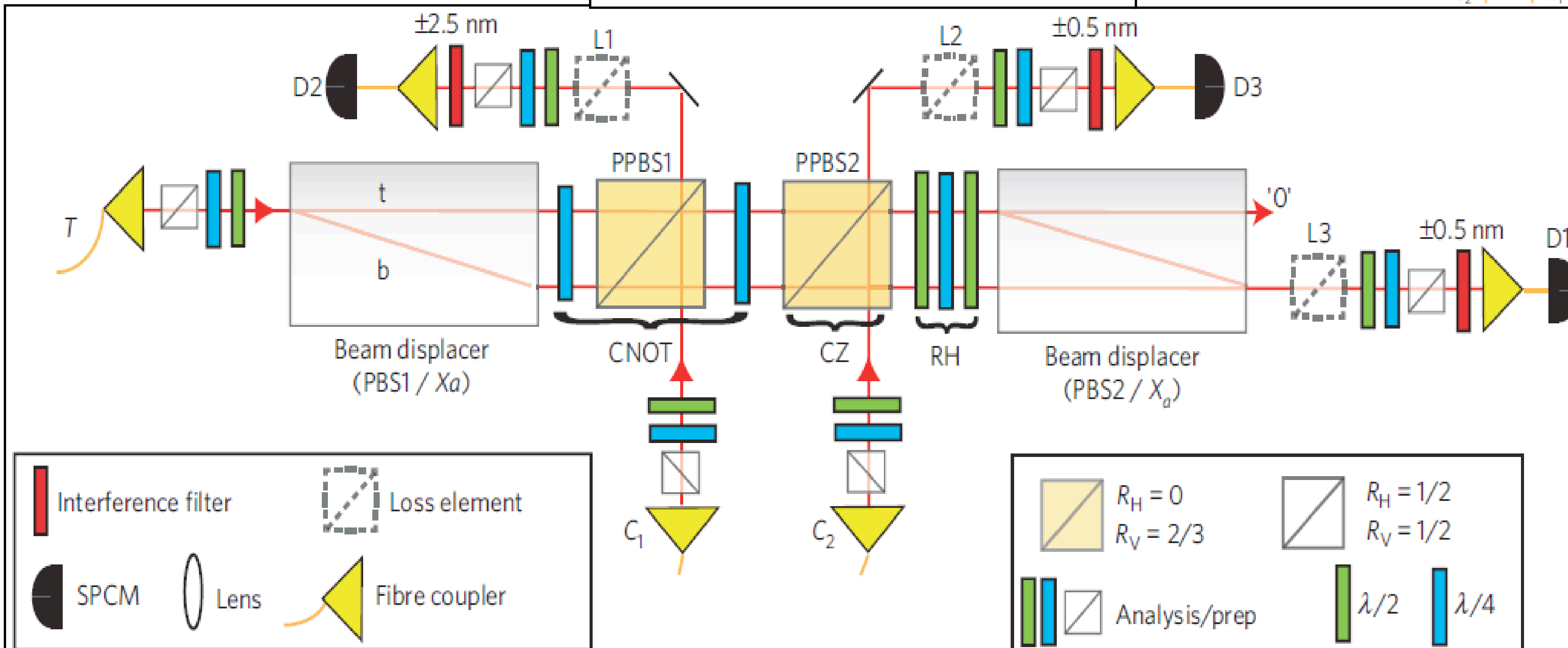


Figure 5 | Toffoli and controlled-unitary experimental layout. a, Conceptual logic circuit. A polarizing beam splitter temporarily expands the Hilbert space of the target information carrier, from a polarization-encoded photonic qubit to a multi-level system distributed across polarization and longitudinal spatial mode. Information in the bottom rail (b) bypasses the two-qubit gates. Detection of a photon at D1 heralds a successful implementation. $R = I$ (the identity) implements a Toffoli. $R = Z_\theta$ (see Fig. 1) implements a C^1U between C_1 and T (in this case, no photon is injected into C_2). **b,c**, Experimental circuit and optical source (see the Methods section). We use an inherently stable polarization interferometer using two calcite beam displacers[3]. PPBS, partially polarizing beam splitter; SPCM, single-photon counting module; PDC, parametric downconversion; SHG, second-harmonic generation.

GATE CHARACTERIZATION (CLASSICAL ACTION)

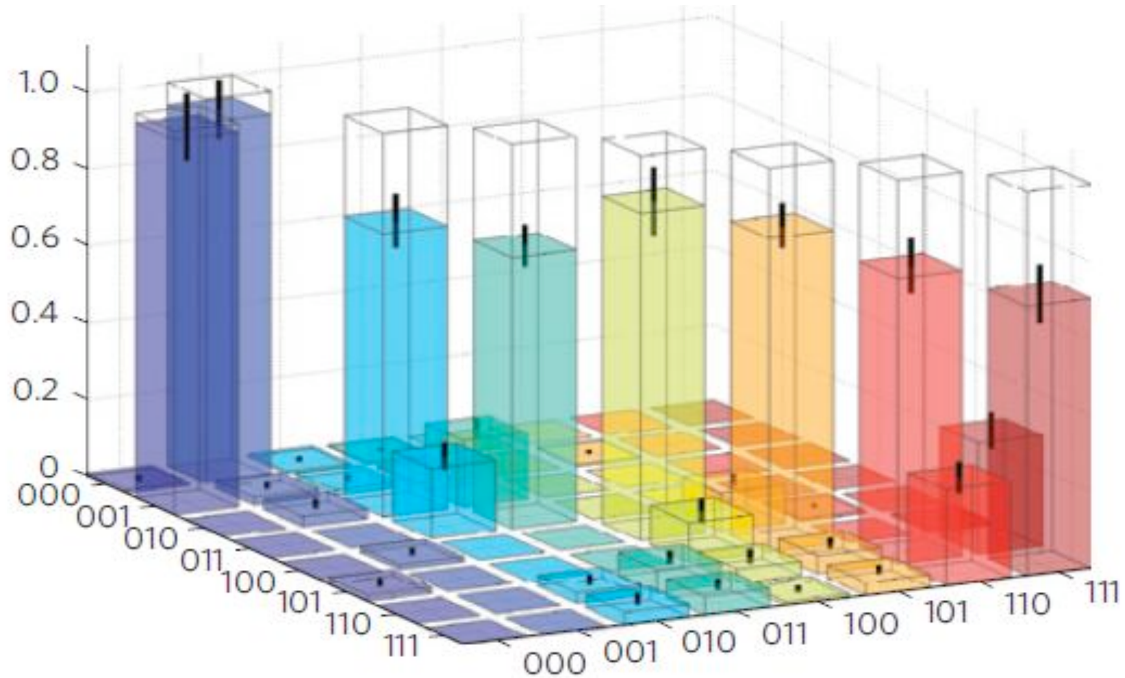


Figure 6 | Experimentally constructed Toffoli logical truth table. The labels on the *x* and *y* axes identify the state $|C_2, C_1, T\rangle$. Ideally, a flip of the logical state of the target qubit (T) occurs only when both control qubits (C_2 and C_1) are in the logical $|\mathbf{0}\rangle$ state. The ideal case is shown as a wire grid and the overlap is $\mathcal{I} = 0.81 \pm 0.03$ (see the Methods section). Error bars are shown representing one standard deviation, calculated from Poissonian photon-counting statistics. The table required four days of measurement.

QUANTUM ACTION OF THE GATE

- Complete state tomography not practical
- Only look at states with interesting outputs

$$|C_2, C_1, T\rangle$$

$$|\mathbf{0}, (\mathbf{0}+\mathbf{1}), \mathbf{0}\rangle/\sqrt{2} \longrightarrow |\mathbf{0},\ \Psi_+\rangle$$

$$|\mathbf{1}, (\mathbf{0}+\mathbf{1}), \mathbf{0}\rangle/\sqrt{2} \longrightarrow |1, (0+1), 0\rangle/\sqrt{2}$$

$$|\Psi_+\rangle = (|0, 0\rangle + |1, 1\rangle)/\sqrt{2}$$

OUTPUT DENSITY MATRICES

$$|\mathbf{0}, (\mathbf{0}{+}\mathbf{1}), \mathbf{0}\rangle/\sqrt{2} \longrightarrow |0,\ \Psi_+\rangle$$

$$|\mathbf{1}, (\mathbf{0}{+}\mathbf{1}), \mathbf{0}\rangle/\sqrt{2} \longrightarrow |1, (0{+}1), 0\rangle/\sqrt{2}$$

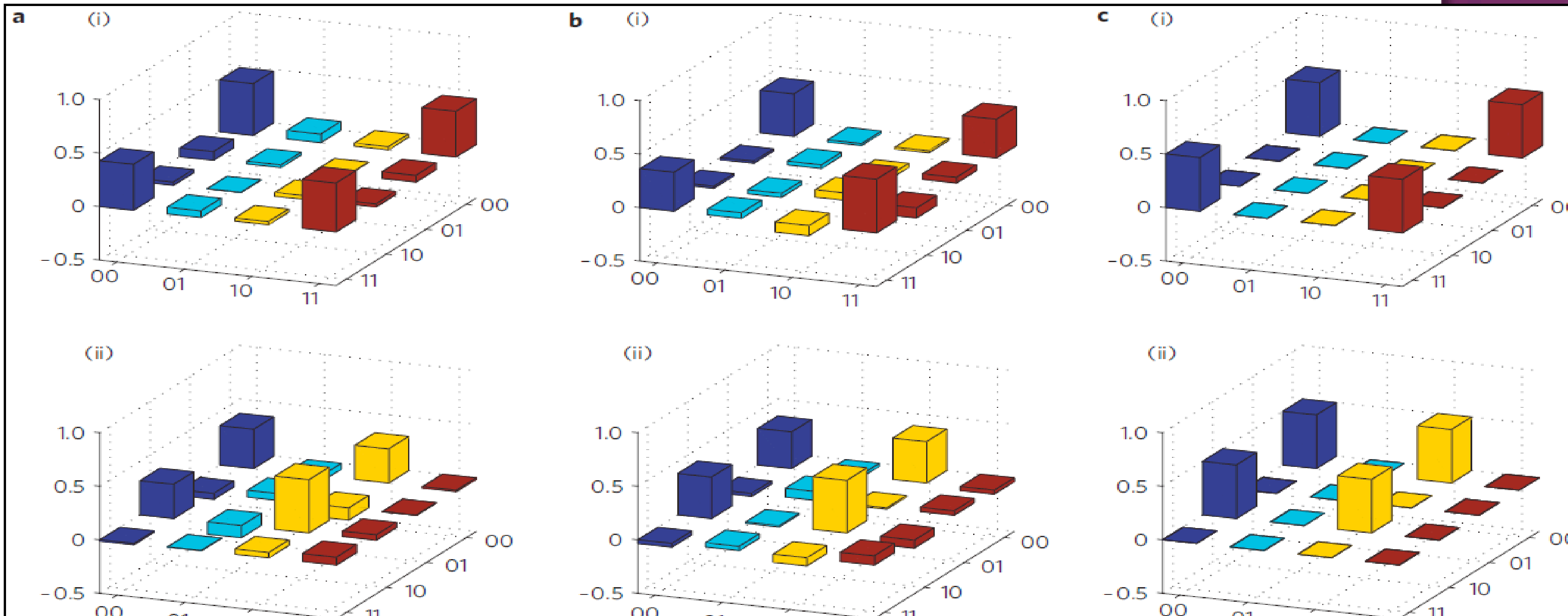
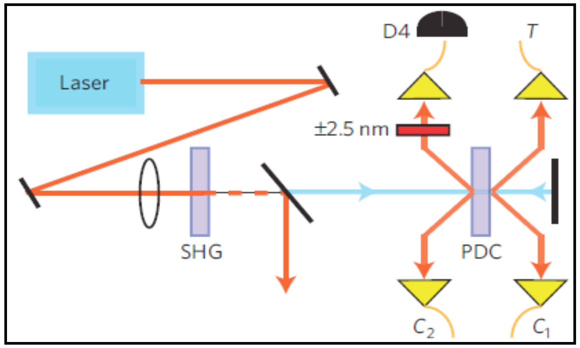


Figure 7 | Experimentally reconstructed Toffoli output density matrices. a, Measured output states of qubits C_1 and T for Toffoli gate inputs; (i) $|\mathbf{0},(\mathbf{0}{+}\mathbf{1}),\mathbf{0}\rangle/\sqrt{2}$; and (ii) $|\mathbf{1},(\mathbf{0}{+}\mathbf{1}),\mathbf{0}\rangle/\sqrt{2}$. We observe fidelities with the ideal states, linear entropies and tangles[39] of (i) {0.90±0.04, 0.21±0.08, 0.68±0.10} and (ii) {0.75±0.06, 0.47±10, 0.04±0.06}, respectively. **b**, As for **a**, but where the roles of C_1 and C_2 have been swapped. We now observe (i) {0.81±0.02, 0.39±0.05, 0.53±0.07} and (ii) {0.80±0.03, 0.40±0.05, 0.01±0.01}. The decrease in tangle in the (i) cases reflects the difference between dependent and independent photon interference, as discussed in the text. **c**, Ideal density matrices. Note, in all cases only real parts are shown; imaginary parts are small. Each density matrix requires 36 separate measurements[28] and takes approximately three days to complete.

EXPERIMENTAL IMPERFECTIONS

- Indistinguishable photons - Two-qubit gates need perfect relative non-classical interference visibility. (Vr)
- Dependent & independent photon interference
- Higher order emission from PDC.



measure $V_r = 100 \pm 1\%$ and $V_r = 92 \pm 4\%$ for the first and second two-qubit gates shown respectively (where $V_r = V_{meas}/V_{ideal}$, $V_{ideal} = 80\%$

MORAL OF THE STORY

- Stepping outside the qubit Hilbert space has practical advantages.
- Other possible simplifications?
- Next : explore small scale quantum algorithms, generate new states, test error correcting codes.