Group Meeting Jan 21 2009

Hardy's Paradox

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What I am going to talk about

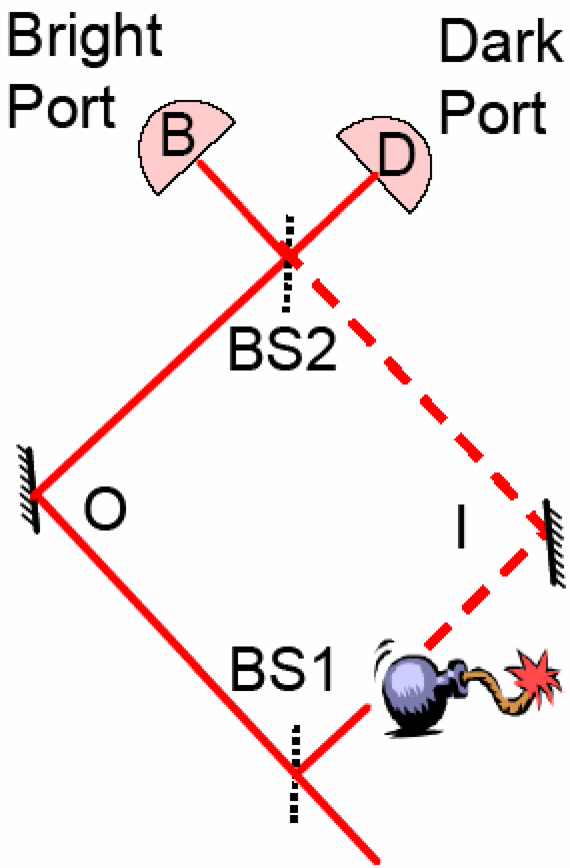
Experimental Joint Weak Measurement on a Photon Pair as a Probe of Hardy's Paradox
J.S. Lundeen and A.M. Steinberg **PRL 102, 020404 Jan 14 2009**

Revisiting Hardy's paradox: counterfactual statements, real measurements, entanglement and weak values
Y. Aharonov, A. Botero, S. Popescu, B. Reznik, J. Tollaksen **Phys Lett. A 301(130) 2002**

Quantum mechanics, local realistic theories, and Lorentz-invariant realistic theories
L Hardy **PRL 68, 2981 Jan 22 1992**

Interaction Free Measurement

Courtesy of Elitzur and Vaidman



Bomb Absent:

Only detector B fires

Bomb Present:

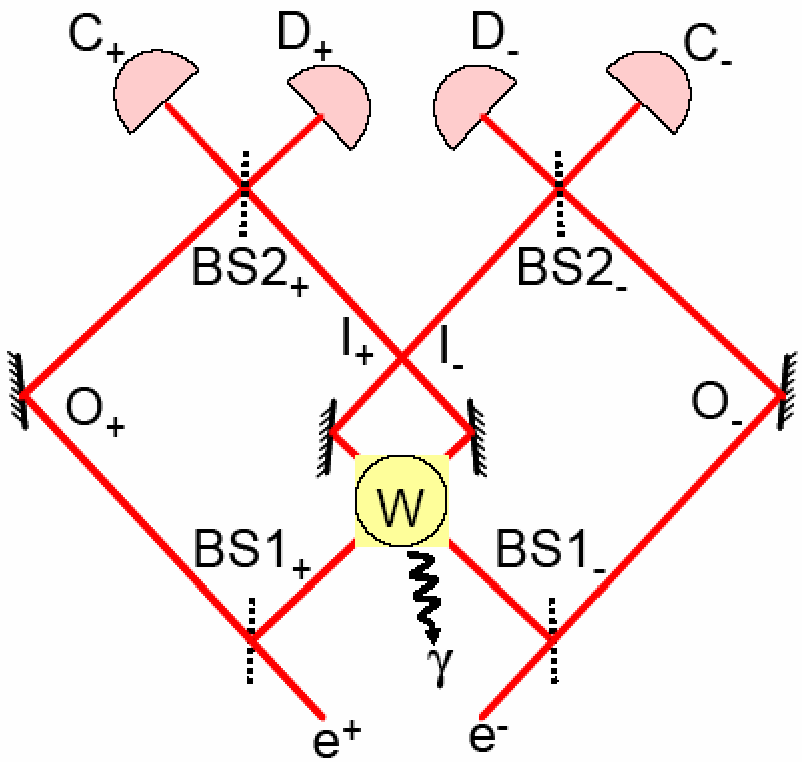
Detector	Prob.	Result
B	¼	None
Neither	½	Bang
D	¼	Present

Path through the paradox

- Introduce Hardy’s paradox
- Real paradox if we really measure
- Weak measurements
- Experimental setup
- Measurement data
- Results
- What can we conclude?

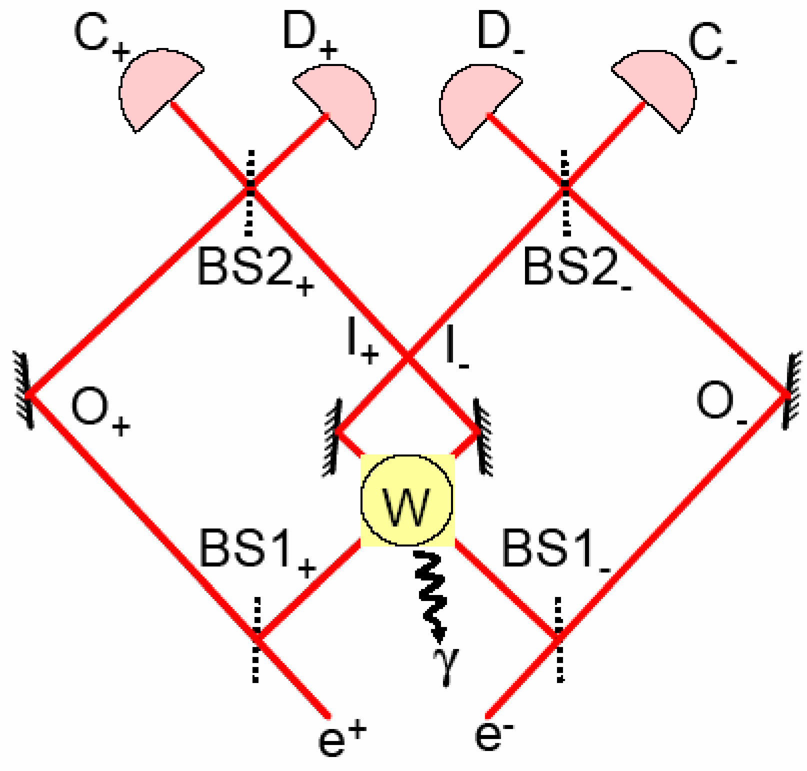
Hardy's Paradox

- Two Mach-Zender Interferometers separately calibrated such that all photons are detected at C ports.
- W: Annihilation Region
- O: Outer Paths
- I: Inner Paths



What happens?

- Sometimes D+ and D- fire simultaneously
- A D click implies particles traverse the inner path.
- If D+ clicked then e- must have been in inner path
- If D- clicked then e+ must have been in inner path
- If both clicked then why did they not annihilate?



Paradox Resolved?

- What happens if we put an actual detector in say the I+ path?
- D+ fires, but it now would anyways because WWI destroys the interferometer.
- Standard measurement makes the paradox go away.

Weak Measurements

- Weak measurements do not collapse a system.
- Trade off required. Information gained is minimal.
- Many repetitions will reveal desired value.
- Mathematical formalism to describe weak measurements (I'll skip and try to make you believe how they work)

Weak Measurements

- Imagine a special pointer

Properties:

- Moves proportional to desired value
- Controllable "coupling"

- Interaction would be:

$$\hat{H}_{\text{int}} = g\hat{P}\hat{A}$$

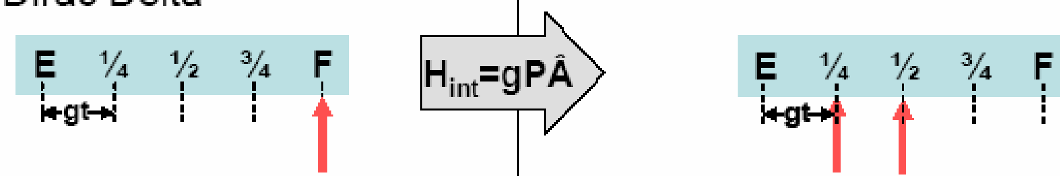
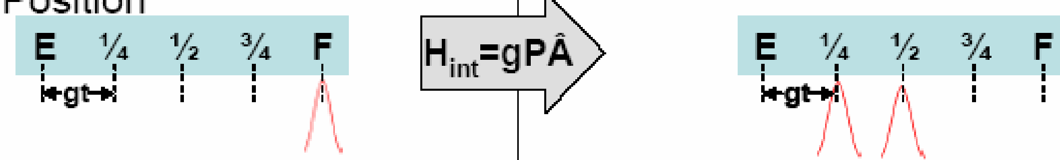
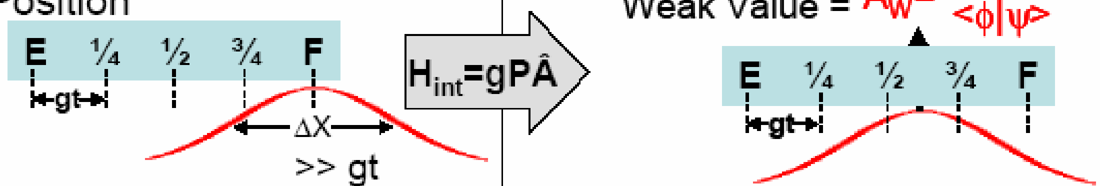
Weak Measurements

- State evolves according to: $\hat{U}_{couple} = e^{-ig\hat{P}\hat{A}}$
- The uncertainty in the pointer position, the coupling strength, and interaction time define the measurement regime
- gt >> σ : Standard projective measurement
- gt << σ : Weak measurement

$$\Psi_{POINT} = \exp\left(-\frac{(x_0 - A_W)^2}{\sigma^2} \right)$$

$$A_W = \frac{\langle \psi | \hat{A} | \varphi \rangle}{\langle \psi | \varphi \rangle}$$

Weak Measurment

	Initial Pointer Position Uncertainty	Pointer shift due to interaction
Ideal	Dirac Delta	
Real	Width << Change in Position	
Weak	Width >> Change in Position	Weak Value = $A_W = \frac{\langle\phi\|A\|\psi\rangle}{\langle\phi\|\psi\rangle}$ 

Generalized Measurement and Post-selection in Optical Quantum Information: Jeff Lundeen 2006

Weak Measurement

- Apply this to hardy's paradox.
- Repeat many times and measure particle number in each arm via pointer.
- Post selecting the double D click final state we can see where the particles went in this situation

Hardy's Paradox

- State(s) after BS1(s)

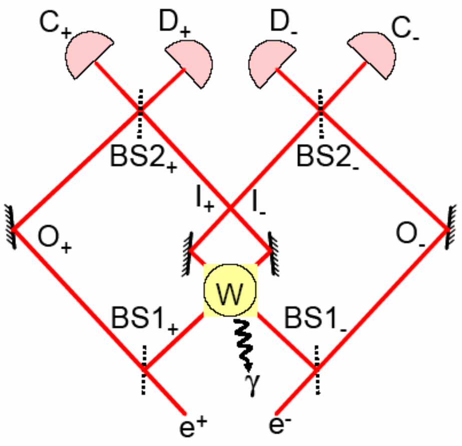
$$|\psi_{Initial}\rangle = \frac{1}{\sqrt{2}}(|I\rangle_+ + |O\rangle_+) \otimes \frac{1}{\sqrt{2}}(|I\rangle_- + |O\rangle_-)$$

- Project out $|I\rangle_+|I\rangle_-$ term (annihilation)

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|O\rangle_+|I\rangle_- + |I\rangle_+|O\rangle_- + |O\rangle_+|O\rangle_-) + |\gamma\rangle$$

- Post select double D detection events corresponds to

$$|\varphi\rangle = \frac{1}{\sqrt{2}}(|O\rangle_+ - |I\rangle_+) \otimes \frac{1}{\sqrt{2}}(|O\rangle_- - |I\rangle_-)$$



- Recall

$$A_W = \frac{\langle\psi|\hat{A}|\varphi\rangle}{\langle\psi|\varphi\rangle}$$

Hardy's Paradox

- Weak Measurement Values

	Op	$WeakVal$
N_O^+	$\vert O^+\rangle\langle O^+\vert$	0
N_I^+	$\vert I^+\rangle\langle I^+\vert$	1
N_O^-	$\vert O^-\rangle\langle O^-\vert$	0
N_I^-	$\vert I^-\rangle\langle I^-\vert$	1

- What does this mean?

Hardy's Paradox

- Joint Measurement Tables

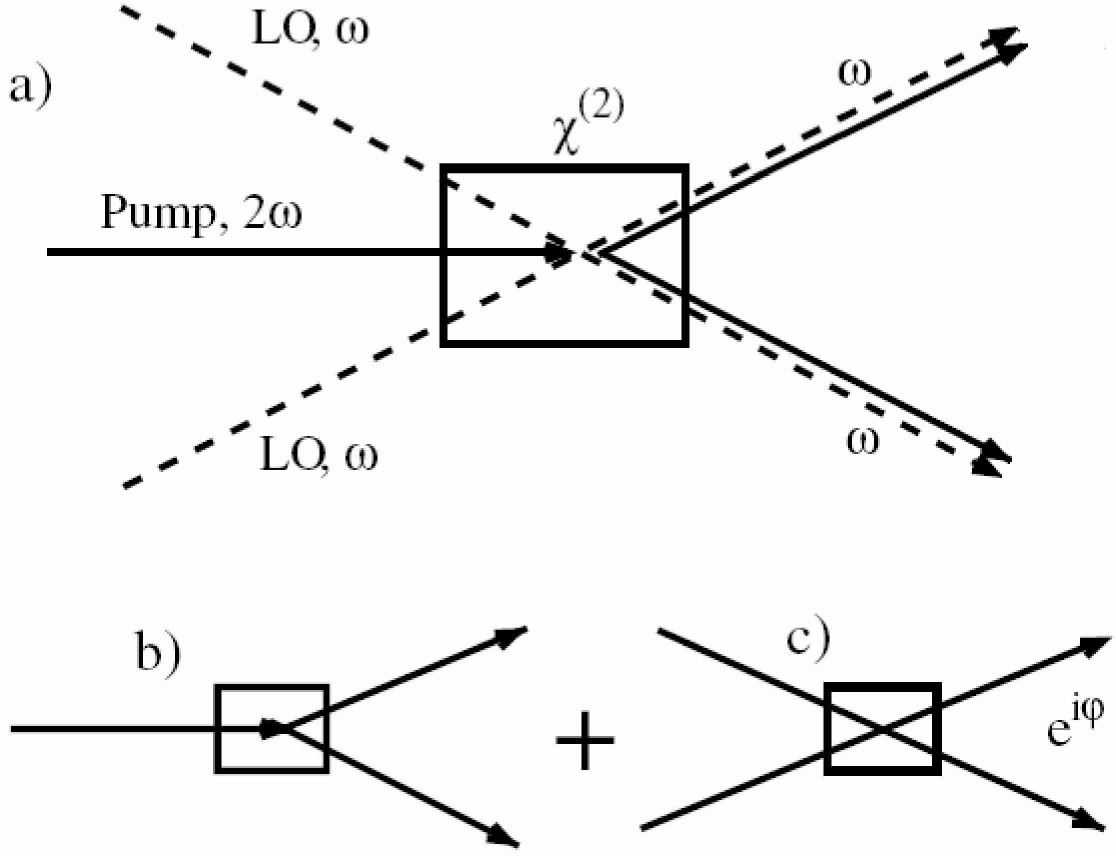
	Op	$WeakVal$
$N_O^+ N_O^-$	$\lvert O^+\rangle\langle O^+\rvert \otimes \lvert O^-\rangle\langle O^-\rvert$	-1
$N_I^+ N_I^-$	$\lvert I^+\rangle\langle I^+\rvert \otimes \lvert I^-\rangle\langle I^-\rvert$	0
$N_I^+ N_O^-$	$\lvert I^+\rangle\langle I^+\rvert \otimes \lvert O^-\rangle\langle O^-\rvert$	1
$N_O^+ N_I^-$	$\lvert O^+\rangle\langle O^+\rvert \otimes \lvert I^-\rangle\langle I^-\rvert$	1

- Quantum mechanics solves the paradox by having -1 particle pairs in the outer arms!

Experimental Implementation

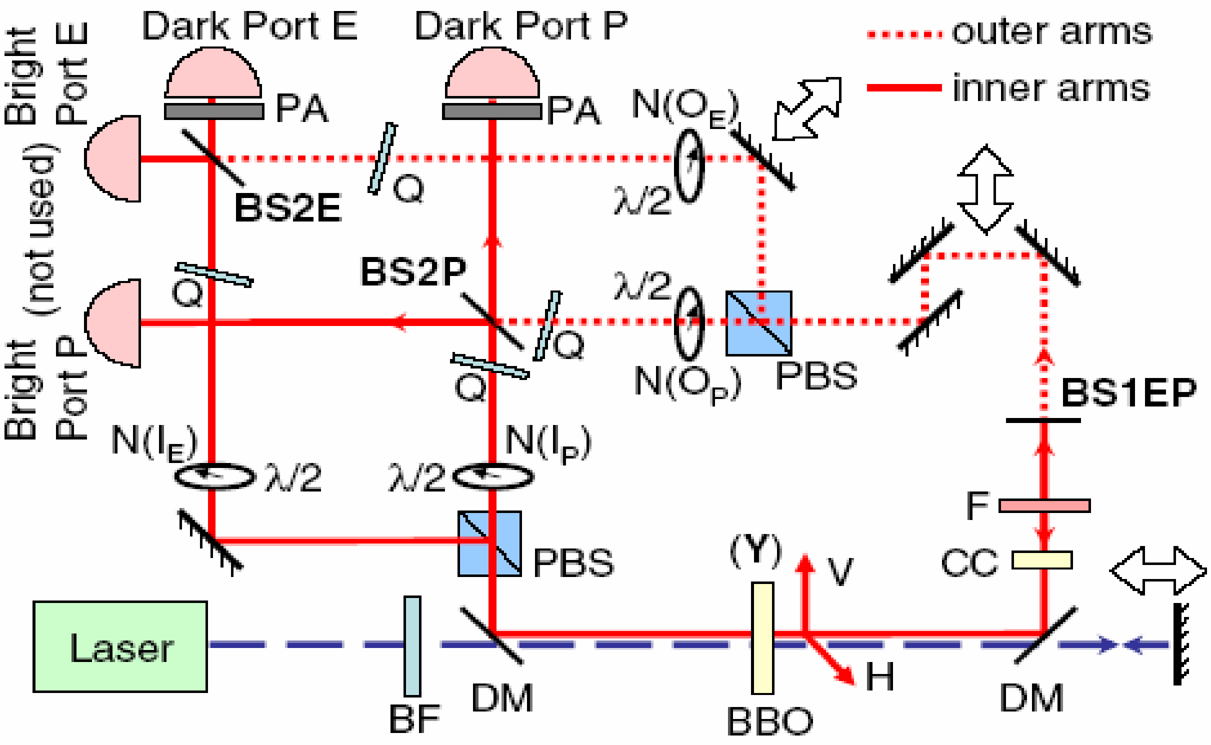
- Particles → Photons
  - + → P
  - - → E
- Annihilation Event → Photon Switch
- Pointer Device → Polarization

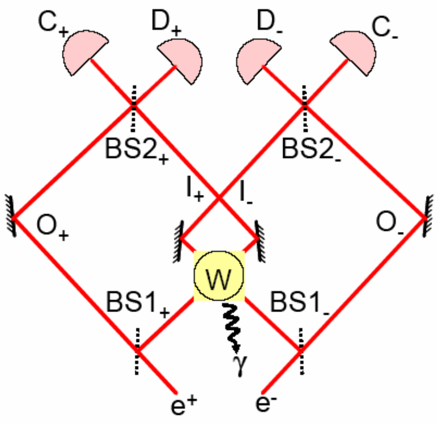
Photon Switch



PRL 87, 123603 (2001) K. Resch, J. Lundeen, & A.M. Steinberg

Experimental Apparatus





PRL 102, 020404 (2009) J. Lundeen & A.M. Steinberg

Measurement

- Weak measurement is done via polarization analysis.
- Wave plates are set to slightly rotate polarization of a passing photon

$$\hat{U}_{couple} = e^{-ig\hat{N}\hat{\sigma}_y}$$

- Different waveplate settings are used for each joint measurement operator

Results

$$\left\langle \hat{N}_K^P \hat{N}_M^E \right\rangle_W = \mathrm{Re}\left\langle \sigma_{ZP}^- \sigma_{ZE}^- \right\rangle = \left\langle \sigma_{XP}\sigma_{XE} \right\rangle - \left\langle \sigma_{YP}\sigma_{YE} \right\rangle \qquad K, M \in \{I, O\}$$

$$\mathrm{Re}\,\langle \hat{\sigma}_{zP}^- \hat{\sigma}_{zE}^- \rangle = \frac{R_{\nearrow\nearrow} + R_{\nwarrow\nwarrow} - R_{\nwarrow\nearrow} - R_{\nearrow\nwarrow}}{R_{\nearrow\nearrow} + R_{\nwarrow\nwarrow} + R_{\nwarrow\nearrow} + R_{\nearrow\nwarrow}} - \frac{R_{\circlearrowleft\circlearrowleft} + R_{\circlearrowleft\circlearrowright} - R_{\circlearrowright\circlearrowright} - R_{\circlearrowright\circlearrowleft}}{R_{\circlearrowleft\circlearrowleft} + R_{\circlearrowleft\circlearrowright} + R_{\circlearrowright\circlearrowright} + R_{\circlearrowright\circlearrowleft}}$$

TABLE I. The measured coincidence rates needed to determine the weak values.

E	P	$R_{\nearrow\nearrow}$	$R_{\nearrow\nwarrow}$	$R_{\nwarrow\nwarrow}$	$R_{\nwarrow\nearrow}$	$R_{\circlearrowleft\circlearrowleft}$	$R_{\circlearrowleft\circlearrowright}$	$R_{\circlearrowright\circlearrowright}$	$R_{\circlearrowright\circlearrowleft}$	g_E	g_P
O	O	556	834	583	730	750	543	666	571	0.674	0.541
I	I	2261	772	115	746	1030	762	913	729	0.635	0.570
I	O	1152	1079	351	179	484	655	452	654	0.635	0.541
O	I	1051	260	329	769	715	609	388	825	0.674	0.570

Results II

TABLE II. The weak values for the arm occupations in Hardy's paradox.

	$N(I_E)$	$N(O_E)$	
$N(I_P)$	0.245 ± 0.068 [0]	0.641 ± 0.083 [1]	$\mathbf{0.926 \pm 0.015}$ **[1]**
$N(O_P)$	0.719 ± 0.074 [1]	-0.759 ± 0.083 [$-$ 1]	$\mathbf{-0.078 \pm 0.02}$ **[0]**
	$\mathbf{0.924 \pm 0.024}$ **[1]**	$\mathbf{0.087 \pm 0.023}$ **[0]**	

- 85% Photon switch efficiency and 95% IFM probabilities are primarcy source of discrepancy

Summary/Conclusions

- Experimental verification indeed verifies all the claims made in the Hardy paradox apparatus.
- Weak values are a valuable tool to measure where standard quantum measurements cannot.
- Hardy’s paradox is resolved with quantum mechanics in a strange way.