Conclusive modification of the inner product of quantum states.

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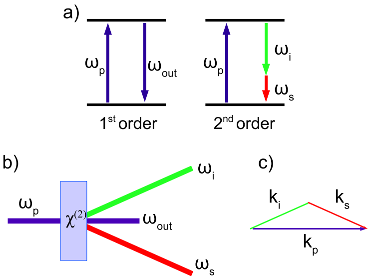
March 4, 2009.

Outline

1. Spontaneous Parametric down conversion process.
   - Generation of twin photons.
   - Types of generation
2. Preparation of quantum states.
   - Manipulation of the quantum states.
3. Mathematical description.
   - Unitary operations.
4. Experimental setup.
   - Setup
   - Results
5. Applications.

S.P.D.C.

The Spontaneous Parametric Down Conversion is a non-linear optic process where a single photon of the pump beam is transformed into two new photons with some conservation laws.



Energy Conservation

$\omega_p = \omega_i + \omega_s$

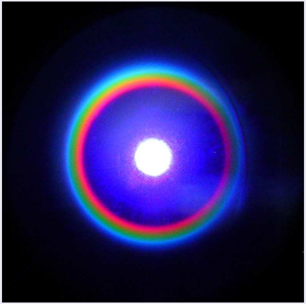
Momentum Conservation

$\vec{k}_p = \vec{k}_i + \vec{k}_s$

Degenerated case

$\omega_i = \omega_s = \frac{\omega_p}{2}$

$\lambda_i = \lambda_s = 2\lambda_p$



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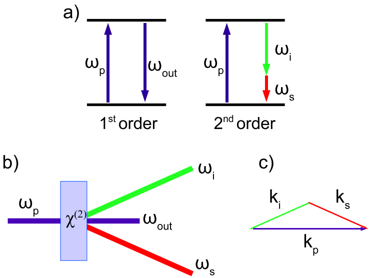

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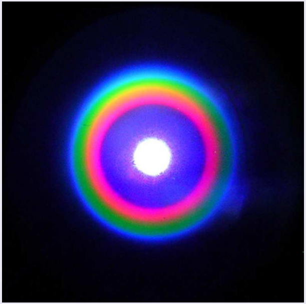

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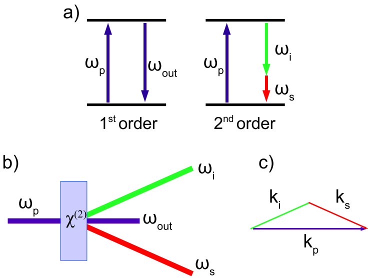

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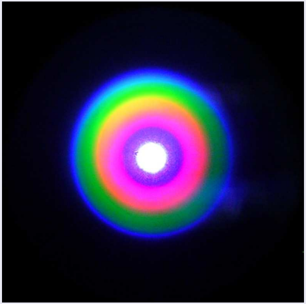
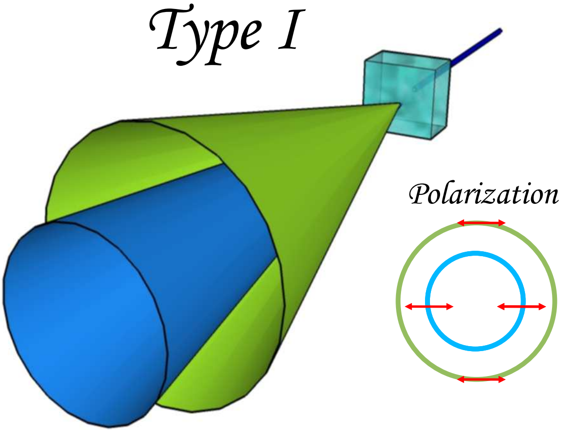
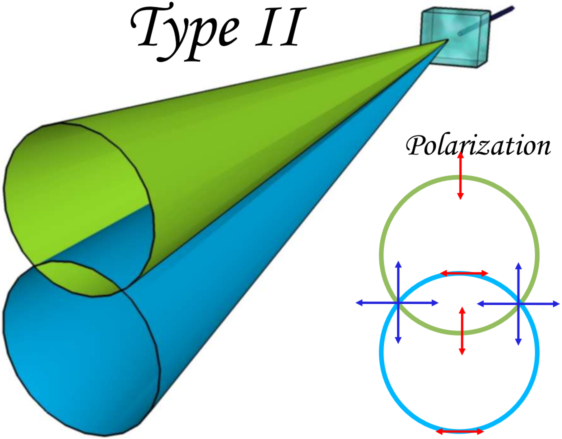


Types of generation: Type I



Types of generation: Type II



Discrimination of quantum states.

Why Discrimination of quantum states?

- Quantum states are not observables... Information encoded on quantum states.
- Minimum Error Discrimination (Helstrom and Holevo). The possible input states, pure or mixed, are identified with some error.
- Unambiguous State Discrimination (pure state). Perfect identification of the states but with the addition of an inconclusive event.

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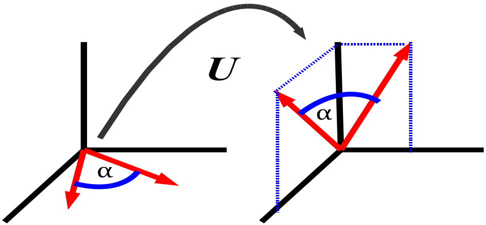

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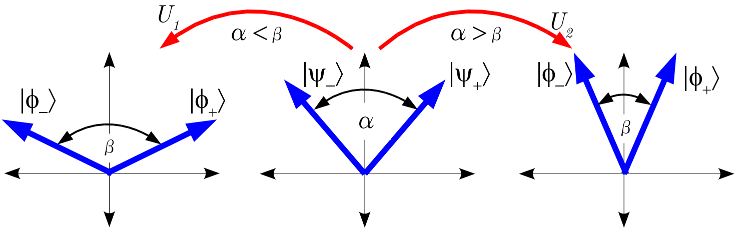


Inner product

Initial states

$$|\psi_\pm\rangle = \cos(\alpha/2)\,|H\rangle \pm \sin(\alpha/2)\,|V\rangle$$

The inner product between this two states is

$$\langle\psi_+|\,\psi_-\rangle = \cos(\alpha)\,, \qquad 0 \leq \alpha \leq \pi \tag{1}$$

After the protocol, the output states changes from $|\psi_\pm\rangle$ to

$$|\phi_\pm\rangle = (\cos(\beta/2)\,|H\rangle \pm \sin(\beta/2)\,|V\rangle) \tag{2}$$

Increasing the angle between the states.

We considerate a unitary operation acting on the basis elements and the ancillary system. That is

$$U_1^{(\gamma_1)} |H\rangle |1\rangle_a = \cos(2\gamma_1) |H\rangle |1\rangle_a + \sin(2\gamma_1) |H\rangle |2\rangle_a \tag{3}$$

$$U_1^{(\gamma_1)} |V\rangle |1\rangle_a = |V\rangle |1\rangle_a \tag{4}$$

The parameter of the transformation is given by

$$\cos(2\gamma_1) = \frac{\tan(\alpha/2)}{\tan(\beta/2)} \tag{5}$$

Using this, the initial state evolve to the state

$$U_1^{(\gamma_1)} |\psi_\pm\rangle |1\rangle_a = \sqrt{P_s^{(1)}} |\phi_\pm\rangle |1\rangle_a + \sqrt{1 - P_s^{(1)}} |H\rangle |2\rangle_a \tag{6}$$

where

$$P_s^{(1)} = \frac{\sin^2(\alpha/2)}{\sin^2(\beta/2)} \tag{7}$$

is the success probability of conclusive modification of the inner product.

Decreasing the angle between the states.

Now, the unitary operation change the elements of the basis as

$$U_2^{(\gamma_2)} |H\rangle |1\rangle_a = |H\rangle |1\rangle_a \quad (8)$$

$$U_2^{(\gamma_2)} |V\rangle |1\rangle_a = \cos(2\gamma_2) |V\rangle |1\rangle_a + e^{i\varphi} \sin(2\gamma_2) |V\rangle |2\rangle_a \quad (9)$$

The parameter of the transformation is given by

$$\cos(2\gamma_2) = \frac{\tan(\beta/2)}{\tan(\alpha/2)} \quad (10)$$

Using this, the initial state evolve to the state

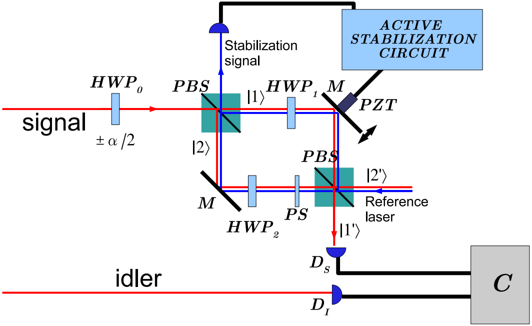
$$U_2^{(\gamma_2)} |\psi_\pm\rangle |1\rangle_a = \sqrt{P_s^{(2)}} |\phi_\pm\rangle |1\rangle_a + e^{i\varphi} \sqrt{1 - P_s^{(1)}} |V\rangle |2\rangle_a \quad (11)$$

where

$$P_s^{(2)} = \frac{\cos^2(\alpha/2)}{\cos^2(\beta/2)} \quad (12)$$

is the success probability of conclusive modification of the inner product.

Setup

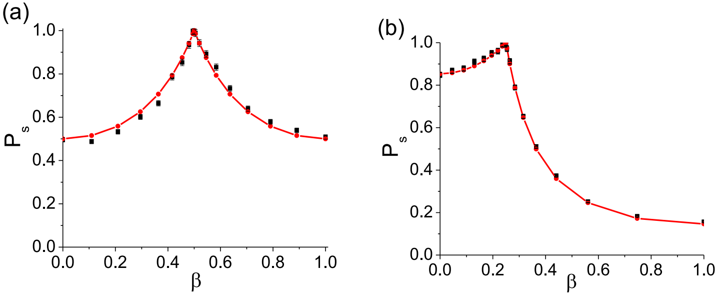


Properties

The setup exhibit the following properties

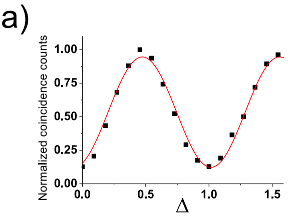
- Each polarization propagates along different path (Conditional operations).
- HWP change the polarization state in each path.
- The path difference is controlled with a thin piece of glass (PS).

Results



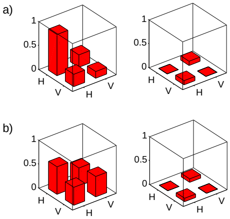
Success probability (black points) as a function of the output angle β in units of π, where the initially superposition was $|\psi_{\pm}\rangle$, with (a) $\alpha = \pi/2$ and (b) $\alpha = \pi/4$. The solid curve corresponds to the success probability $P_{\mathrm{s}}^{(1)}$ (Eq. 7) for β satisfying $\alpha \leq \beta \leq \pi$ and $P_{\mathrm{s}}^{(2)}$ (Eq. 12) for β satisfying $0 \leq \beta \leq \alpha$.

Results



$\Delta = l_1 - l_2$. A linear polarizer rotated at an angle of $\pi/4$ was inserted in front of the detector D_{s1}.

Tomographic reconstruction:
(a)Input state $|\psi_+\rangle$ with $\alpha = \pi/4$ and (b)Output state $|\phi_+\rangle$ with $\beta = 0.44\pi$.
Fidelities:
$F_a = 0.94 \pm 0.03$ $F_b = 0.88 \pm 0.04$



Conclusive Modification of Entanglement.

If the initial state is

$$\begin{aligned}\left|\Psi\right\rangle &= \cos\left(\frac{\alpha}{2}\right)\left|H_s\right\rangle\left|H_i\right\rangle + e^{i\delta}\sin\left(\frac{\alpha}{2}\right)\left|V_s\right\rangle\left|V_i\right\rangle \\ &= \frac{1}{\sqrt{2}}\left(\left|\psi^{(+)}\right\rangle_s\left|+\right\rangle_i + \left|\psi^{(-)}\right\rangle_s\left|-\right\rangle_i\right)\end{aligned}$$

where

$$\left|\psi^{(\pm)}\right\rangle = \cos\left(\frac{\alpha}{2}\right)\left|H_s\right\rangle \pm e^{i\delta}\sin\left(\frac{\alpha}{2}\right)\left|V_s\right\rangle, \qquad \left|\pm_i\right\rangle = \frac{1}{\sqrt{2}}\left(\left|H_i\right\rangle \pm \left|V_i\right\rangle\right)$$

Degree of entanglement and inner product.

$$E(\Psi) = \sqrt{1-\psi^2}, \qquad \left\langle \psi^{(+)} \middle| \psi^{(-)} \right\rangle = \gamma = \cos(\alpha)$$

If we rotate both plates inside the interferometer, then the state changes to

$$\begin{aligned}
|\Phi\rangle &= \sqrt{N_1}\left|\Psi^{(1)}\right\rangle - i\sqrt{N_2}\left|\Psi^{(2)}\right\rangle \\
N_1 &= \cos^2\left(\frac{\alpha}{2}\right)\cos\left(2\gamma_1\right) + \sin^2\left(\frac{\alpha}{2}\right)\cos\left(2\gamma_2\right) \\
N_2 &= 1 - N_1 \\
\left|\Psi^{(1)}\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|\phi_1^{(+)}\right\rangle_s |+\rangle_i + \left|\phi_1^{(-)}\right\rangle_s |-\rangle_i\right) \\
\left|\Psi^{(2)}\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|\phi_2^{(+)}\right\rangle_s |+\rangle_i + \left|\phi_2^{(-)}\right\rangle_s |-\rangle_i\right)
\end{aligned}$$

where the states $\left|\phi_{12}^{(\pm)}\right\rangle$ are

$$\begin{aligned}
\left|\phi_1^{(\pm)}\right\rangle_s &= \frac{1}{\sqrt{N_1}}\left(\cos(\alpha/2)\cos(2\gamma_1)\,|H_1\rangle_s \pm e^{i\delta}\sin(\alpha/2)\cos(2\gamma_2)\,|V_1\rangle_s\right) \\
\left|\phi_2^{(\pm)}\right\rangle_s &= \frac{1}{\sqrt{N_2}}\left(\cos(\alpha/2)\cos(2\gamma_1)\,|V_1\rangle_s \pm e^{i\delta}\sin(\alpha/2)\cos(2\gamma_2)\,|H_1\rangle_s\right)
\end{aligned}$$

The entanglement of the states $\left|\Psi^{(1,2)}\right\rangle$ are given by

$$E(\Psi^{(1)}) = E(\Psi)\frac{|\cos(2\gamma_1)\cos(2\gamma_2)|}{N_1},$$

$$E(\Psi^{(2)}) = E(\Psi)\frac{|\sin(2\gamma_1)\sin(2\gamma_2)|}{N_1},$$

Some cases

- $E(\Psi) = 0$. State at the output port will be still separable. Local operations cannot create entanglement.
- $E(\Psi) \neq 0$, $\gamma_1 = \gamma_2 = 0, \pi/4$, then $E_{out} = E(\Psi)$.
- $E(\Psi) \neq 0$, $\gamma_1 = \gamma_2 \neq 0$, then $E_{out} = E(\Psi)$.
- $E(\Psi) \neq 0$, $\gamma_1 = \gamma_2 \neq 0$, then $E_{out} = E(\Psi)$.

if

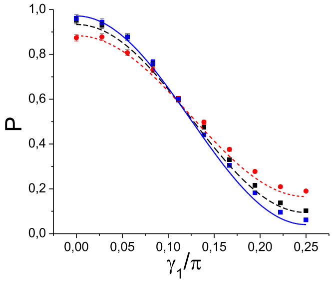
$$\cos^2(\alpha/2) = \frac{1+\epsilon}{2}, \qquad \epsilon \in [-1, 1]. \tag{13}$$

then

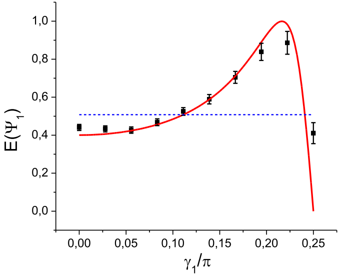
$$\epsilon \geq \frac{|\cos(2\gamma 2) - \cos(2\gamma_1)|}{|\cos(2\gamma 2) + \cos(2\gamma_1)|} \tag{14}$$

when $|\cos(2\gamma_2)| \leq |\cos(2\gamma_1)|$ or

Measured success probability, $N1$, of generating output state $|\Psi^{(1)}\rangle$ as a function of the angle γ_1 at HWP_1, while $\gamma_2 = \pi/9$, for different values of entanglement of the initial states. Points are experimental results and lines are theoretical fit $E(\Psi) = 0.51$ (triangles and solid line); $E(\Psi) = 0.74$ (squares and dashed line); $E(\Psi) = 0.90$ (circles and dotted line).



Measured degree of entanglement of the output state $|\Psi^{(1)}\rangle|$ as a function of the angle γ_1 at HWP1, while $\gamma_2 = \pi/9$. Points are experimental results and the solid line is the theoretical fit given by Eq. (18). The straight solid line represents the entanglement of the initial state, $E(\gamma^{(1)}) = 0.51$. The probability of generation for each state is given by the triangles.



Conclutions

- We have experimentally demonstrated the conclusive modification of non-orthogonal quantum states.
  - The case where the inner product between the initial states is larger than that between the final states.
  - The case where the inner product between the initial states is shorter than that between the final states.
- This protocol can by used for establishing a common secret key in the so-called "4+2" quantum cryptographic protocol.
- This experimental setup also is well suited for the controlled generation of mixed states in polarization for two qubits as well as for their unambiguous discrimination.

This experiment has been performed using a separable polarization two-photon state. A similar experiment could be conducted using an arbitrary entangled polarization two-photon state. The possibility of mapping non-orthogonal states onto non-orthogonal allows one to increase or decrease the degree of entanglement of the initial state at will.

Acknowledgment

Milenio ICM P06-067F
FONDECyT 1061046.
Department of Physics, University of Concepción
MECESUP

C.Q.O.Q.I.


Summer
Season in
Concepción
Thanks...
Omar
Jimenez
Benjamin
Burgos
José
Aguirre
Aldo
Delgado
Fabián
Torres
Rodrigo
Fuentes
Carlos
Saavedra
Sebastiao
De Pádua
Gustavo
Lima

Experimental Team

In the picture... from left to right

G. Lima, A. Delgado, J. Aguirre, S. Pádua, C. Saavedra and F. Torres as photographer

Thanks

Center for Quantum Optics and Quantum Information