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Experimental Quantum Process Discrimination

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Discrimination between unknown processes chosen from a finite set is experimentally shown to be possible even in the case of nonorthogonal processes. We demonstrate unambiguous deterministic quantum process discrimination of nonorthogonal processes using properties of entanglement, additional known unitaries, or classical communication. Single qubit measurement and unitary processes and multipartite unitaries (where the unitary acts nonseparably across two distant locations) acting on photons are discriminated with a confidence of $\geq 97\%$ in all cases.

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Outline

- Introduction
- Measurement Quantum Process Discrimination (QPD)
- Orthogonal Unitary Operations
- Entanglement assisted unitary QPD
- Unitary QPD without entanglement
- Multi-partite QPD without entanglement
- Conclusions

Introduction

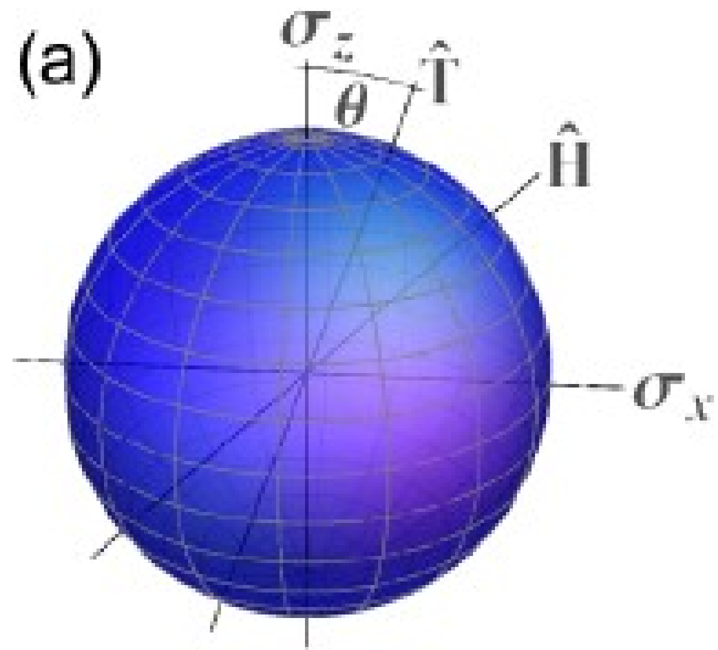
- Quantum process transforms a quantum system
  - Measurements, logic gates, decoherence etc.
- In QPD want to identify an unknown process from a set of nonorthogonal processes
- Key differences between QPD and quantum state discrimination
  - States can only be 'used' once

Measurement QPD

- Distinguish between measurements along z axis and T
- Simple case:

$$\theta = 2\arctan\left(\frac{1}{\sqrt{n-1}}\right)$$

- Can use n-qubit W state to distinguish the states perfectly



$$W^{(n)} \equiv (|0\ldots01\rangle + |0\ldots10\rangle + \ldots + |100\ldots\rangle)/\sqrt{n}$$

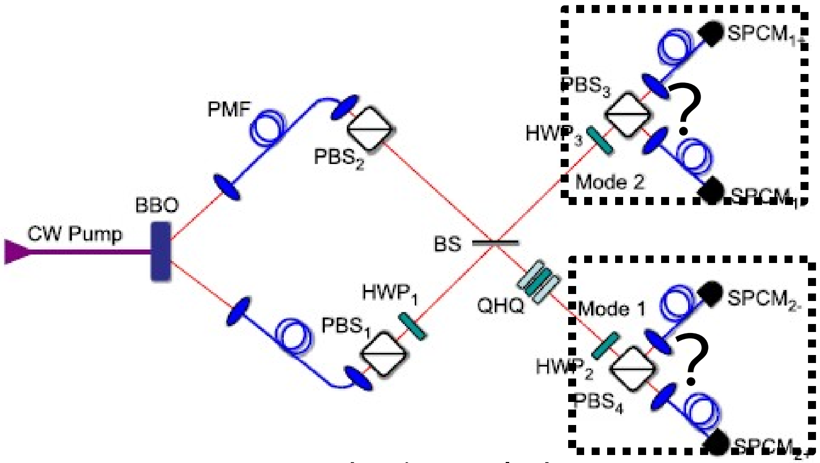
Measurement QPD

simple example with n=3

- Now: $\theta = 70.5^o$
- **T** becomes: $T = \frac{1}{3}\sigma_z + \frac{2\sqrt{2}}{3}\sigma_x$
- In the z-basis: $|W^3> = (|001> + |010> + |100>)/\sqrt{3}$
- In the t-basis: $|W^3> = (2|000> - |011> - |101> - |110> - \sqrt{2}|111>)/3$
- So a measurement of σz will yield exactly one -1 eigenvalue, and a measurement of T will yield something else, but never exactly one -1 eigenvalue

Measurement QPD

The Experiment with n=2



Type I SPDC, two horizontal photons are coupled into fiber. A half-wave plate (HWP1) rotates the photon in mode 1 from horizontal (H) to vertical (V), and the two orthogonally polarized photons impinge on the input ports of the central 1/2 reflectivity beam splitter (BS) creating the state |H1V1>+|H1V2>+|V1H2>+ |V2H2>.

The optic axes of HWP2 and HWP3 are set to 0 or 22.5 to perform z or H, respectively. Orthogonal polarizations are detected by a PBS followed by two single photon counting modules (SPCMs) in each mode.

- Angle of 90 i.e. measure in H/V (Bloch z axis) and A/D(Bloch x axis)
- The 2 qubit W-state in H/V:

$$|\psi\rangle = (|H1,V2\rangle + |V1,H2\rangle)/\sqrt{2}$$

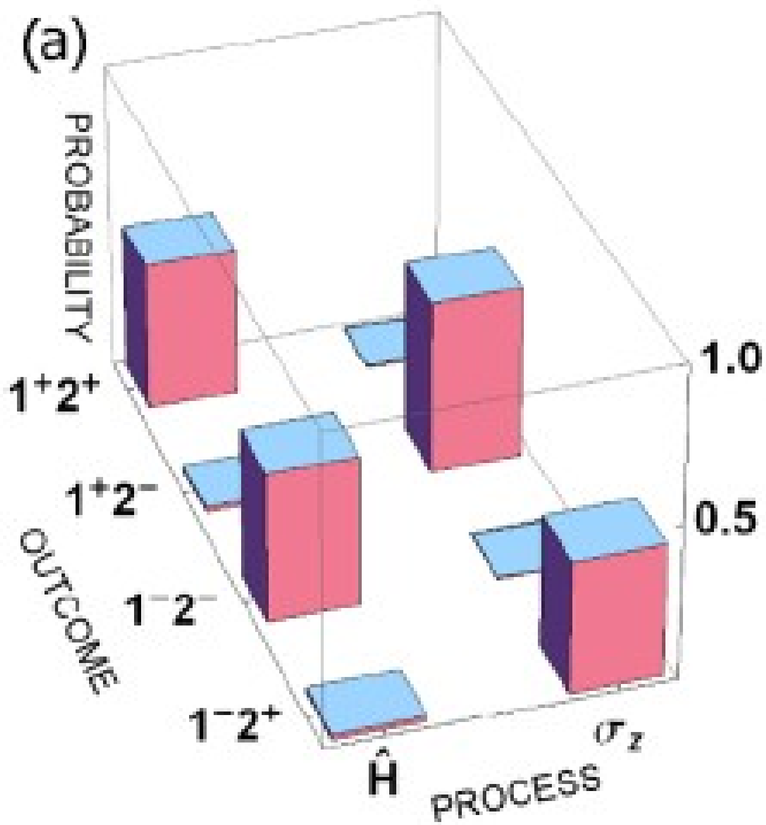
- In A/D:

$$|\psi\rangle = (|D1,D2\rangle - |A1,A2\rangle)/\sqrt{2}$$

- Correlated results
  - Means measurement in A/D
- Anti-correlated results
  - Means measurement in H/V

Measurement QPD

The Results



- When the HWP is set to 22.5 in measure A/D see correlations
- When the HWP set to 0 measure in H/V and see correlations
- Distinguish with 97.8 ± 0.05% confidence

Nonorthogonal Unitaries

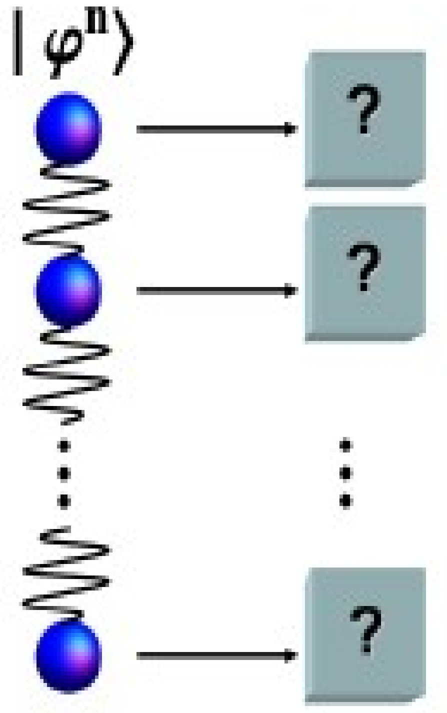
- Want to distinguish **U** and **V**
- If for some |ϕ>, **U**|ϕ> and **V**|ϕ>are orthogonal

$$\langle\phi|V^{\dagger}U|\phi\rangle=0$$

- It was proven for some |ϕ> and finite integer n:

$$\langle\phi|(V^{\dagger}U)^{n}|\phi\rangle=0$$

- Where |ϕ> is an n-partite entangled state



Nonorthogonal Unitaries

General Procedure for Discrimination

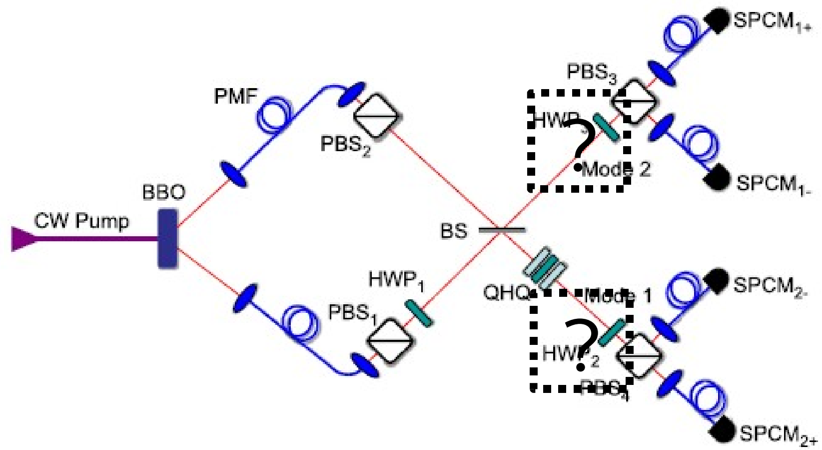
0) Find N and |ϕ> for your unknowns

1) prepare an N-qubit entangled state

2) apply the unknown operation to each qubit

3) preform projective measurements to distinguish the possible final states

Entanglement Assisted Unitary QPD



- Now draw the black box only around the HWP.
  - Discriminate **σz** and **H**
- Input state:

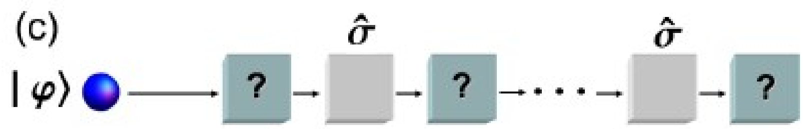
$$|\psi\rangle=(|H1,V2\rangle+|V1,H2\rangle)/\sqrt{2}$$

- Measure in H/V
- Depending on the unknown process:

$$H|\psi\rangle=(|H1,H2\rangle-|V1,V2\rangle)/\sqrt{2} \quad \text{OR} \quad \sigma_z|\psi\rangle=(|H1,V2\rangle+|V1,H2\rangle)/\sqrt{2}$$

- Again look for correlations or anti-correlations

Unitary QPD without entanglement



- If the unknown can be applied sequentially no entanglement is needed
- Start with **U**|ϕ> and **V**|ϕ>
  - if not orthogonal apply some other unitary, σ, followed by the unknown
  - If still not orthogonal repeat
- Final orthogonal states:

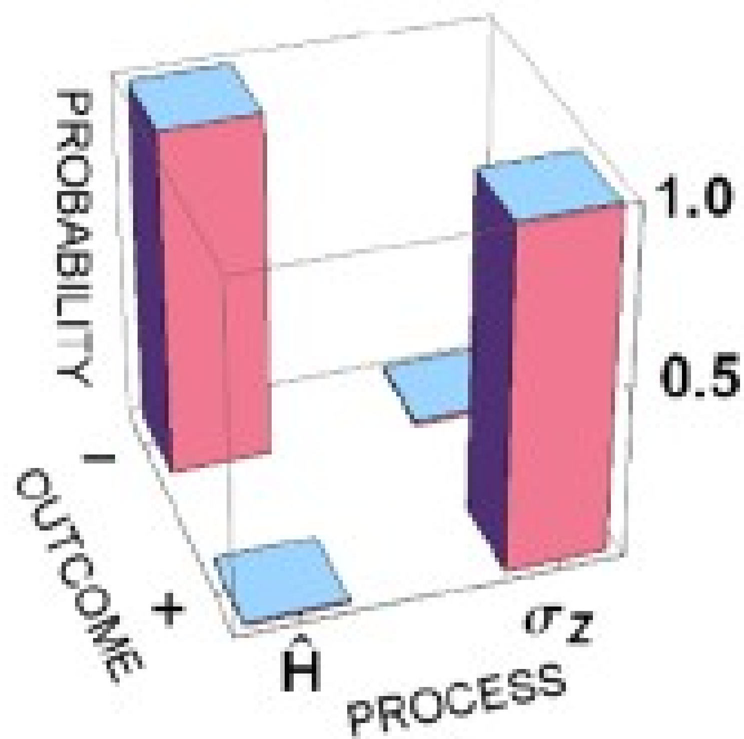
$$|\psi_U\rangle = UXU.....UXU\,|\psi\rangle$$
$$|\psi_V\rangle = VXV.....VXV\,|\psi\rangle$$

- Instead of using some N-qubit entangled state apply the unknown N times
- This N is the same as the entangled parallel case

Unitary QPD without entanglement

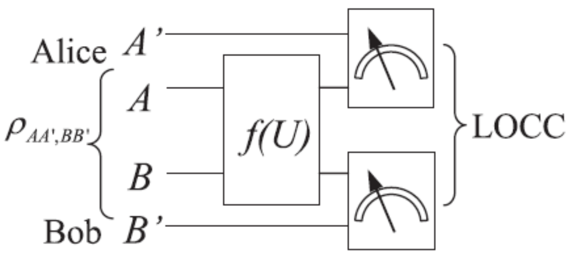
The Experiment

- To discriminate **σz** and **H** find N=2 and **σ = σz**
- The 2 possible outcomes:
  - $H\sigma_z H|\psi>$ and $\sigma_z\sigma_z\sigma_z|\psi>=\sigma_z|\psi>$
- If: $|\psi>=|0>$
  - $\sigma_z|0>=|0>$ and $H\sigma_z H|0>=|1>$
- Experimentally
  - 3 HWP's followed by a polarization analyzer in H/V
- Discrimination with 99.0±0.02% confidence



Multipartite QPD without entanglement

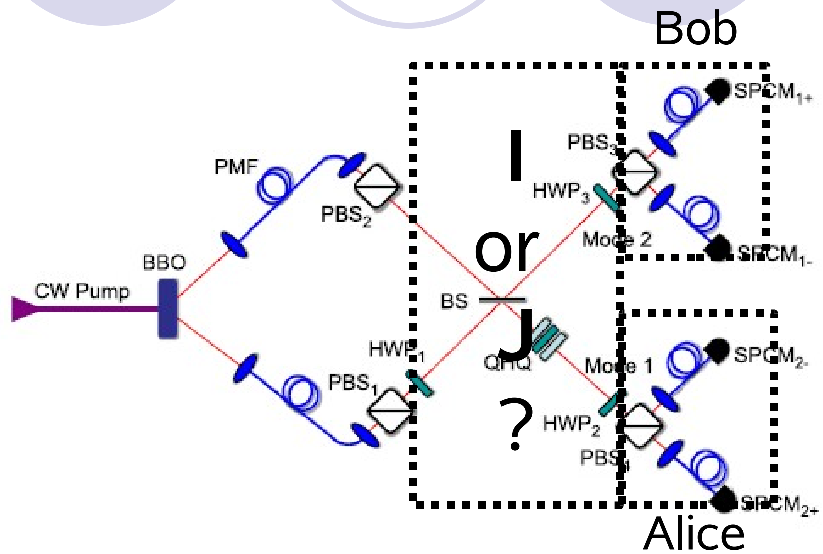
- “The problem of distinguishing multipartite unitary operations naturally arises when several parties share a unitary operation but forget its real identity.” (Duan, R. *et al*)



- Unknown unitary acting on multiple nonlocal qubits only use LOCC to discriminate
- Results of theory paper (Duan, R. *et al.* and Zhou, X.F. *et al.*)
  - We don't need any a prior nonlocal entanglement
  - At worst one party is required to prepare local entanglement
- “Although we cannot present the optimal quantum circuits and input states, we prove that, in principle, any two unitary operations U and V can be perfectly identified locally.” (Zhou, X.F. *et al.*)

Multipartite QPD without entanglement

The Experiment



- New Basis:

$$|0>_A = |0>_1 \qquad |0>_B = |0>_2$$
$$|1>_A = |1H>_1 \qquad |1>_B = |1H>_2$$
$$|2>_A = |1V>_1 \qquad |2>_B = |1V>_2$$
$$|3>_A = |2H>_1 \qquad |3>_B = |2H>_2$$
$$|4>_A = |1H,1V>_1 \qquad |4>_B = |1H,1V>_2$$
$$|5>_A = |2V>_1 \qquad |5>_B = |2V>_2$$

- New Operators:

$\hat{I} = I^{(6)} \otimes I^{(6)}$ and $\hat{J} \equiv (H^{(6)} \otimes H^{(6)}).\hat{B}$

$$H^{(6)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & 0 & 0 & 0 \\ 0 & & \hat{H} & 0 & 0 & 0 \\ 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & H^{(3)} & \\ 0 & 0 & 0 & & & \end{pmatrix}$$

$$H^{(3)} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}$$

- Initial state:

$$|\psi\rangle = |2\rangle_A \otimes |1\rangle_B$$

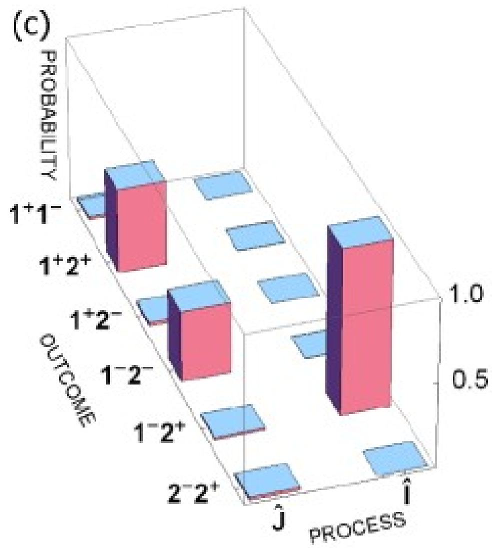
- **J** takes it to an orthogonal state:

$$\frac{1}{2\sqrt{2}}(|0\rangle_A|3\rangle_B - |0\rangle_A|5\rangle_B + |1\rangle_A|1\rangle_B - |2\rangle_A|2\rangle_B + |3\rangle_A|0\rangle_B - |5\rangle_A|0\rangle_B$$

- Look for correlations and anti-correlations

Multipartite QPD without entanglement

The Results



Discrimination with 96.6 ± 0.4% confidence

Conclusions

- Looked at schemes for discriminating:
  - PVM measurements
  - Unitary operations with entanglement
  - Unitary operations without entanglement
  - Nonlocal unitary operations using only LOCC
- In general it is always possible to discriminate even non local unitaries only using LOCC

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