

Group Meeting

9/21/11

Dylan Mahler

Universal digital quantum simulation with trapped ions

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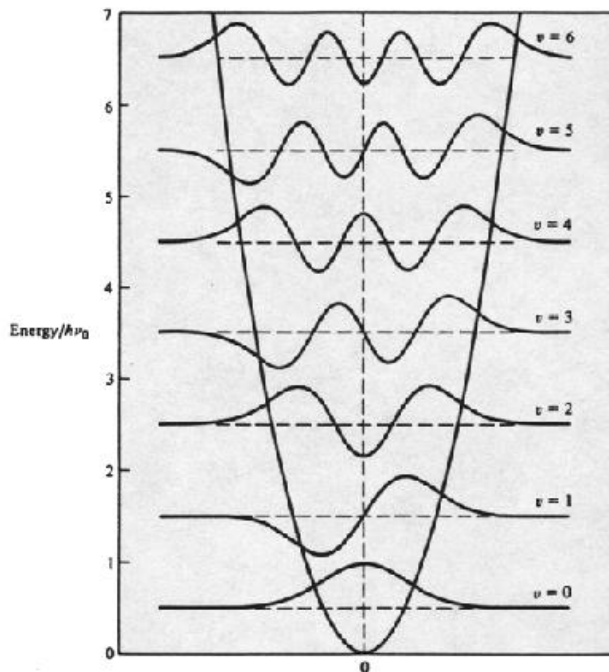
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Outline

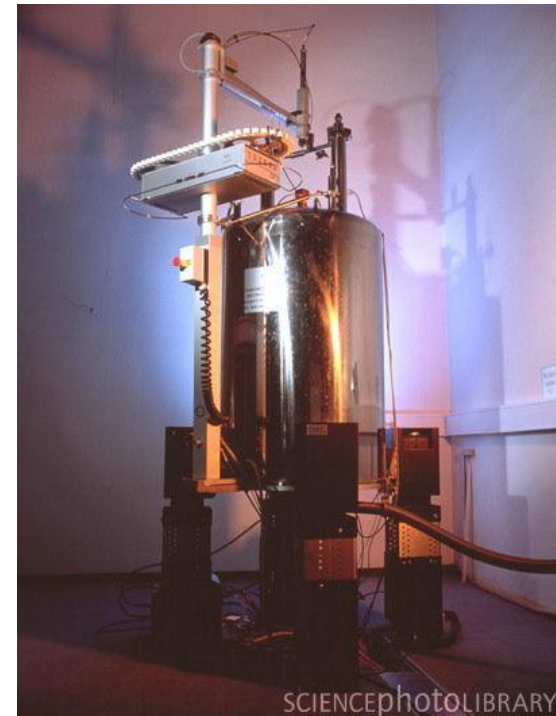
- Simulators: Analog or Digital?
- The Trotter Approximation
- Trapped Ions
- Results: Digital Simulation of 6 Spins with Ising-type interactions

Quantum Simulations

Quantum System



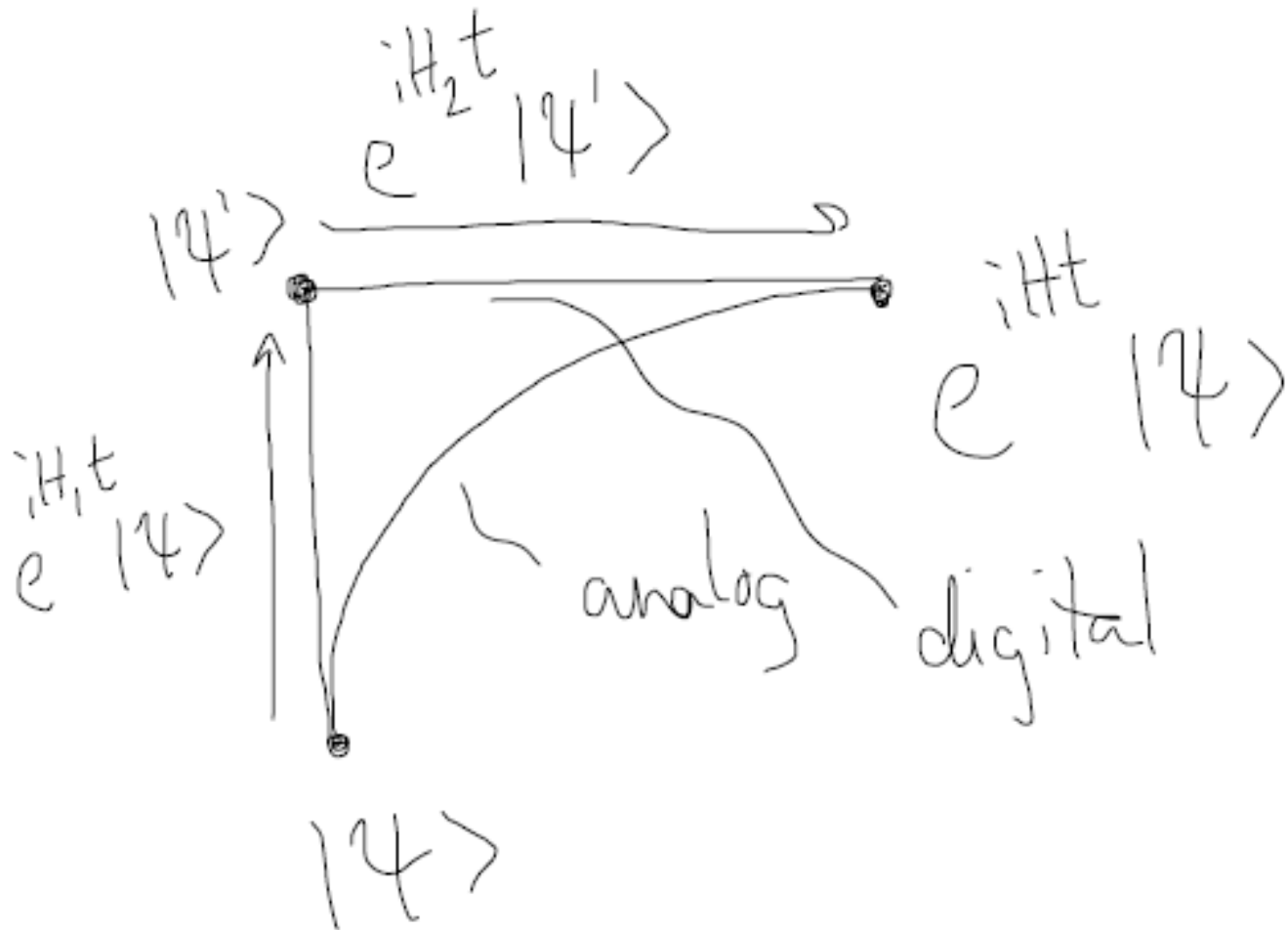
Simulator



Analog vs. Digital

- Analog:
 - One to one mapping from system to simulator
 - Target interaction exists in simulator already
- Digital
 - Not necessarily one to one mapping
 - Interaction is “built up” by discrete jumps in Hilbert space

Hilbert Space



Problem!

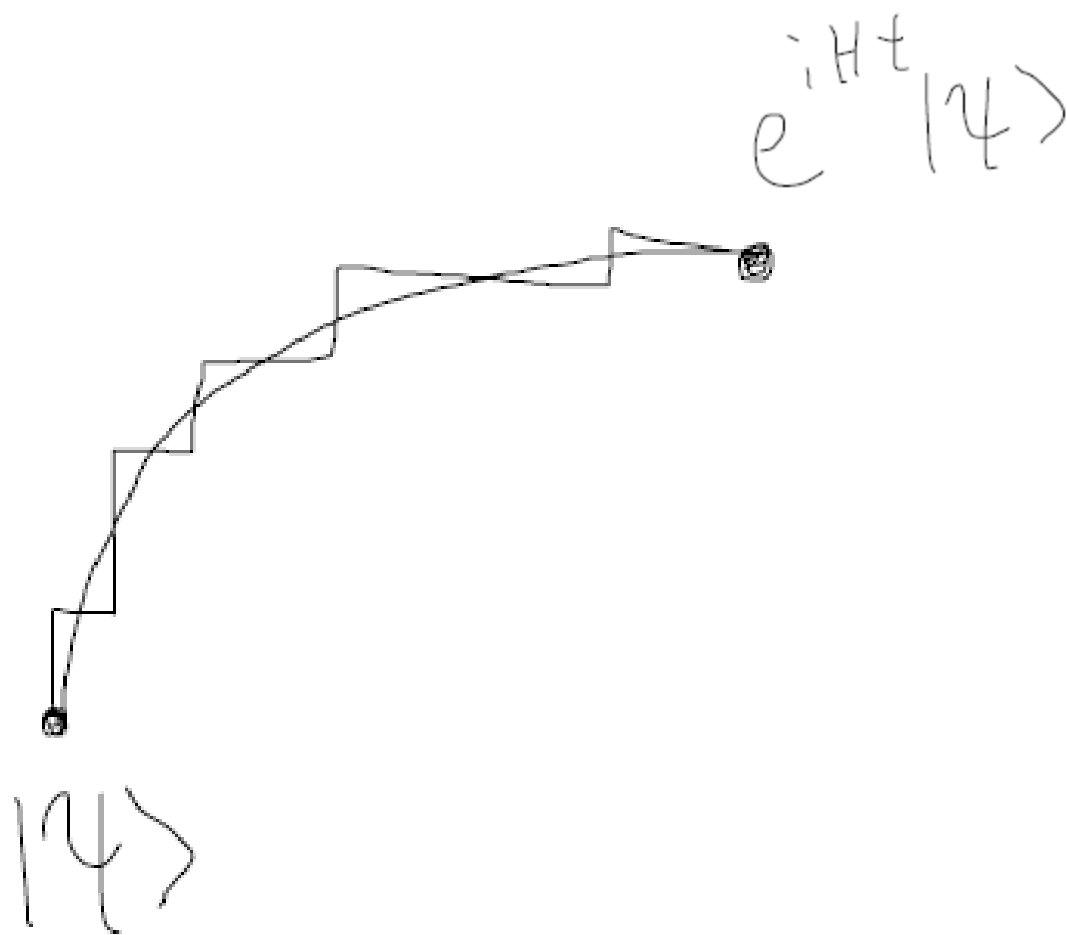
If:

$$H = \sum_{i=1}^{\ell} H_i,$$

Then in general:

$$U \neq \prod_k e^{-iH_k t/\hbar}$$

Problem: Fixed!



The Trotter Approximation

$$e^{-iHt} = \lim_{n \rightarrow \infty} \left(\prod_k e^{-\frac{i}{\hbar} H_k t / n} \right)^n$$

- Nice things:
 - Error is strictly bounded
 - This is efficient (linear in the number of systems), as long as the number of different H_k 's is on the order of N
 - This also efficient in time!


Trapped Ions

20. H. F. Trotter, *Proc. Amer. Math. Soc.* **10**, 545 (1959).

21. S. Lloyd, L. Viola, *Phys. Rev. A* **65**, 010101 (2001).

22. Materials and methods are available as supplementary online material.

23. A. Sørensen, K. Mølmer, *Phys. Rev. Lett.* **82**, 1971 (1999).

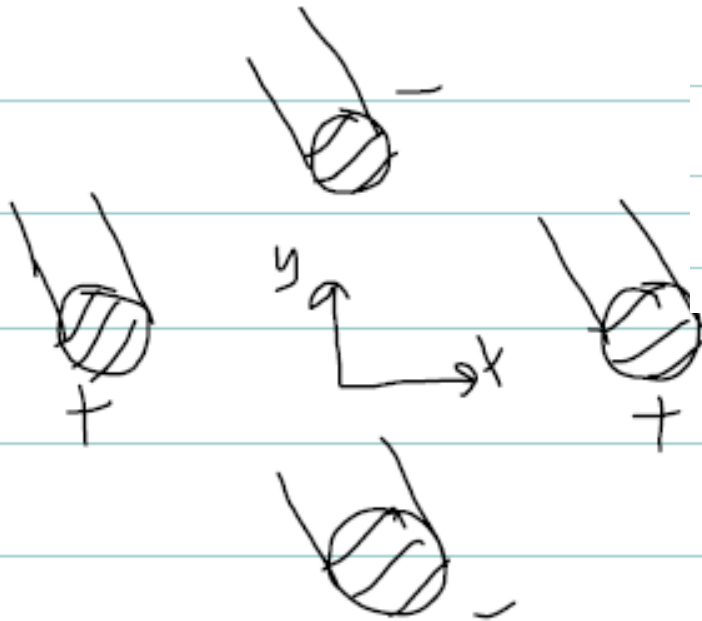


?!?!%&?#@%\$!!!! (No⁸
supplementary materials for a
preprint)

Basics of Ion Trapping

- Goal:
 - Trap an ion (charged particle!)
 - Use it for quantum computing (we need a good qubit + local unitaries + CNOTS)
- Naively use a static electric field?
 - Nope! No stable equilibrium exists (Earnshaw's Theorem)
- Instead, use oscillating electric fields

Quadrupole Trap (Paul Trap)



$$V_+(t) = U_0 + V_0 \cos(\Omega t)$$
$$V_-(t) = -V_+(t)$$

$$x(t) = X_0 [\cos(\omega_x t)] (1 + f(t))$$

↳ "Micromotion"

↳ Harmonic motion

"Effective potential"

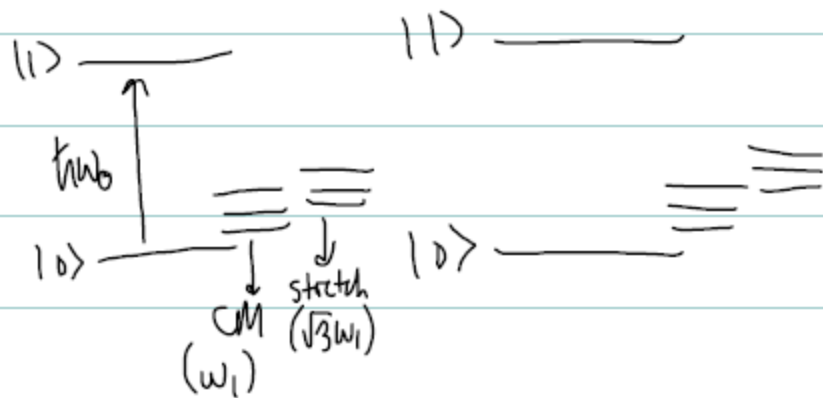
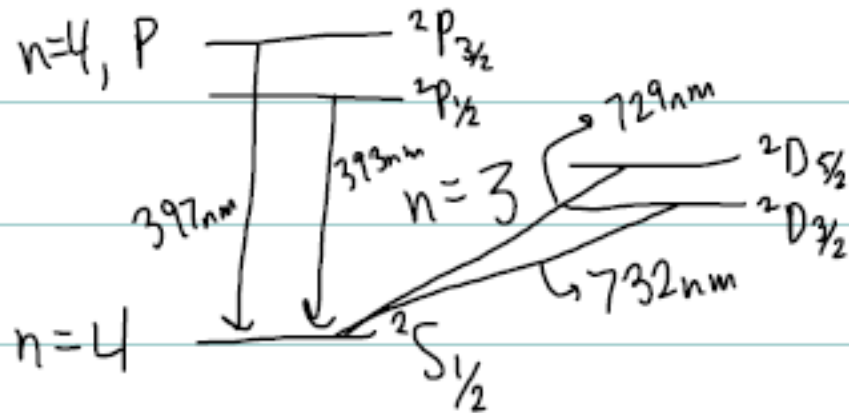
$$U(x, y, z) = \frac{1}{2} m (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

r.f. radial
trapping direction

↳ E/m trapping

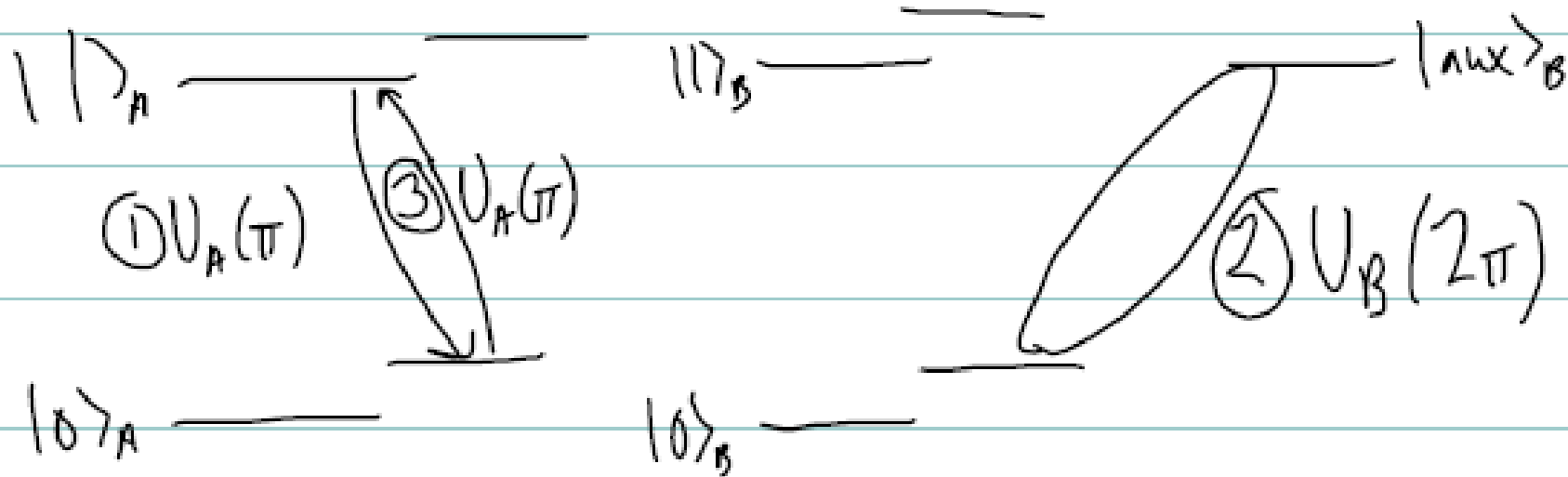
Typically, $\omega_x, \omega_y \gg \omega_z$

Recall for Ca^+ :



- sidebands from external motion.

Cirac-Zoller Quantum Gate



$$|00\rangle \xrightarrow{\textcircled{1}} |00\rangle \xrightarrow{\textcircled{2}} |00\rangle \xrightarrow{\textcircled{3}} |00\rangle$$

$$|01\rangle \rightarrow |01\rangle \rightarrow |01\rangle \rightarrow |01\rangle$$

$$|10\rangle \rightarrow |00\rangle|1\rangle \rightarrow -|00\rangle|1\rangle \rightarrow |10\rangle$$

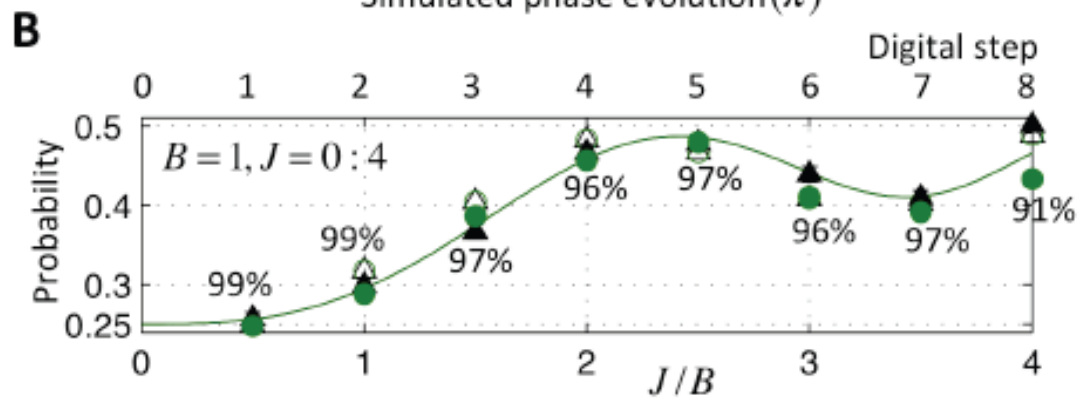
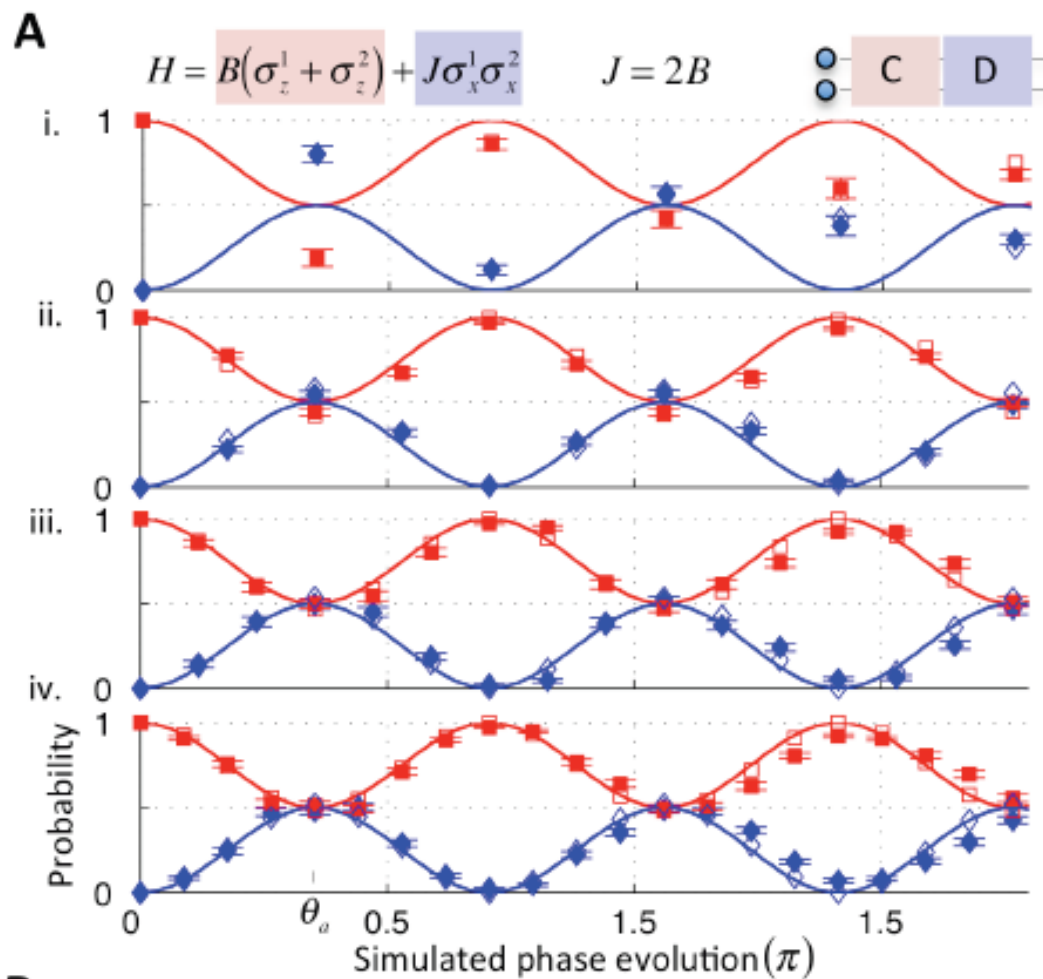
$$|11\rangle \rightarrow |01\rangle|1\rangle \rightarrow |01\rangle|1\rangle \rightarrow -|11\rangle$$

} C-Z Gate

Ising Interaction (2 Spins)

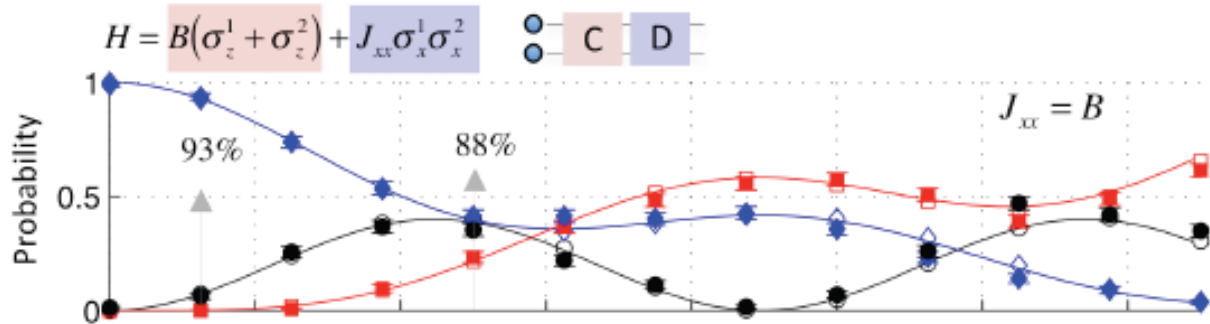
$$\tilde{H}_{\text{Ising}} = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2.$$

- First two terms are single qubit rotations (easy to implement)
- Last term is a “qubit-qubit interactions” which is accomplished with CNOTS and single qubit unitaries

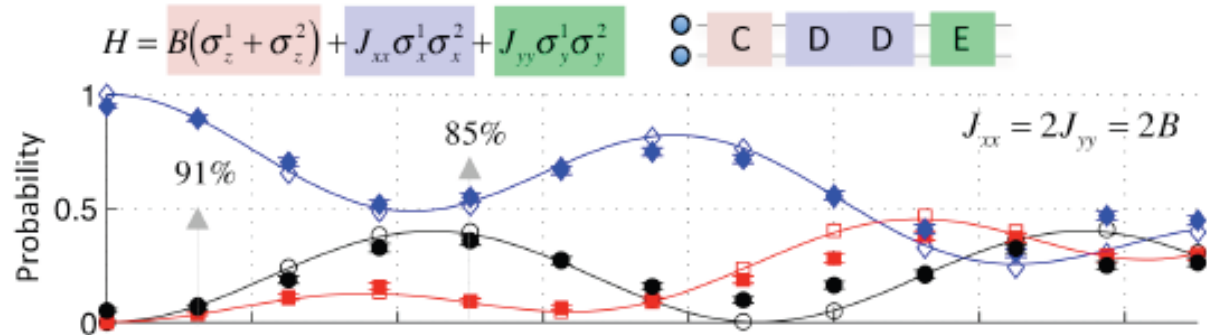


Cool, but its been done before...

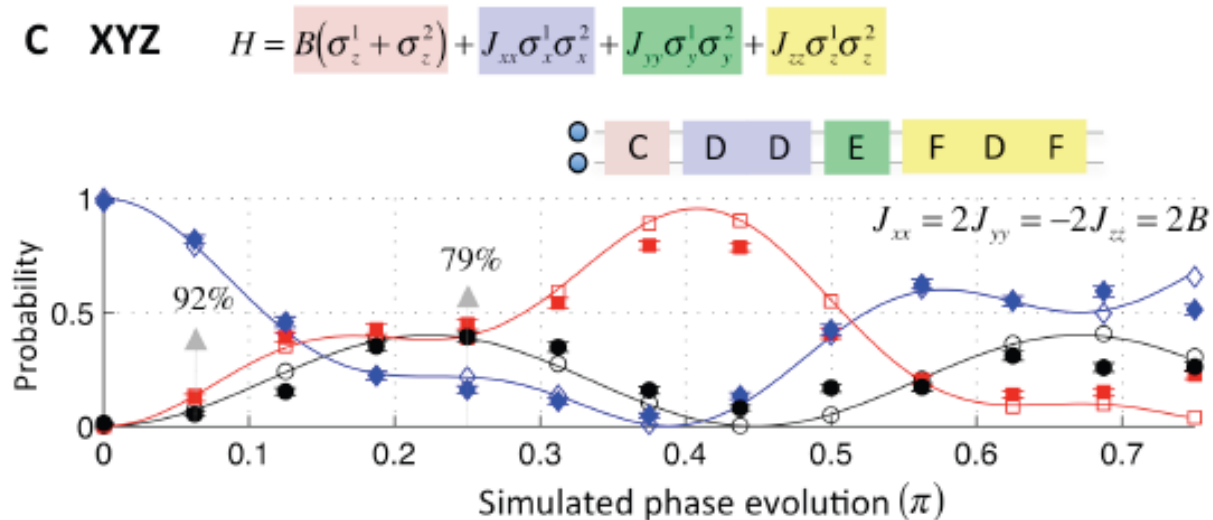
A Ising



B XY

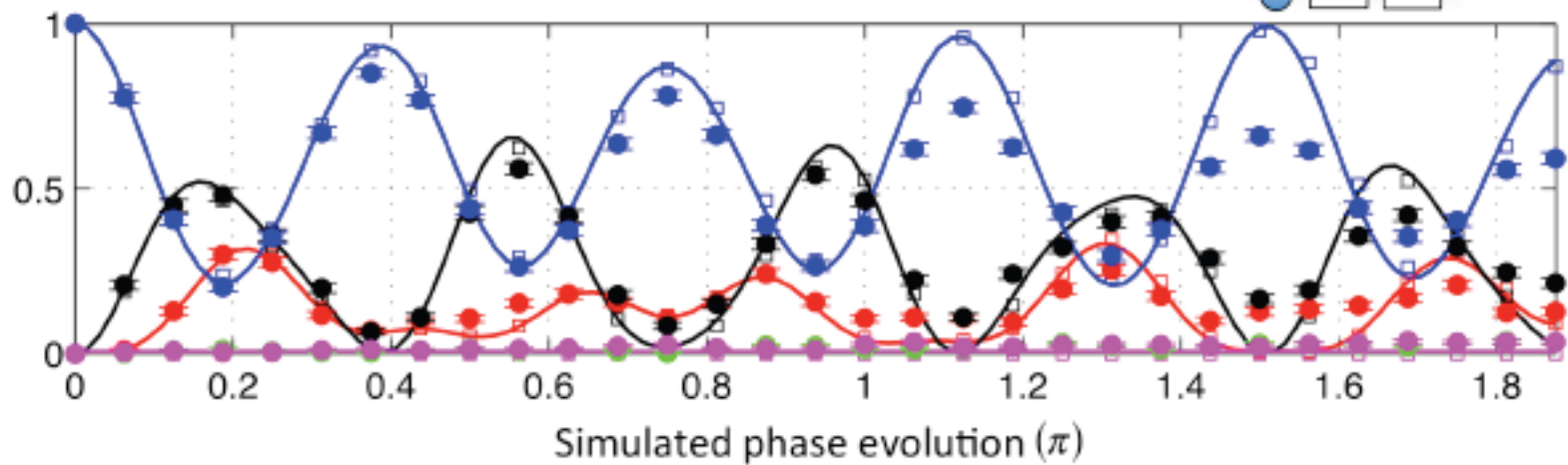
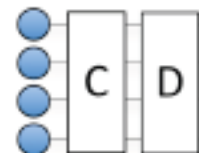


C XYZ

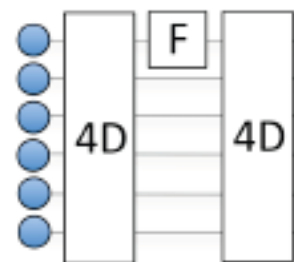
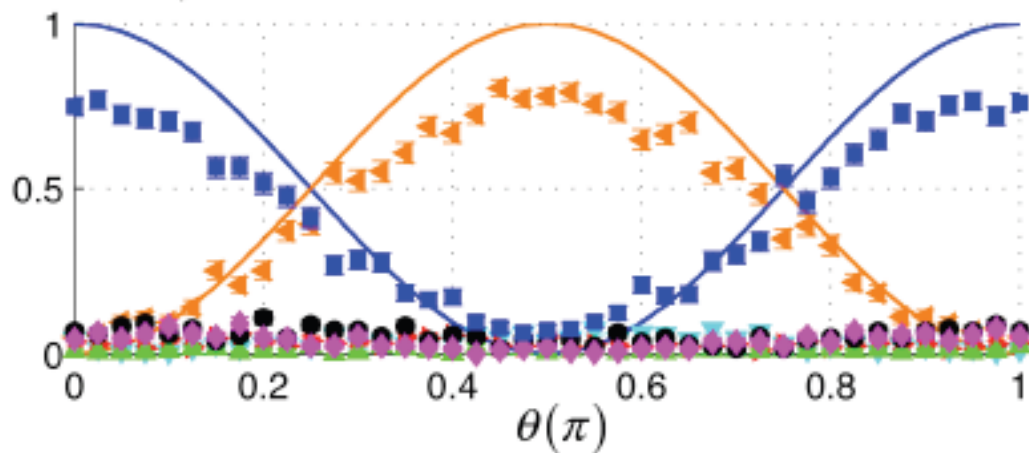


A

$$H = J \sum_{i < j} \sigma_x^i \sigma_x^j + B \sum_k \sigma_z^k \quad J = 2B$$

**B**

$$H = \sigma_y^1 \sigma_x^2 \sigma_x^3 \sigma_x^4 \sigma_x^5 \sigma_x^6$$



Conclusion

- Trapped ions can be quantum simulators!
- Main source of error is intensity fluctuations in laser: “This is not currently a fundamental limitation and, once properly addressed, should enable an increase in simulation capabilities.”
- Unfortunately, the 6-qubit example with >100 gates is in the supplementary material...