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LETTERS

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# Experimental high-dimensional two-photon entanglement and violations of generalized Bell inequalities

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**Quantum entanglement[1,2] plays a vital role in many quantum-information and communication tasks[3]. Entangled states of higher-dimensional systems are of great interest owing to the extended possibilities they provide. For example, they enable the realization of new types of quantum information scheme that can offer higher-information-density coding and greater resilience to errors than can be achieved with entangled two-dimensional systems (see ref. 4 and references therein). Closing the detection loophole in Bell test experiments is also more experimentally feasible when higher-dimensional entangled systems are used[5]. We have measured previously untested correlations between two photons to experimentally demonstrate high-dimensional entangled states. We obtain violations of Bell-type inequalities generalized to *d*-dimensional systems[6] up to $d = 12$. Furthermore, the violations are strong enough to indicate genuine 11-dimensional entanglement. Our experiments use photons entangled in orbital angular momentum[7], generated through spontaneous parametric down-conversion[8,9], and manipulated using computer-controlled holograms.**

by local realist theories[19], the violation of Bell-type inequalities may be used to demonstrate the presence of entanglement. Bell-type experiments have been carried out using two-dimensional subspaces of the OAM state space of photons[20,21] and experimentalists have demonstrated two-dimensional entanglement using up to 20 different two-dimensional subspaces[22]. Careful studies have also been carried out to describe how specific detector characteristics bound the dimensionality of the measured OAM states in photons generated by SPDC using Shannon dimensionality[23].

Our experimental study of high-dimensional entanglement is based on the theoretical work of Collins *et al.*[6], which was applied in experiments for qutrits encoded in the OAM states of photons[18,24]. We encode qudits using the OAM states of photons, with eigenstates defined by the azimuthal index $\ell$. These states arise from the solution of the paraxial wave equation in its cylindrical co-ordinate representation, and are the Laguerre–Gaussian modes $LG_{p,\ell}$, so called because they are light beams with a Laguerre–Gaussian amplitude distribution.

In our set-up (Fig. 1), OAM entangled photons are generated through a frequency-degenerate type-I SPDC process, and the

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# Outline

- Bell’s Inequality
  - d-Dimensional generalization
- Orbital Angular Momentum
  - Entanglement
  - Measurement
- Results

# Bell’s Inequality

- Alice and Bob each get one particle
  - Alice measures A1 or A2
  - Bob measures B1 or B2
  - Each measurement can have one of d outcomes
- ASSUME: there are 4 local hidden varibles
  - j – result if Alice measures A1
  - k – result if Alice measures A2
  - l – result if Bob measures B1
  - m – result if Bob measures B2
  - And for a given trial there exist fixed values of j,k,l & m

# Bell's Inequality

- Construct the quantities
  - $r = B1-A1 = l-j$
  - $s = A2-B1 = k-l$
  - $t = B2-A2 = m-k$
  - $u = A1-B2 = j-m$
  - $r+s+t+u=0$
- $P(Aa=Bb+k)$ = prob. Aa and Bb differ by k
- $I = P(A1=B1)+P(A2=B2)+P(B2=A1)+P(B1=A2+1)$
- Our assumptions say: $<3$

# QM Prediction d=2

- If we do some experiment and $I$ >3 then nature can’t be described by a realist non-local theory.
- Alice and Bob share: $|00\rangle + |11\rangle$
  - A1 = $|0\rangle \pm |1\rangle$
  - A2 = $|0\rangle \pm i|1\rangle$
  - B1 = $|0\rangle \pm \exp(i\pi/4)|1\rangle$
  - B2 = $|0\rangle \pm \exp(-i\pi/4)|1\rangle$
- $I$ = 3.4

# Now Set Arbitrary d

d-dimensional entanglement: $\psi = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} | j \rangle_A \otimes | j \rangle_B$

More general inequality:

$$I_d \equiv \sum_{k=0}^{[d/2]-1} \left(1 - \frac{2k}{d-1}\right) \{ + [P(A_1 = B_1 + k) + P(B_1 = A_2 + k + 1) + P(A_2 = B_2 + k) + P(B_2 = A_1 + k)] - [P(A_1 = B_1 - k - 1) + P(B_1 = A_2 - k) + P(A_2 = B_2 - k - 1) + P(B_2 = A_1 - k - 1)] \}. \quad (6)$$

Now: Id ≤2

But if Alice and Bob measure :

$$|k\rangle_{A,a} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp\left(i \frac{2\pi}{d} j(k + \alpha_a)\right) | j \rangle_A ,$$

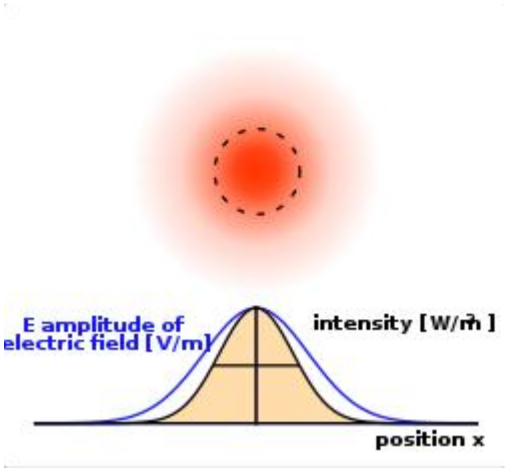
$$|l\rangle_{B,b} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp\left(i \frac{2\pi}{d} j(-l + \beta_b)\right) | j \rangle_B ,$$

Id =2.6981, as d→∞

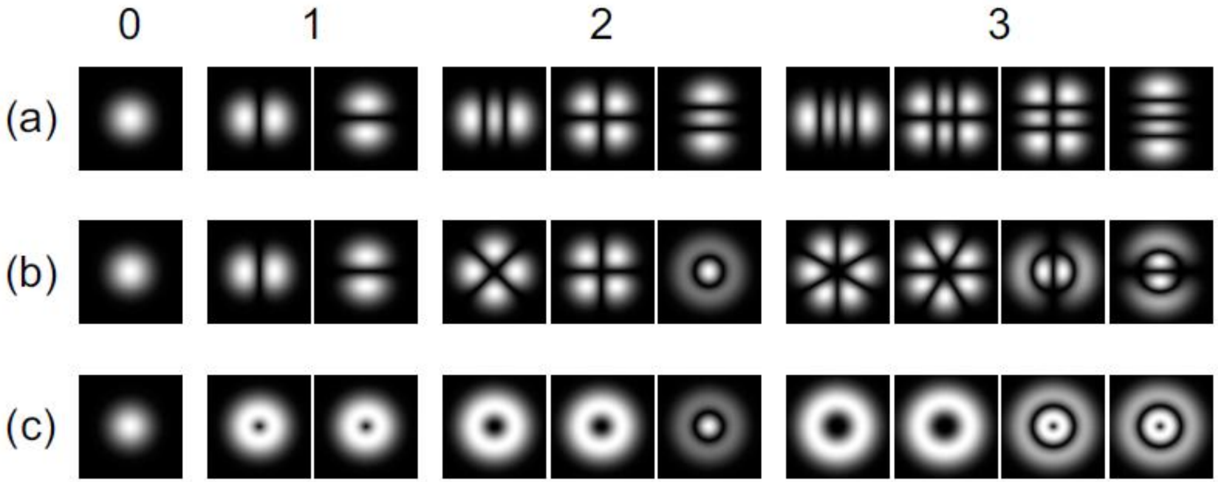
# Transverse Spatial Modes

## Gaussian Beam:

A solution to Helmholtz equation in the paraxial approx.



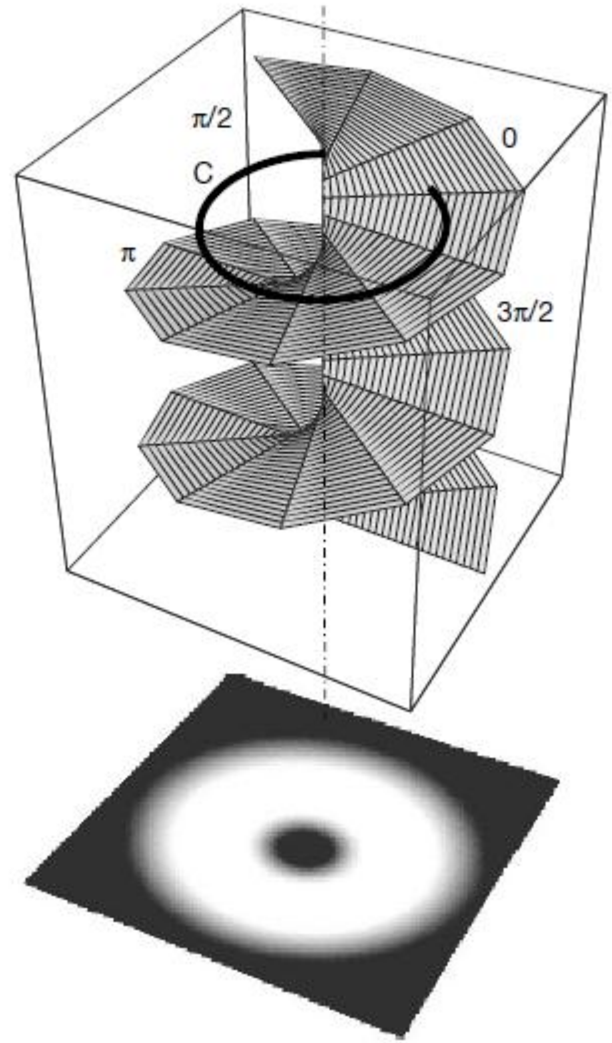

## Higher Order Solutions:



# Laguerre-Gauss Vortex Modes

$$u_{pl}^{LGV}(r,\theta,z) = u_g(r,\theta,z)\; e^{i(2p+|l|)\psi(z)} \frac{1}{\sqrt{2}} A_p^{|l|}(r,z)\; e^{il\theta}$$

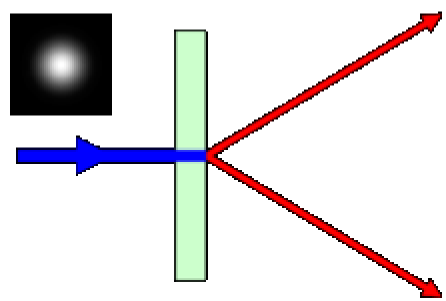
Each photon in the beam carries ħl units of orbital angular momentum



# Our d-dimensional Entangled System

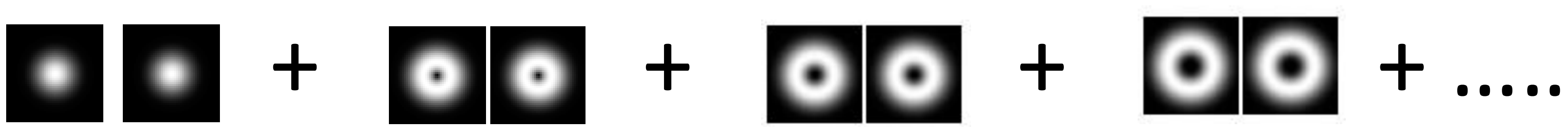
Spontaneous Parametric Down-Conversion!!!!

Pump with l=0 light



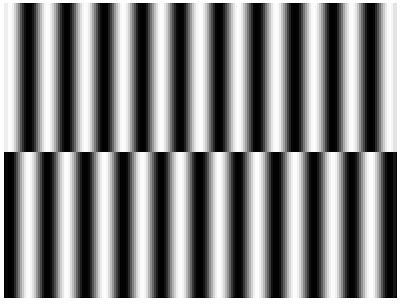
Conservation of angular momentum:

c0|0,0> + c1|+1,-1> + c2|+2,-2> + c3|+3,-3> +…….



# Transforming Transverse Modes

Use grating with variable phase



Output field: $u_{\text{out}}(\mathrm{s}, z) = \tau(\mathrm{s})\, u_{\text{in}}(\mathrm{s}, z)$

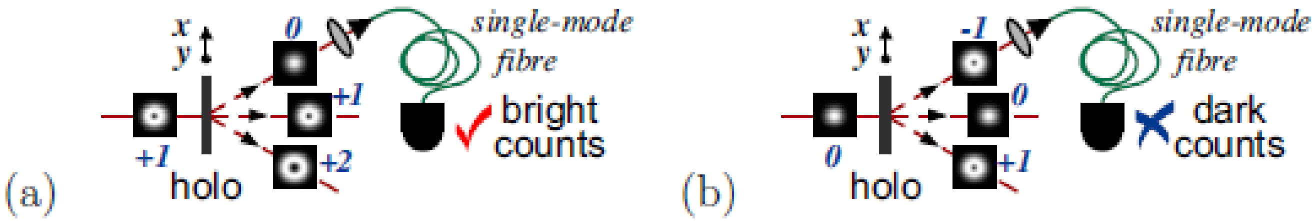
Consider: $\tau(\mathrm{s}) = e^{i\frac{1}{2}\theta_\varepsilon \cos[k_x x - \phi(\mathbf{s})]} = \sum_{n=-\infty}^{\infty} J_n\left(\frac{1}{2}\theta_\varepsilon\right) i^n e^{-iny}$

$k_x = k \sin\alpha$  $y = k_x x - \phi(\mathrm{s})$

Then: $u_{\text{out}}(\mathrm{s}) = \sum_{n=-\infty}^{\infty} J_n\left(\frac{1}{2}\theta_\varepsilon\right) i^n e^{in\phi(\mathbf{s})} u_g(\mathrm{s}, 0) e^{-ink_x x}$

The nth diffraction order picks up a transverse phase, Φ(**S**)

# Measuring Transverse Spatial Modes




- Convert desired mode into TEM00, then couple to single mode fiber
- ‘Undo’ phase and amplitude changes of desired mode

# Experiment

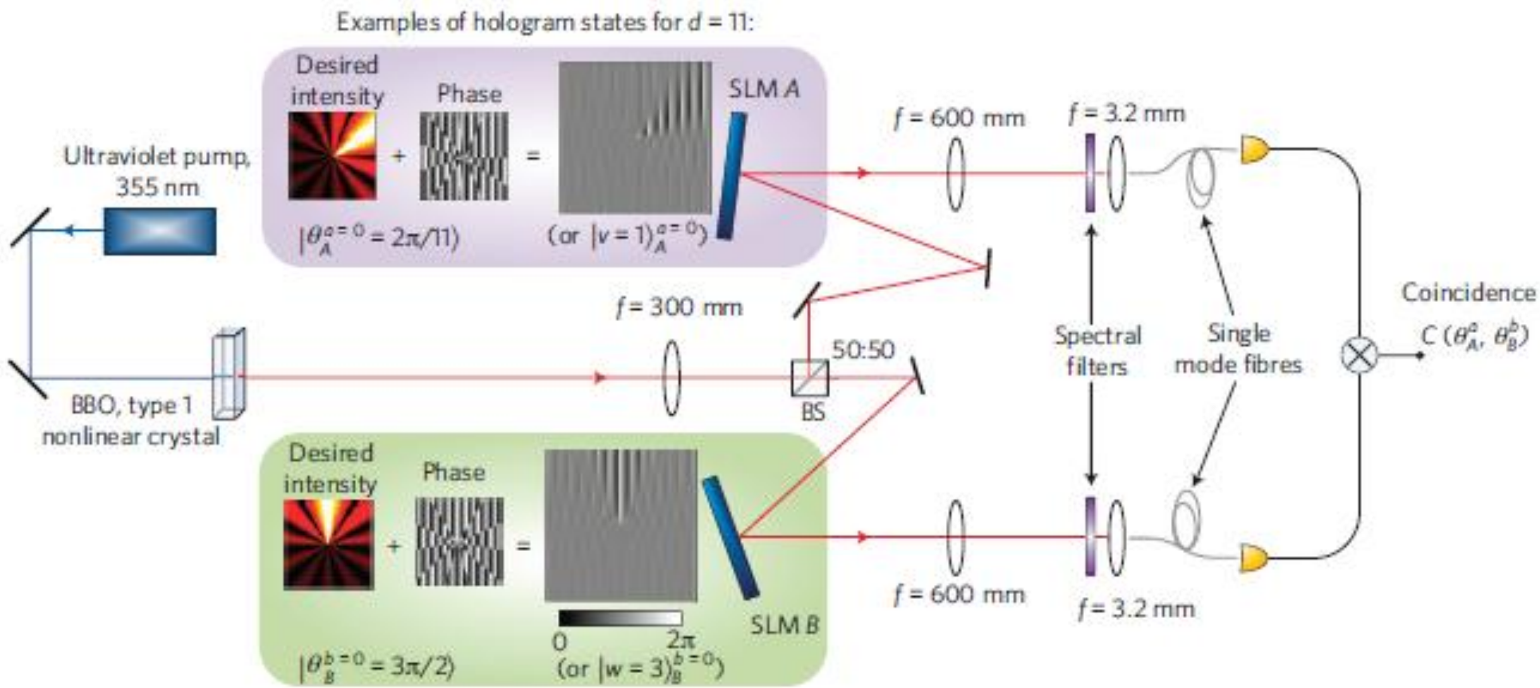



**Figure 1 | Schematic representation of experimental set-up for violations of Bell-type inequalities.** $C(A_a = v, B_b = w)$ or $C(\theta_A^a, \theta_B^b)$ is the coincidence count rate when SLM *A* is in state $|v\rangle_a^A$ or $|\theta_A^a\rangle$ and SLM *B* is in state $|w\rangle_b^B$ or $|\theta_B^b\rangle$ respectively.

# Filtering the Output to Increase Entanglement

Ideal state has equal amplitudes

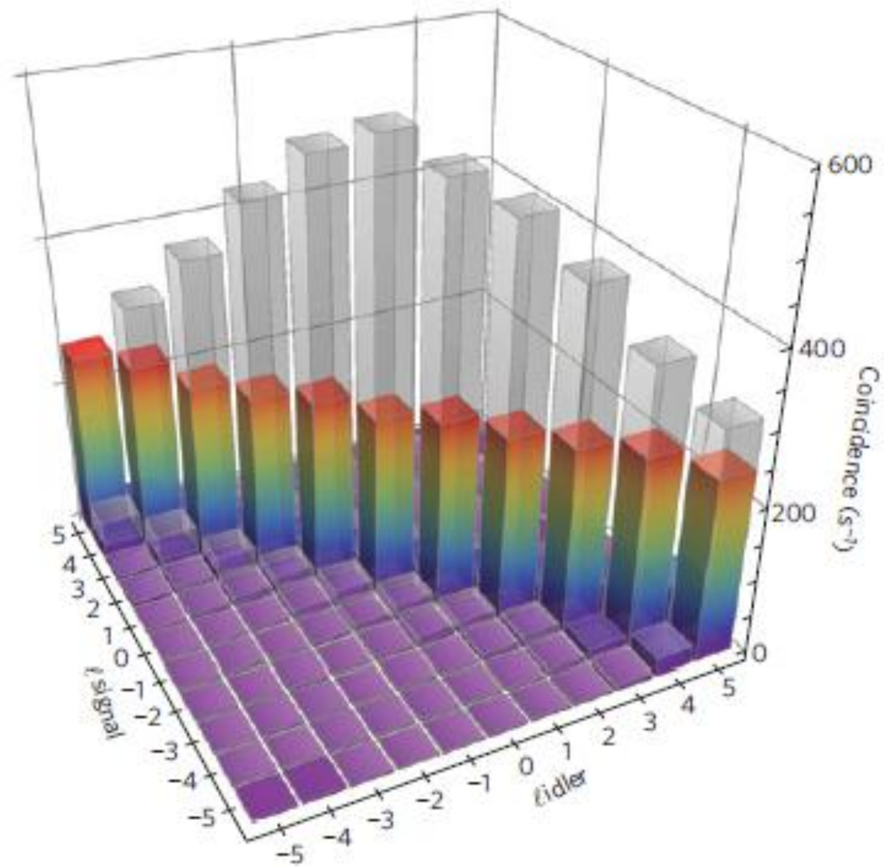



**Figure 4 | Experimental coincidence rates proportional to the probability of measuring the state $|\ell_s\rangle \otimes |\ell_i\rangle$ with $\ell_s, \ell_i = -5, \ldots, +5$.** The coloured and greyed-out bars depict the measurement results with and without the application of Procrustean filtering respectively. The measurement time was 20 s for each combination of $\ell_s$ and $\ell_i$.

# The measurements:

- You all remember:

$$|v\rangle_a^A = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp\left[ i \frac{2\pi}{d} j (v + \alpha_a) \right] |j\rangle$$

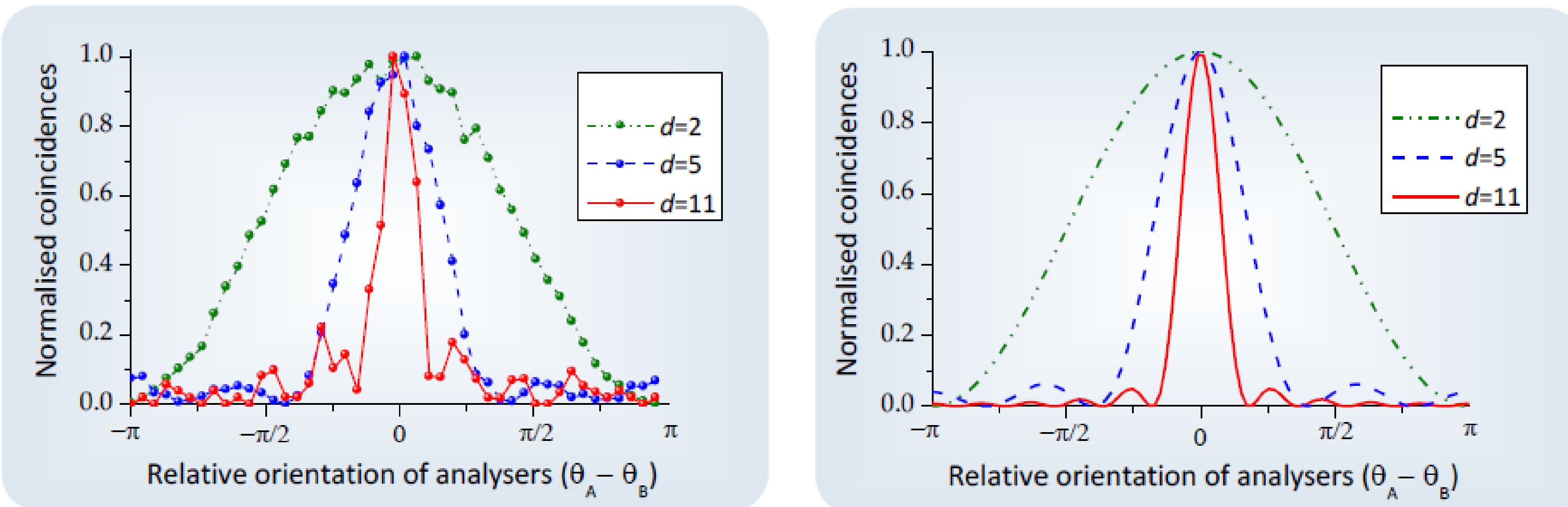
$$|w\rangle_b^B = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp\left[ i \frac{2\pi}{d} j (-w + \beta_b) \right] |j\rangle$$

- Clearly: $|v\rangle_a^A \equiv |\theta_A^a\rangle = \frac{1}{\sqrt{d}} \sum_{\ell=-[d/2]}^{\ell=+[d/2]} \exp\left[ i\theta_A^a g(\ell) \right] |\ell\rangle$, and

$$|w\rangle_b^B \equiv |\theta_B^b\rangle = \frac{1}{\sqrt{d}} \sum_{\ell=-[\frac{d}{2}]}^{\ell=+[\frac{d}{2}]} \exp\left[ i\theta_B^b g(\ell) \right] |\ell\rangle$$

- Where θ are angles of mode analyzers

# Raw Data




(a) Experiment (b) Theory

FIG. S1. Coincidence count rates (self normalised) as a functions of relative orientation angles between state analysers ($\theta_A - \theta_B$). The figures depict (a) experimental coincidence curves and (b) theoretical prediction for maximally entangled states of two $d$-dimensional systems with mode analyser settings defined in Eq. S15 using $d = 2, 5,$ and 11, as examples. Only modes with $p = 0$ are used here. The observed fringes are typical of genuine 2-, 5-, and 11-dimensional entanglement, respectively.

# Results:

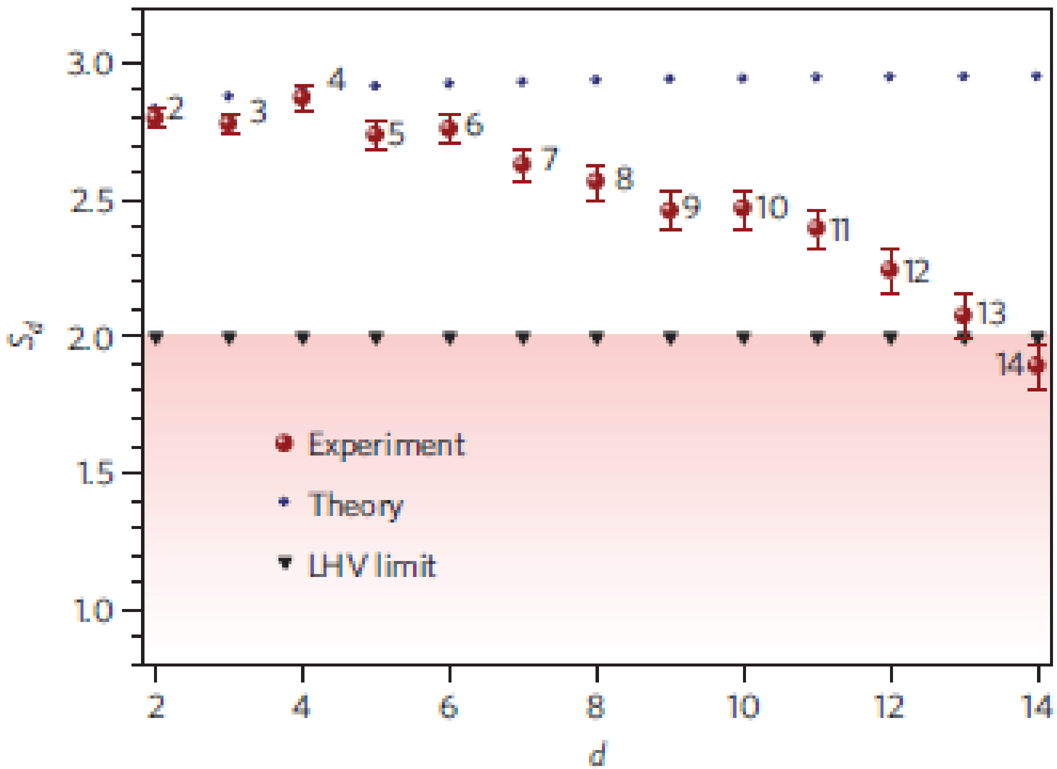



**Figure 3 | Experimental Bell-type parameter $S_d$ versus number of dimensions *d*.** $S_d > 2$ violates local realism for any $d \geq 2$. The plot compares the theoretically predicted violations by a maximally entangled state and the LHV limit with the experiments. Violations are observed for up to $d = 12$. Errors were estimated assuming Poisson statistics.

# Does it Really Prove all 11 dimensions are entangled?

- Yes….
- S11 = 2.67±0.22
  - They know the diagonal elements
  - Assuming perfect coherence between first 10 and no coherence between 11:
    - S'11 = 2.14

- The 11th dimension is at least partially entangled

# Conclusions

- Violate 12-dimensional Bell Inequality
  - (with 11 entangled dimensions)

- Demonstrated 11-dimensional entanglement