The quantum state cannot be interpreted statistically

Matthew F. Pusey,[1,*] Jonathan Barrett,[2] and Terry Rudolph[1]

[1] *Department of Physics, Imperial College London, Prince Consort Road, London SW7 2AZ, United Kingdom*

[2] *Department of Mathematics, Royal Holloway, University of London, Egham Hill, Egham TW20 0EX, United Kingdom*

(Dated: November 14, 2011)

Quantum states are the key mathematical objects in quantum theory. It is therefore surprising that physicists have been unable to agree on what a quantum state represents. There are at least two opposing schools of thought, each almost as old as quantum theory itself. One is that a pure state is a physical property of system, much like position and momentum in classical mechanics. Another is that even a pure state has only a statistical significance, akin to a probability distribution in statistical mechanics. Here we show that, given only very mild assumptions, the statistical interpretation of the quantum state is inconsistent with the predictions of quantum theory. This result holds even in the presence of small amounts of experimental noise, and is therefore amenable to experimental test using present or near-future technology. If the predictions of quantum theory are confirmed, such a test would show that distinct quantum states must correspond to physically distinct states of reality.

Lee Rozema

QO group meeting

November 23, 2011

‘Quantum theorem shakes foundations’ – Nature

‘seismic’ – Antony Valentini

‘the most important result in the foundations of quantum mechanics’ in 15 years – David Wallace

‘fascinating’, ‘correct’ and ‘It’s very important and beautiful in its simplicity’ – Rob Spekkens

‘A truly major advance’ – Aephraim Steinberg

‘I don't understand why it is a big deal’ – Alex Hayat

‘it’s a profound triviality, something that most people who thought about quantum mechanics already knew, but probably didn’t *know* they knew.’ – Scott Aaronson

Outline

- What is a statistical interpretation of QM (background)
  - Epistemic vs. ontic states
- Why it's a big deal
  - No new predictions -- so who cares?
- How it shows what it shows
  - The actual paper

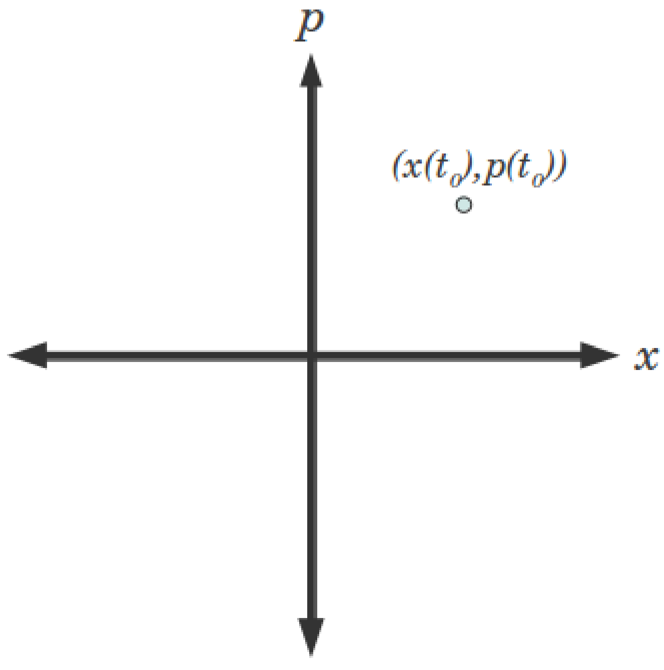
http://mattleifer.info/2011/11/20/can-the-quantum-state-be-interpreted-statistically/

Classical Ontic States

- For a 1-D point particle the ontic(real) state is (x,p)

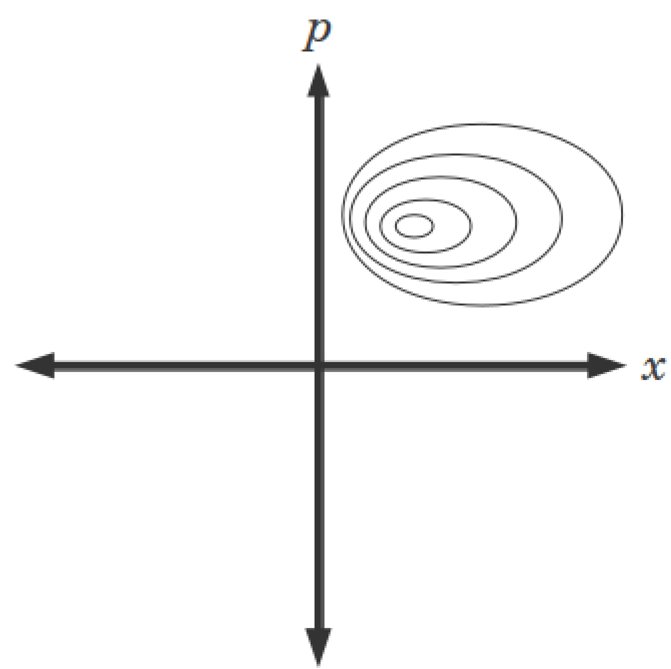
- The particle is real and exists at x with momentum p

- This is a 'state of reality'

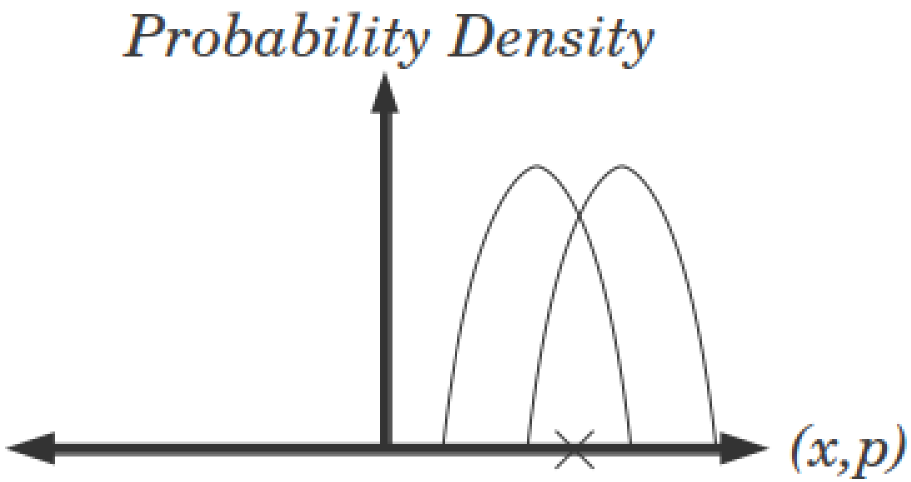


Classical Epistemic States

- If we lack knowledge about our system we assign a probability distribution over phase space
- We still think particle is really at some (x,p) we just don't know where
- Probability distribution is the epistemic state, describes our lack of knowledge



Classical Epistemic States (cont.)

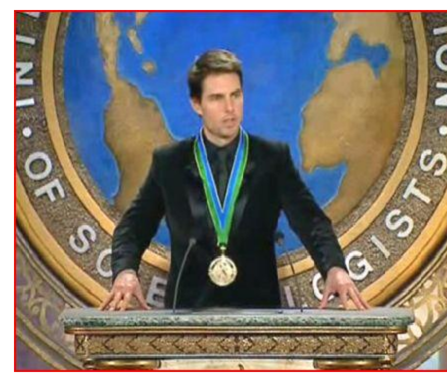


- Probability distributions can overlap
- Different epistemic states can be consistent with the same ontic state of reality

Quantum States – What is ψ?

- ψ-epistemologists
  1. the quantum state represents ordinary statistical uncertainty about some underlying “real” object (which isn’t *itself* a quantum state)
  2. the quantum state represents statistical uncertainty, but there is no “real” object
     e.g. Copenhagenists , Qbayesians and instrumentalists

- ψ-ontologists
  – the quantum state *is* the “real” object (Bohmians, may-worlds, collapse models)

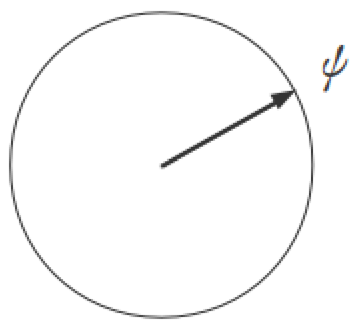


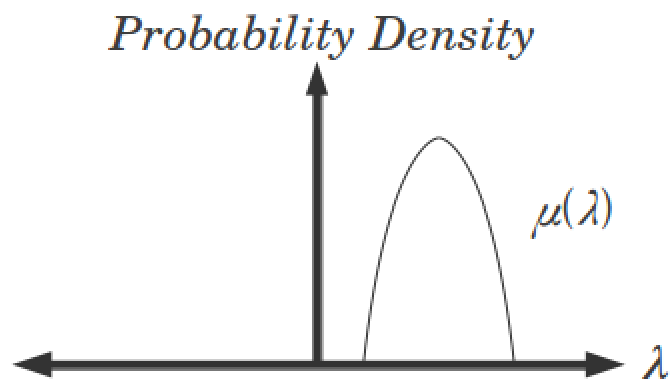
(From Scott Aaronson on Matt Liefers blog)

No new predictions, so why is it a big deal?

- Like Bells theorem this result gives an experimentally measurable quantity to test what was previously left to philosophers to argue about
  - Of course if you believe quantum mechanics you don’t expect any surprises here, just as you wouldn’t have for Bells inequality

The Structure of Hidden Variable Models (ψ-epistemologists of type-I)

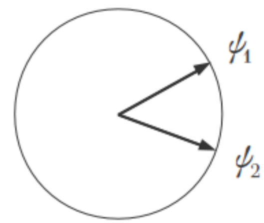


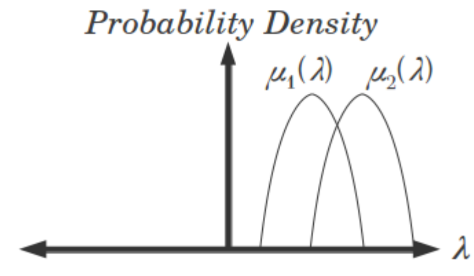


- Bell's framework:
  - Hidden variable, λ, describes ontic state
  - Distribution (related to ψ): μ(λ)
  - So each preparation of ψ has a different λ, distributed according to μ(λ)

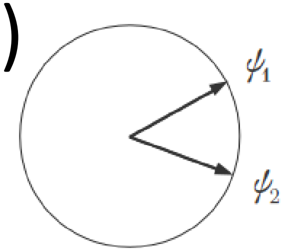
The Result

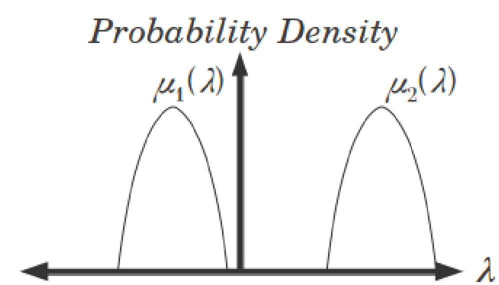
- Consider two (non-orthogonal) states, if μ1 and μ2 can overlap we will say ψ is epistemic, because there may be some real (ontic state) λ out there





- If μ1 and μ2 can NEVER overlap, for any two ψs, then each λ uniquely specifies μ(λ) and thus ψ, so we might as well call ψ the ‘real thing’ or part of the real thing





Hidden Variable and Measurements

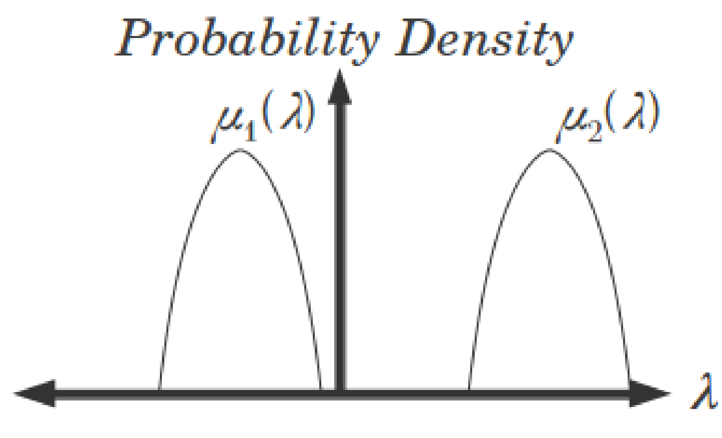
- $M(k|\lambda)$
  - for a measurement M, the probability of outcome k, given the hidden variable is λ
  - $\Sigma_k M(k|\lambda) = 1$ (normalization)
- Now if we are in $|\psi\rangle$ with some $\mu(\lambda)$ the prob. of outcome k is
  - $P(k|\psi) = \int d\lambda\, M(k|\lambda)\mu(\lambda)$
- To be consistent with QM, if the kth outcome of M corresponds to the projector Pk
  - $\langle\psi|P_k|\psi\rangle = \int d\lambda\, M(k|\lambda)\mu(\lambda)$

The Proof Begins

- RECALL: $<\psi|Pk|\psi> = \int d\lambda\, M(k|\lambda)\mu(\lambda)$
- For measurement M, if there is some outcome, k, that QM says occurs with probability 0 then
  - $M(k|\lambda) = 0$ for all λ such that $\mu(\lambda) \neq 0$
- Imagine 2 states, $|0>$ & $|1>$, & $M=\sigma_z$
  - We know $<0|1><1|0>=0$ and $<1|0><0|1>=0$
  - If there exists some λ such that $\mu_0(\lambda) \neq 0$ and $\mu_1(\lambda) \neq 0$ then $M(0|\lambda)=0$ and $M(1|\lambda)=0$

  BUT normalization requires $M(1|\lambda)+M(2|\lambda)=1$, so if there is a λ consistent with $|0>$ and with $|1>$ we can't have $<\psi_1|P_1|\psi_1> \neq 0$ and $<\psi_2|P_2|\psi_2> \neq 0$

- So there is no λ consistent with |0> and |1>
- But we can’t find such measurements for any two non-orthogonal states



Try $|0\rangle$ and $|+\rangle$

- Consider multiple copies of the two states:
  - $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$, $|+\rangle \otimes |0\rangle$ and $|+\rangle \otimes |+\rangle$
- And a 4-outcome measurement such that
  - $\langle 00|M1|00\rangle=0$, $\langle 0+|M2|0+\rangle=0$, $\langle +0|M3|+0\rangle=0$ and $\langle ++|M4|++\rangle=0$
- Assume system a, with hidden variable $\lambda 1$, is independent of b, with hidden variable $\lambda 2$, so for each state the distribution factorizes:
  - $\mu 0(\lambda 1)\mu 0(\lambda 2)$, $\mu 0(\lambda 1)\mu +(\lambda 2)$, $\mu +(\lambda 1)\mu 0(\lambda 2)$, $\mu +(\lambda 1)\mu +(\lambda 2)$

- Now if there is some λ, such that $\mu_0(\lambda) \neq 0$ and $\mu_+(\lambda) \neq 0$, then any of the 4 states should have non-zero probability being prepared in λ.
- Then to obey the QM predictions

$$M(1|\lambda)=0,\ M(2|\lambda)=0,\ M(3|\lambda)=0 \text{ and } M(4|\lambda)=0$$

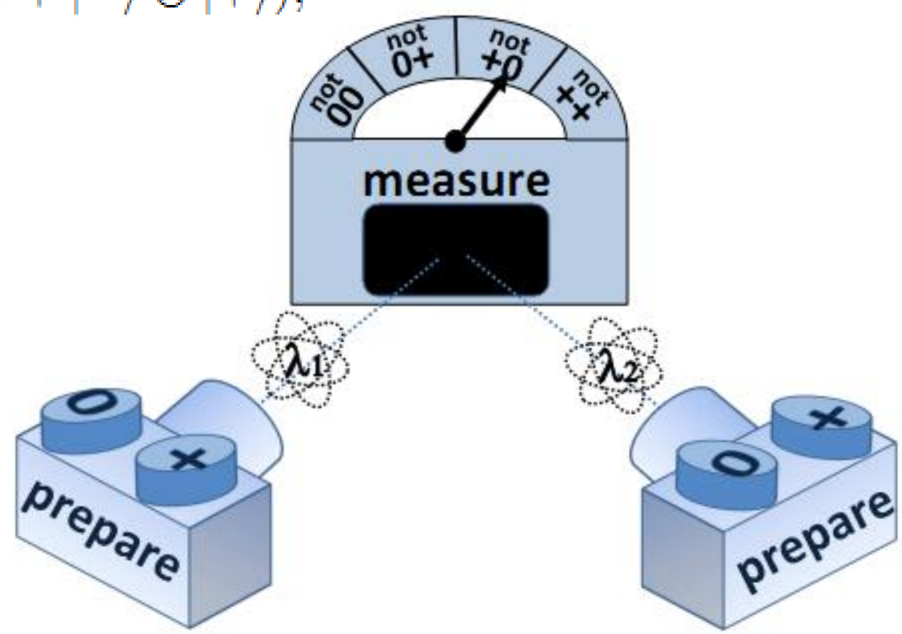
violating normalization.

- In terms of measurements in the lab this means if |0>|0> is prepared and μ_0 and μ_+ overlap we would never get a zero for a measurement of M, as QM would predict

- $\psi 1 = |0\rangle$, $\psi 2 = |+\rangle = |0\rangle + |1\rangle$
- Measure:

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle),$$
$$|\xi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle),$$
$$|\xi_3\rangle = \frac{1}{\sqrt{2}}(|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle),$$
$$|\xi_4\rangle = \frac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle),$$

- Now
  - $\langle 00|\xi 1\rangle = 0$
  - $\langle 0+|\xi 2\rangle = 0$
  - $\langle +0|\xi 3\rangle = 0$
  - $\langle ++|\xi 4\rangle = 0$



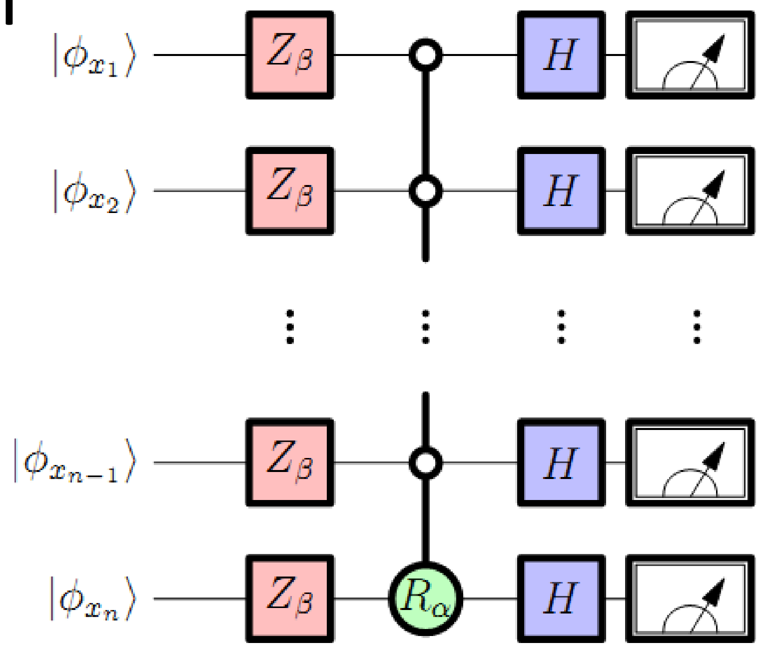
What about other non-orthogonal states?

- The less orthogonal the two states get the more copies of the two states are needed
- If our states are of the form:

$$|\phi_0\rangle = \cos(\theta/2)\,|0\rangle - \sin(\theta/2)\,|1\rangle$$
$$|\phi_1\rangle = \cos(\theta/2)\,|0\rangle + \sin(\theta/2)\,|1\rangle$$

- Need n such that

$$2\arctan\left(2^{1/n} - 1\right) \leq \theta$$



Conclusions

- If I've confused you (and you're still interested), read this blog: mattleifer.info/2011/11/20/can-the-quantum-state-be-interpreted-statistically/, and listen to this talk pirsa.org/11050028/
- The assumptions:
  - We can make pure states
  - We can make uncorrelated states
  - Measurements depend only on the system being measured
- Wave functions are real OR something in our hidden variable framework is wrong OR we should stop asking these questions