Lecture 3 -Griffin

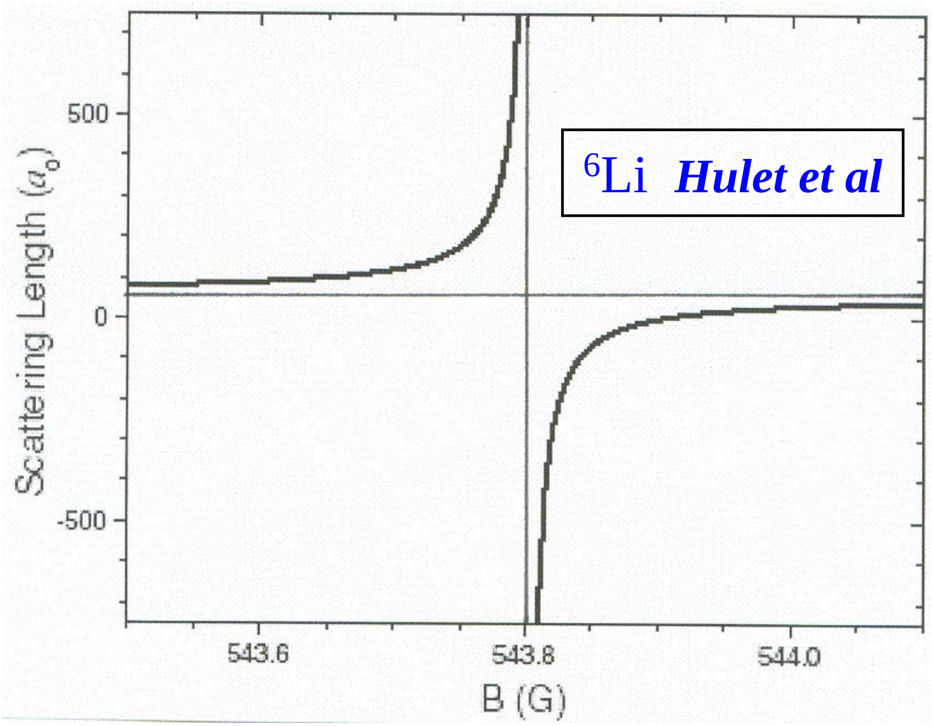
Model for interacting Fermi atoms and Boson molecules - putting everything together!

We need a microscopic model that includes:

- ✓ **Feshbach resonance in the two-body potential**
- ✓ **BCS Cooper pair formation**
- ✓ **BCS-BEC crossover as interaction increases**

The model (due to Timmermans, Holland, Drummond and coworkers) explicitly includes the Fermi atoms, the Bosonic molecules formed from these atoms, and the Feshbach resonance coupling term. This theory is often now called **resonance superfluidity**, a term introduced By Holland

Feshbach resonance: two body physics



Stable molecules form when $a_{2b} > 0$. This is equivalent to $2\nu < 0$ or $B < B_0$.

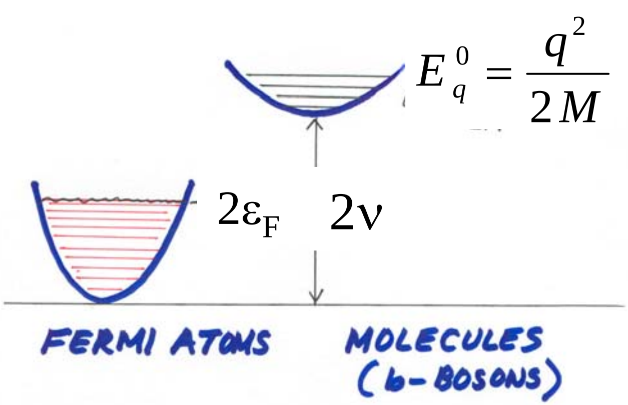
$$2\nu \propto B - B_0$$

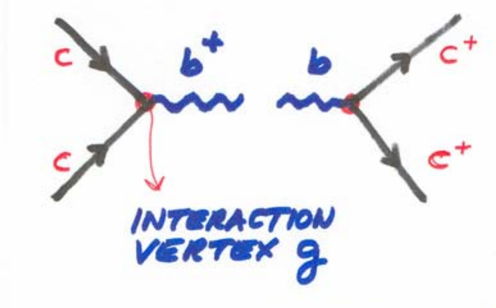
$$-\frac{4\pi\hbar^2 a_{2b}}{m} \equiv U + \frac{g^2}{2\nu}$$

--->

$$a_S = a_{bg}\left(1 + \frac{w}{B_0 - B}\right)$$

$$\mathcal{H} = \sum_{\boldsymbol{p}\sigma} \varepsilon_p c^\dagger_{\boldsymbol{p}\sigma} c_{\boldsymbol{p}\sigma} + \sum_{\boldsymbol{q}} (E^0_q + 2\nu) b^\dagger_{\boldsymbol{q}} b_{\boldsymbol{q}}$$
$$- U \sum_{\boldsymbol{p},\boldsymbol{p}'} c^\dagger_{\boldsymbol{p}\uparrow} c^\dagger_{-\boldsymbol{p}\downarrow} c_{-\boldsymbol{p}'\downarrow} c_{\boldsymbol{p}'\uparrow} + g_{\mathrm{r}} \sum_{\boldsymbol{p},\boldsymbol{q}} [b^\dagger_{\boldsymbol{q}} c_{-\boldsymbol{p}+\boldsymbol{q}/2\downarrow} c_{\boldsymbol{p}+\boldsymbol{q}/2\uparrow} + \mathrm{h.c}].$$





- The atom-molecule interaction is denoted by g_r
- The non-resonant attractive interaction is **- U**

The molecular bound state energy **2ν** can be **tuned**. Molecules(with finite lifetime) start to form when **$2\nu \le 2\varepsilon_F$** and will **not** be able to **decay** when **$2\nu < 0$.**

$$N = \langle \sum_{p\sigma} c^{\dagger}_{p\sigma} c_{p\sigma} \rangle + 2 \langle \sum_{q} b^{\dagger}_{q} b_{q} \rangle$$
$$\equiv N_{\mathrm{F}} + 2N_{\mathrm{B}}.$$

A crucial feature of this Hamiltonian is that the b-molecules are formed from the Fermi atoms. There is thus only one chemical potential, with

$$H - \mu N = H - \mu N_F - 2\mu N_B, \text{ with } \mu_B = 2\mu.$$

This coupled Hamiltonian modifies the effect of the bare two-body Feshbach resonance. Two atoms are now part of an interacting system in the presence of a filled Fermi sea.

First thing to do is to solve our coupled FB model in a mean field approximation, allowing for Cooper pairs and a Bose condensate (q = 0) of b-molecules:

$$H_{FB} - \mu N = \sum_{p,\sigma} (\varepsilon_{p,\sigma} - \mu) c^{+}_{p,\sigma} c_{p,\sigma} + \phi_m^2 (E_{q=0} - 2\nu - 2\mu)$$

$$-U \sum_{p} (\phi_C c^{+}_{p\uparrow} c^{+}_{-p\downarrow} + h.c.) + g \sum_{p} (\phi_m c^{+}_{p\uparrow} c^{+}_{-p\downarrow} + h.c.)$$

ϕ_C **= Cooper pair condensate-** see previous lectures

ϕ_m **= Molecular condensate = < $b_{q=0}$>**

Both condensates are dependent on each other. We end up with a BCS-type theory but now with a composite order parameter:

$$\tilde{\Delta} = U\phi_C - g\phi_m$$

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This order parameter is the sum of contributions from two mechanisms:

Pair wavefunction = scattering channel
+ molecular channel

However, they are strongly coupled to each other and one determines the other:

$$\phi_m = -\frac{g}{2\nu - 2\mu}\phi_C$$

Note that μ is a strong function of molecule energy 2ν.

The number of Bose condensed b-molecules is given by $N_b = |\phi_m|^2$.

The composite BCS order parameter reduces to:

$$\tilde{\Delta} = U\phi_C - g\phi_m = \left(U + \frac{g^2}{2\nu - 2\mu} \right)\phi_C \equiv U_{eff}\,\phi_C$$

The physics is clear. The attractive interaction - *U* between the Fermi atoms in the open channel is now renormalized to U_{eff} by the resonant coupling to the b-molecules in the closed channel. Calculation also shows that we always have $2\nu > 2\mu$

One can speak in terms of a Bose condensate of BCS Cooper pairs or in terms of a molecular BEC of b-molecules, on both sides of the Feshbach resonance.

Note we are now dealing with a renormalized Feshbach resonance for atoms interacting in a superfluid Fermi gas, not two atoms in a vacuum. The b-molecules are described by a propagator

$$D_0(q,\omega) = \frac{1}{\omega - \left(E_q^0 + 2\nu - 2\mu\right)}$$

For coupling to Cooper pairs with $q = 0$, $\omega = 0$, this b-molecule propagator reduces to

$$D_0(0,0) = -\frac{1}{\left(2\nu - 2\mu\right)}$$

The b-molecules play the role of phonons in metals, leading to a large pairing interaction as $(2\nu - 2\mu) \rightarrow 0$.

It is no surprise that the renormalized energy gap is given by a BCS-type gap equation but now with an enhanced attractive interaction U_{eff}

$$\tilde{\Delta}(T) = \left[U + \frac{g^2}{2\nu - 2\mu}\right] \sum_k \frac{\tilde{\Delta}(T)}{2E_k} \tanh\left(\frac{1}{2}\beta E_k\right)$$

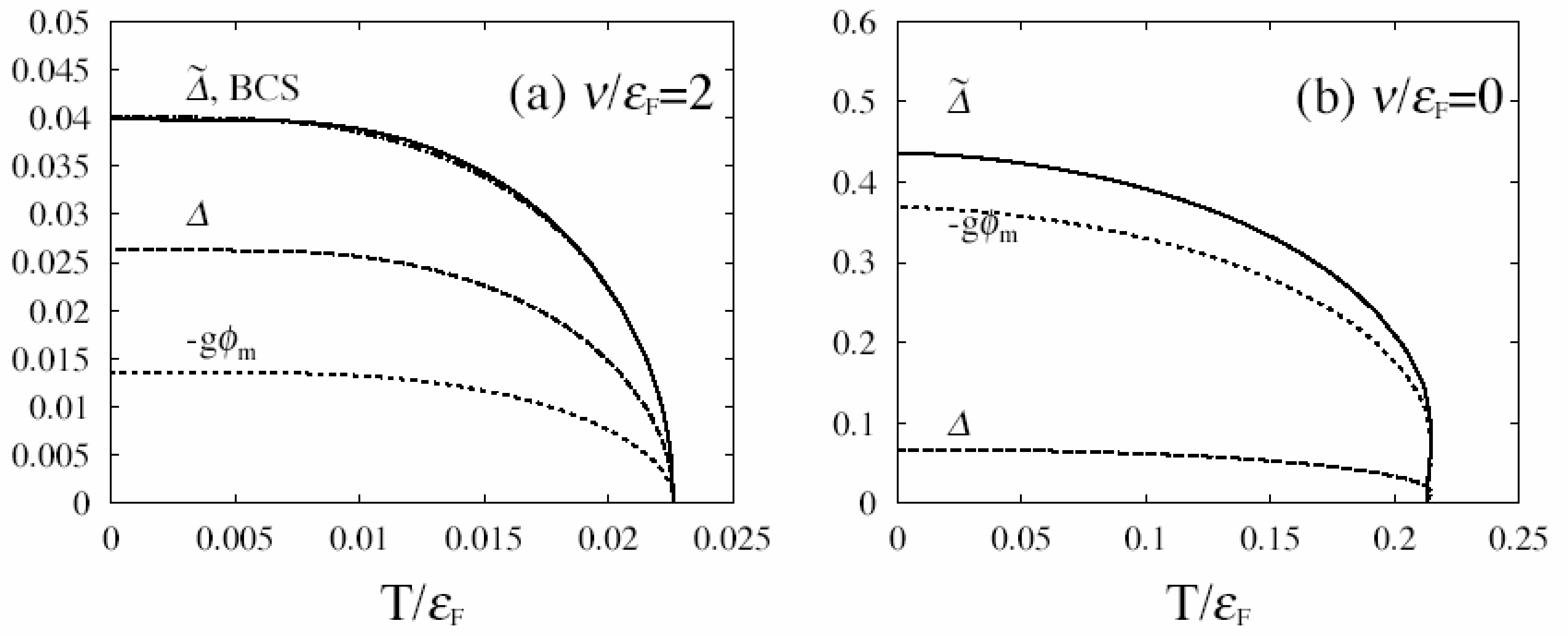
$E_k = \sqrt{[\epsilon_k - \mu]^2 + \tilde{\Delta}^2}$ = BCS quasiparticle spectrum with energy gap at $\tilde{\Delta}$

At $T = T_{BCS}$, $\tilde{\Delta}(T) \to 0$ and above equation reduces to BCS equation for T_{BCS},

$$1 = \left[U + \frac{g^2}{2\nu - 2\mu}\right] \sum_k \frac{\tanh(E_k/2k_B T_{BCS})}{2(\epsilon_k - \mu)}$$

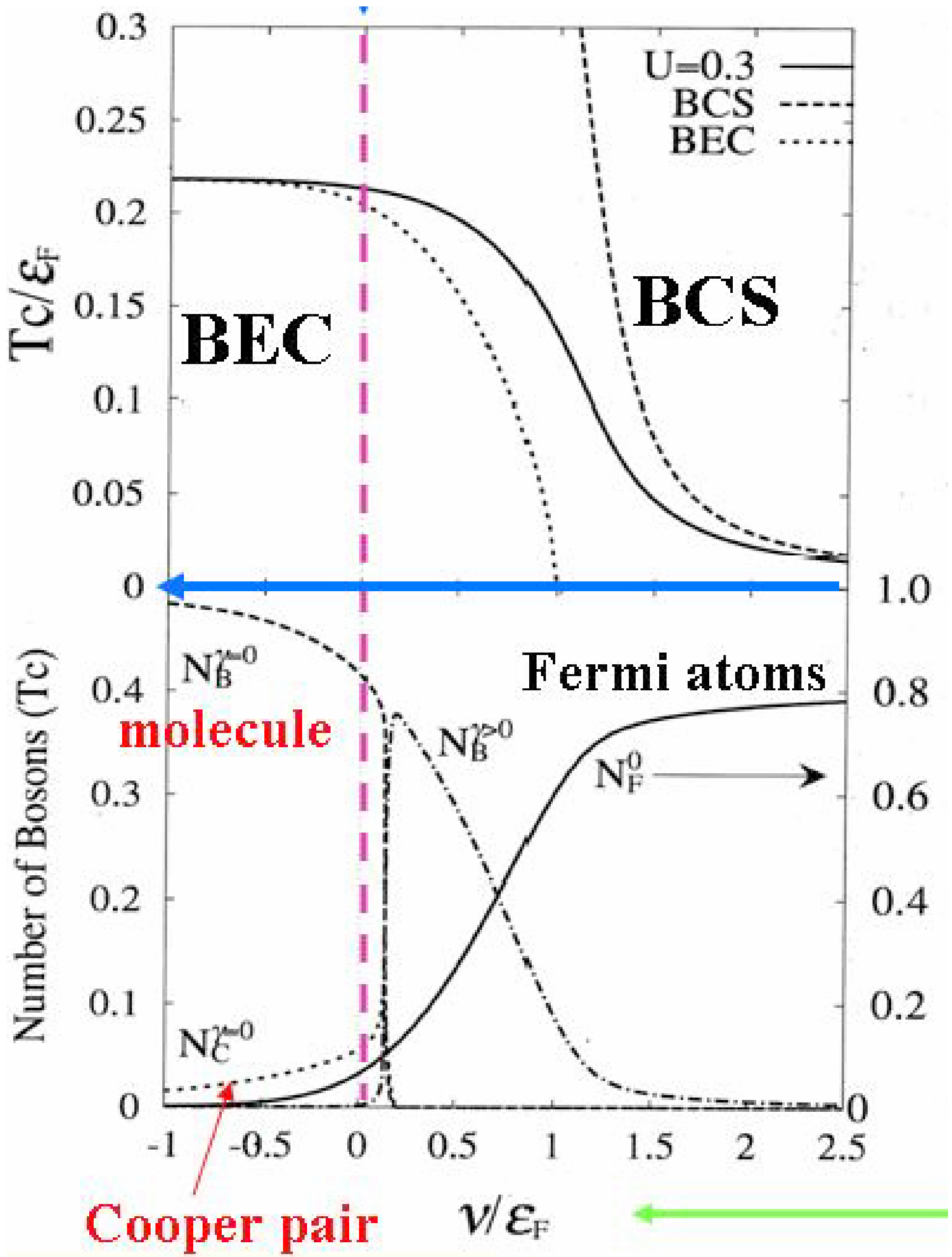
We already see the possibility that T_C will be large. However, μ is also dependent on the value of 2ν .

BCS limit   In crossover region



The **relative weight** of the b-molecules and the Cooper pairs in the **composite Bose condensate** is shown as a function of the temperature(**uniform gas**).

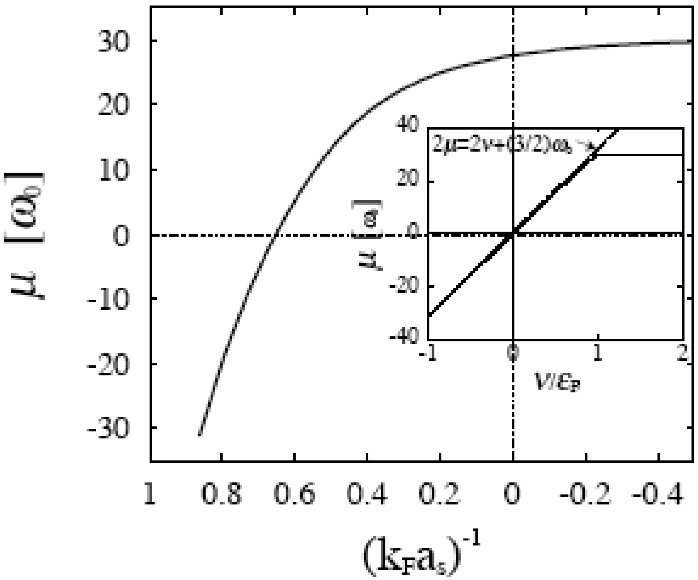
Ohashi and Griffin , PRA, 2003



Uniform gas results at T_c

One has long-lived b-molecules on the BCS side($\nu > 0$) and stable Cooper pairs on the BEC side ($\nu < 0$).

Similar results are obtained at $T < T_c$ and in trapped gases.



<= ε_F

Chemical potential (in units of the trap frequency ω_0) at T = 0.

How parameters change through crossover

BEC	**vs**	**BCS**
$a_s > 0$		$a_s < 0$
$2\nu < 2\varepsilon_F$		$2\nu > 2\varepsilon_F$
$\mu < 0$		$\mu > 0$ $(\approx \varepsilon_F)$
$E_g = [\mu^2 + \Delta^2]^{1/2}$		$E_g = \Delta$
Momentum distribution spread out		**Momentum distribution has sharp Fermi surface**

Energy gap in uniform gas -> (applies to the E_g row)

To calculate the chemical potential μ **and** the order parameter $\tilde{\Delta}$ in a self-consistent way a function of as, one has to include the **fluctuations** around the **BCS - Gorkov MFA :**

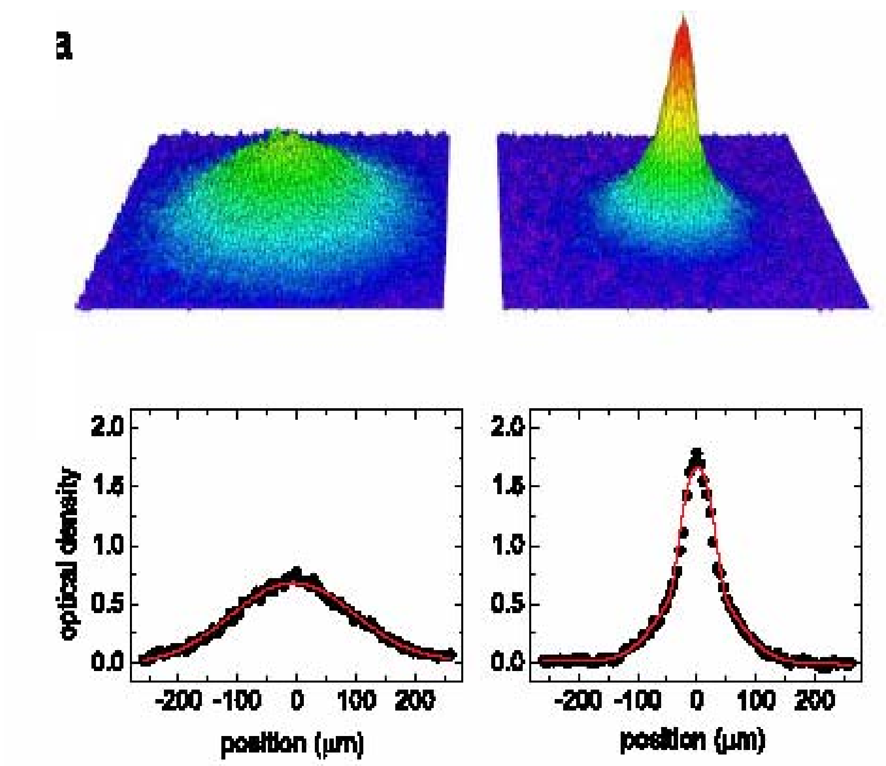
- ✓ The Cooper pairs **outside** the BCS condensate
  - Nozieres and Schmitt-Rink (1985) at T_c .
- ✓ The b-molecules **outside** the molecular condensate
  - Ohashi and Griffin (2002) at T_c .
- ✓ Both effects included **below** T_c by O&G (2003).

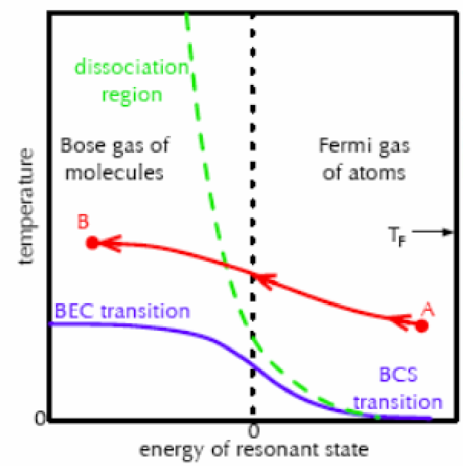
The **number** of b-molecules **and** Cooper pairs is self-consistently adjusted as **2ν is decreased,**

$$N_F = N_{atoms} + 2N_{Cooper\ pairs} + 2N_{b\text{-}molecules}$$

A molecular Bose condensate formed by SLOWLY ramping the magnetic field from just above ($a_s < 0$) to just below ($B - B_0 = -0.56G$) the resonance ($a_s > 0$).

$T = 0.19T_F$ $N_C = 0\%$ $T = 0.06T_F$ $N_C = 12\%$



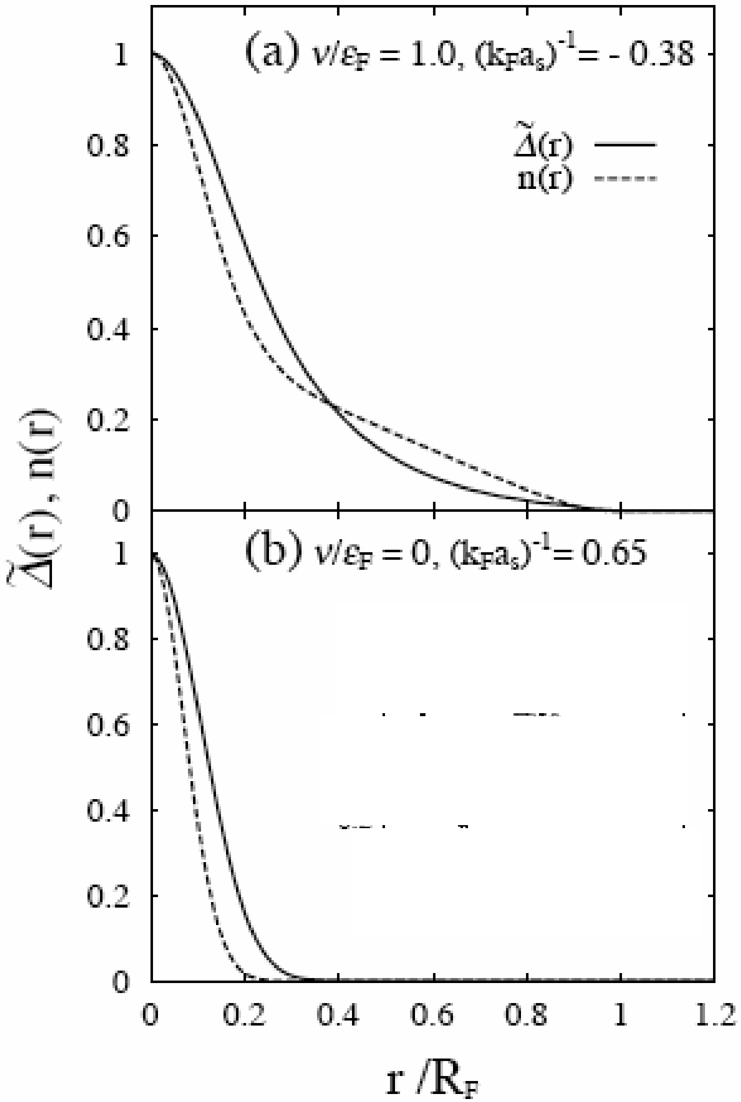


$2\nu \propto B - B_0$

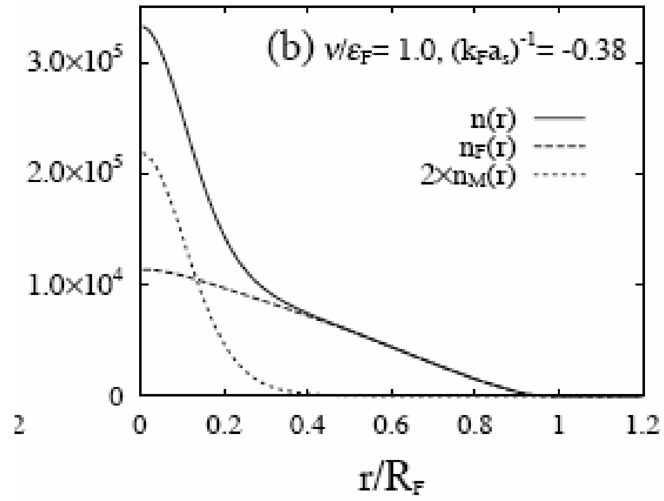
© Geoffrey Wheeler
Deborah Jin, Markus Greiner and Cindy Regal (left to right)

The density profile of the free Fermi atoms is not shown. The molecular condensate has the same profile as an atomic BEC, except M = 2m.

Greiner, Regal and Jin, Nature, Nov.7, 2003



T = 0



Relative number of free **Fermi** atoms and **Bosonic** bound states as a function of position in trap.

Self-consistent solutions of the BdG equations for the local density of atoms n(r) and the local order parameter $\tilde{\Delta}(r)$ in a harmonic trap. Normalized to values at center of trap.

THE BIG PICTURE THAT EMERGES

In a trapped Fermi gas, we can form two-particle bound states (dimers) which are **Bosons** and hence they can Bose-condense, forming a **Fermi superfluid**.

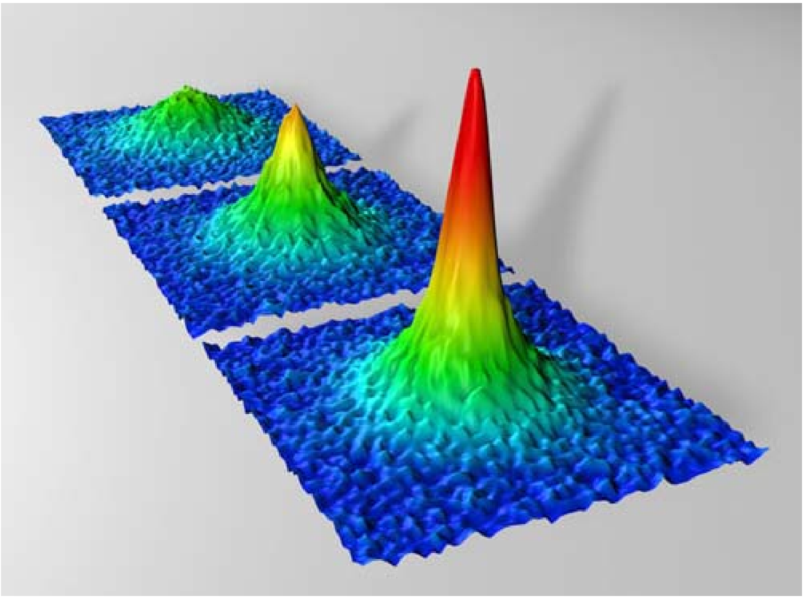
In the **crossover**, we go from a region where Cooper pairs dominate to one where real molecules dominate. However, the **entire region** can be described by the **same BCS type formalism**, built on a condensate of pairs.

The **unbound or free Fermions** swim around in this condensate and are renormalized by the order parameter. In a trap, these **single particle excitations** have an energy gap E_g and a spectrum that depends on both $\Delta(r)$ and μ in a **complicated way**(compared to usual BCS theory).

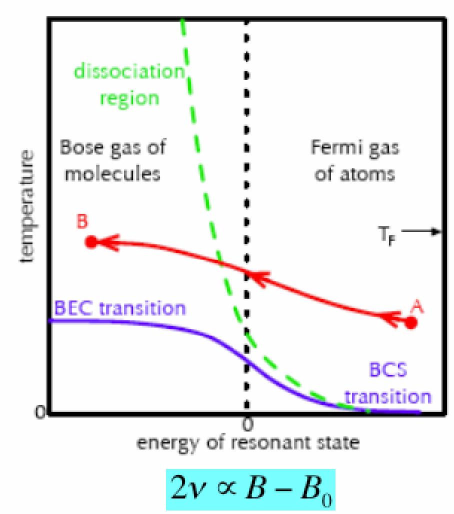
The holy grail has been to find evidence for a BCS Cooper pair condensate above the resonance, where $a_s < 0$. The **bimodal profile** shows evidence for the appearance of a Bose condensate of **Cooper pairs**. This experiment is done by ramping **RAPIDLY** from BCS region to BEC region so that real molecules do **not have time** to Bose-condense.

$B - B_0 = 0.55G$
$N_C = 1\%$ ---->

$B - B_0 = 0.12G$
$N_C = 10\%$ ---->

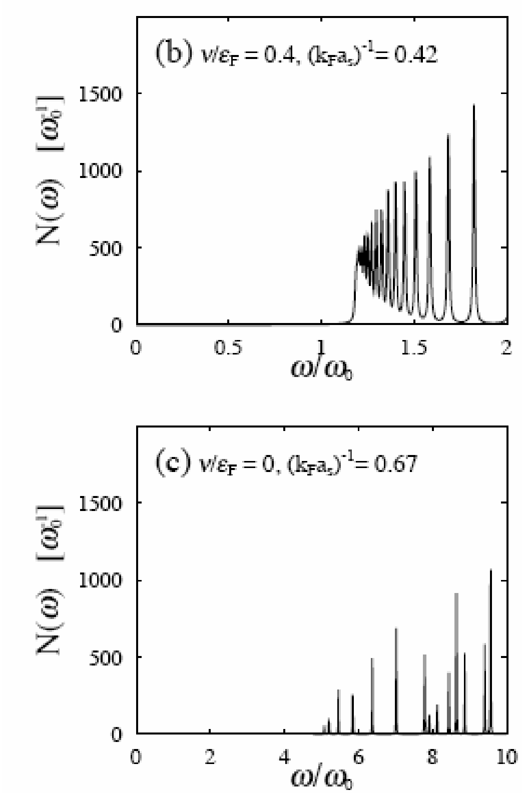


Regal, Greiner and Jin, PRL, Jan 30, 2004.

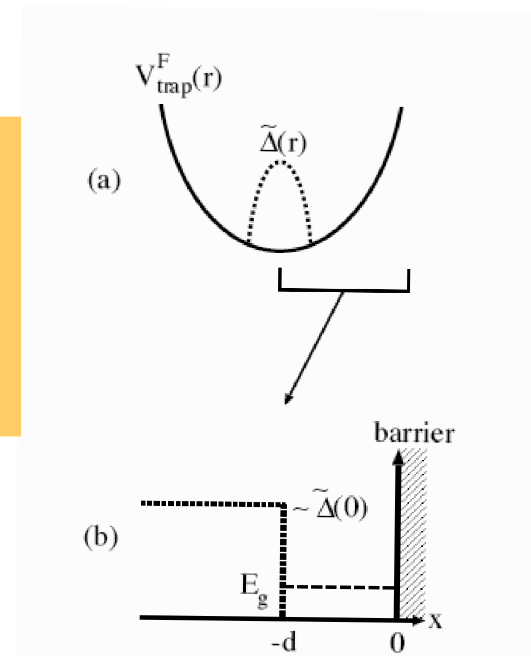


The density profile of the **free atoms** is **not** shown.

What about the single-particle **Fermi excitations** of these Fermi superfluids ? Their spectrum can be used to probe the underlying **Bosonic condensate**. This is the hot area of research now in ultracold atom physics.

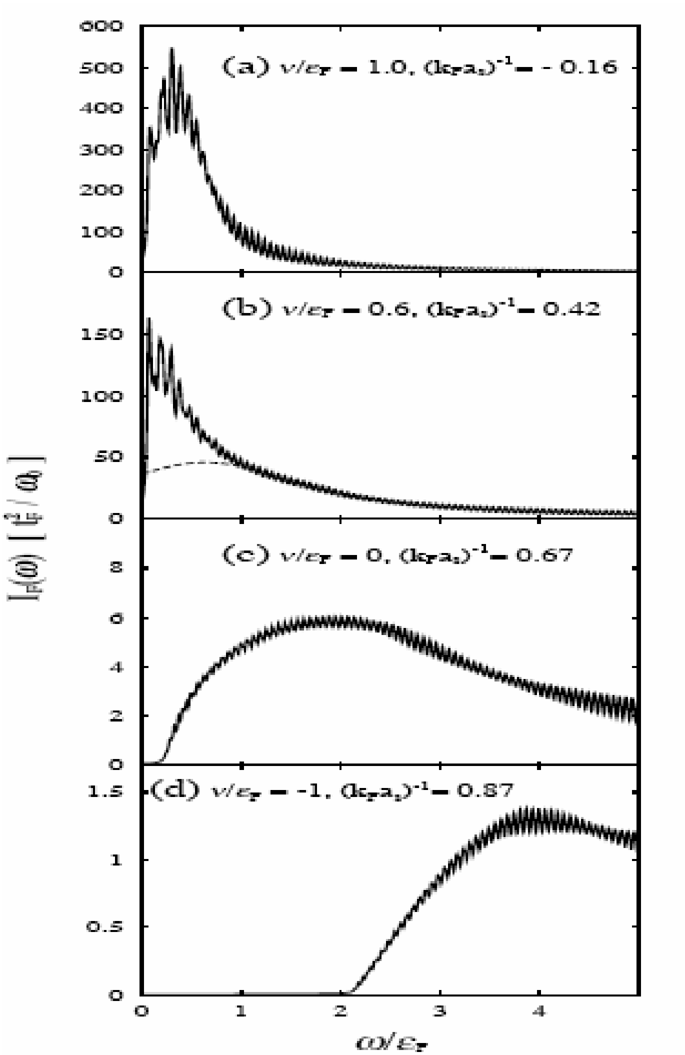


The sharp peaks at low energies come from the analogue of Andreev states at edge of trapped gas.



Graph of single-particle density of states *N(ω)* in a trap (in units of the trap frequency ω_0).

A standard calculation of the **tunneling current** gives it in terms of the single-particle Green's functions:

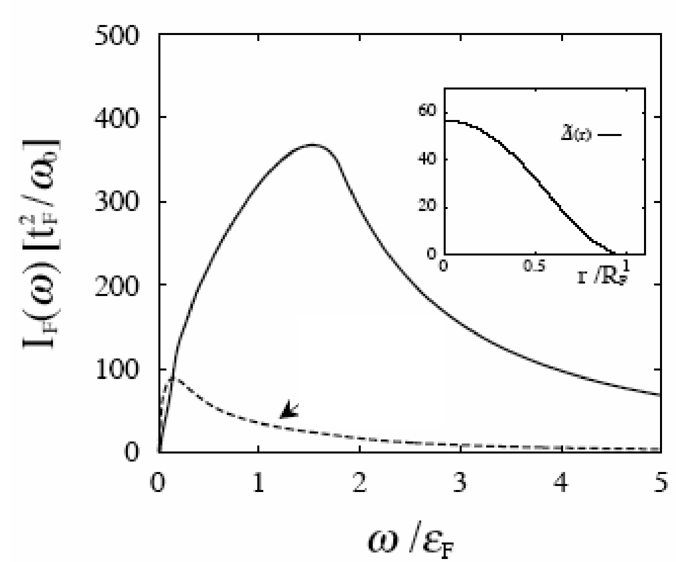


$$I_F(\omega) = \langle \hat{I}_F(\omega) \rangle = 2t_F^2 Im \int d\mathbf{r} d\mathbf{r}' \Pi_F(\mathbf{r}, \mathbf{r}', -\omega)$$

where the effective detuning frequency is

$$\omega \equiv \omega_L - \omega_a - \mu + \mu_a$$

$$\Pi_F(\mathbf{r}, \mathbf{r}', i\nu_n) \equiv -\int_0^\beta d\tau e^{i\nu_n\tau} \langle T_\tau \{ \Psi_a^\dagger(\mathbf{r},\tau)\Psi_\uparrow(\mathbf{r},\tau)\Psi_\uparrow^\dagger(\mathbf{r}')\Psi_a(\mathbf{r}')\} \rangle$$
$$= \frac{1}{\beta} \sum_{i\omega_m} G_{11}(\mathbf{r}, \mathbf{r}', i\omega_m + i\nu_n) G_a(\mathbf{r}', \mathbf{r}, i\omega_m).$$



Ohashi & Griffin, cond-mat/0410220

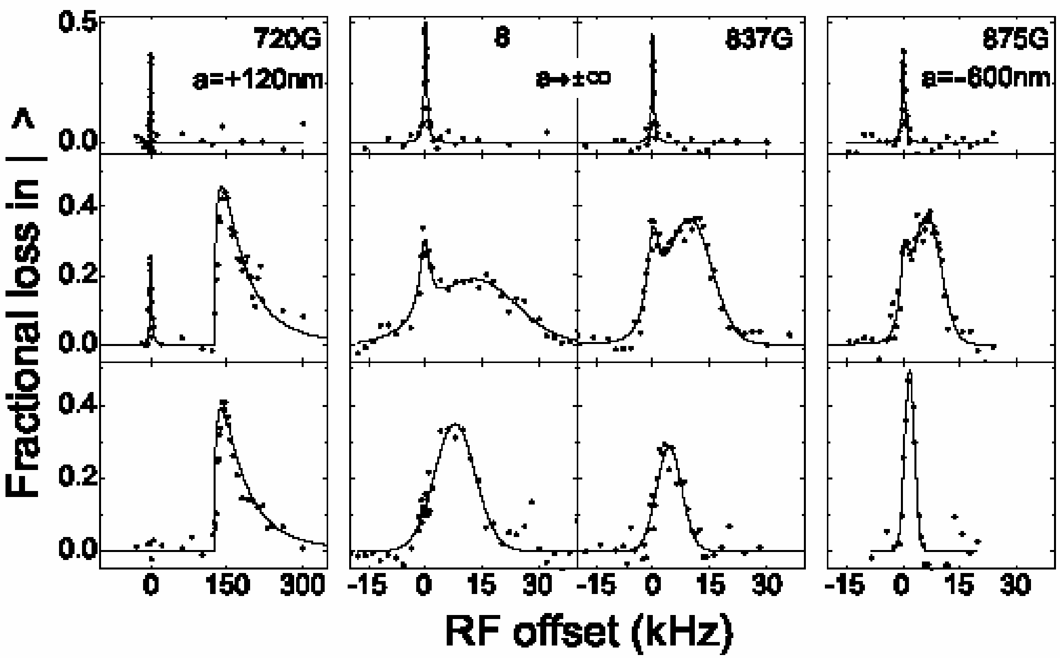
Conclusions

- **There is no fundamental difference between a molecular condensate in the BEC limit and a Cooper pair condensate in the BCS limit.**

- **The single particle Fermi excitations have the BCS Bogoliubov spectrum with an energy gap. However, this energy gap is now longer simply related to the order parameter even in the BCS region, but is due to low energy Andreev states localized near the edge of the trap.**

- **These single particle excitations can be directly probed using rf-tunneling into another atomic state.**

Recently Grimm and coworkers at Innsbruck have used **rf-tunneling** of Fermi atoms into **another** atomic state. This type of measurement is the **analogue** of tunneling from a superconductor to a normal metal. It gives information about the **spectral density of the quasiparticle excitations** of the Fermi superfluid.



T >T_C

T << T_C

BEC Crossover region **BCS**

- **With these atomic superfluid Fermi gases, we can study the effect of a pair condensate in a direct way compared to usual BCS superfluids, since the pairing interaction can be varied.**

- **So far *s-wave* interactions have been mainly studied. However, *p-wave* and *d-wave* atomic Fermi superfluids are now being considered by theorists (Ohashi, cond-mat / 0410516 ; Ho and Diener, cond-mat / 0408468) and by experimentalists. Stay tuned!**

My home city of Toronto, Canada in summer time!

2005 Banff Cold Atom Meeting:
10th Anniversary of Bose-Einstein Condensation

Banff, Alberta, Canada
February 24-27, 2005

Organized by

Allan Griffin, Department of Physics, University of Toronto (Director)
David Feder, Department of Physics and Astronomy, University of Calgary (Co-Director)

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