

BCS EXCITATIONS IN A SUPERFLUID GAS OF TRAPPED FERMI ATOMS

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Since 1995, we have become familiar with **BEC in trapped gases of Bose atoms**. **Trapped Fermi gases** have become the hot topic in **ultracold atom physics** in the last year. This talk will discuss why.

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THE ESSENTIAL IDEAS

Interactions between the Fermi atoms can produce **bound states** (pairs) which act as **Bosons**. These **molecules** can then Bose condense below a certain transition temperature, giving rise to what is called a **Fermi superfluid**.

These **molecular Bose condensates** are the analogue of **metallic superconductors**, which involves a Bose condensate of **Cooper pairs**.

There is **no** real difference between a **Fermi superfluid** with a **molecular condensate** and one with a **BCS Cooper pair condensate**. This is what is involved in the famous **BCS-BEC crossover**.

Trapped Atomic Fermi Gases

- Many groups can now cool Fermi gases down to $0.05T_F$, where T_F is the Fermi temperature (10^{-6} K).
- Two different atomic hyperfine states $|F, m_F\rangle$ are used, where $m_F = -F, \dots, -1, 0, 1, \dots, F$ denotes the different Zeeman levels. F denotes the total spin of the atom (nuclear and electronic). For **Fermions**, F must be an **odd multiple** of $1/2$.
- ^{40}K ($F = 9/2$) - Jin (JILA, Boulder).
- ^6Li ($F = 1/2$) - Grimm (Innsbruck), Ketterle (MIT), Hulet (Rice), Salomon (ENS, Paris).
- ^6Li is a nice Fermi atom since $m_F = 1/2, -1/2$

Fermi superfluids: molecules and Cooper pairs

Interactions between Fermi atoms can form **two-particle bound states** (or pair) states. These dimers arise because of two **separate** reasons:

1. Two-body potential has a bound state. As a result, **molecules** can form in a gas of Fermi atoms.
2. An **attractive** interaction in a Fermi gas always leads to the formation of **Cooper pairs**, strictly as a many-body effect. This was first shown by **Bardeen, Cooper, and Schrieffer** (BCS) in 1957 in their theory of **superconductivity in an electron gas**.

Overview

These **bound states** in **interacting** Fermi gases are **Bosonic** in nature and hence can **Bose condense**, just like Bose atoms can. As a result, in our discussion of trapped Fermi gases, the well-known description of Bose condensate will appear once again, except that it now describes a **molecular Bose condensate** or a **Cooper pair condensate**, immersed in the gas of unpaired Fermi atoms.

In the **extreme limit**, all N Fermi atoms can form $N/2$ bound states. This is the **BEC limit** of an interacting Fermi gas. It is effectively a Bose-condensed gas of $N/2$ **molecules**, each with mass $M = 2m$.

Interesting as this **pure molecular BEC** is , new physics occurs when some Fermi atoms have **not** paired up to form molecules. As mentioned earlier, the other extreme is the **BCS superfluid phase**, where Cooper pairs play the role of the molecules. This leads us to the **crossover** between the BCS and BEC phases of superfluid Fermi gases.

Great progress in studying this **BCS-BEC crossover** has been achieved in the last **15 months**, both by experimentalists and theorists. It is the **most important discovery** in ultracold atoms since BEC in 1995.

NOTE: For clarity, I will often discuss the results for a **uniform** Fermi gas. The physics is similar for a trapped Fermi gas. The actual calculations are done for a **trapped gas**.

More information on ultracold matter:

Two excellent textbooks have been published by top theorists:

**BOSE-EINSTEIN CONDENSATION IN
DILUTE GASES**, by Pethick and Smith (Cambridge , 2002).

BOSE-EINSTEIN CONDENSATION,
by Pitaevski and Stringari (Oxford, 2003).

All my work on **Fermi superfluids** is done in collaboration
with **Prof. Yoji Ohashi**, University of Tsukuba, Japan.

For more details on my research, see
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Interacting quantum gases

At **ultralow** temperatures, a **tremendous** simplification occurs because the atoms have such low energy. Only the **lowest partial wave scattering** contribution is not **frozen out**.

Thus for two identical Bose atoms or two Fermi atoms in **different** hyperfine states, we need only can keep the $l = 0$ **partial wave**, ie, the **s-wave** scattering length a . Moreover, standard **two-body scattering theory** shows that one can replace the **real interatomic** potential with an effective short-range pseudopotential:

$$v_{\uparrow\downarrow}(r - r') = \frac{4\pi\hbar^2 a_{\uparrow\downarrow}}{m} \delta(r - r') \equiv -U\delta(r - r')$$

The s-wave scattering length a is the **only** parameter needed to describe the interactions in ultracold **Bose gases** as well as in two-component **Fermi gases**. This is one of the reasons why cold quantum gases are so interesting.

Using this pseudopotential for the s-wave interaction between spin up and spin down atoms, the interaction energy is

$$\begin{aligned}
 V &= \sum \int dr \int dr' \psi_{\downarrow}^{\dagger}(r) \psi_{\uparrow}^{\dagger}(r') v_{\uparrow\downarrow}(r-r') \psi_{\uparrow}(r') \psi_{\downarrow}(r) \\
 &= -U \int dr \psi_{\downarrow}^{\dagger}(r) \psi_{\uparrow}^{\dagger}(r) \psi_{\uparrow}(r) \psi_{\downarrow}(r)
 \end{aligned}$$

$$= -U \sum_{p,q} c_{p\downarrow}^{\dagger} c_{-p\uparrow}^{\dagger} c_{-q\uparrow} c_{q\downarrow}$$

$$\psi_{\alpha}(r) = \sum_p e^{ip \cdot r} c_{p\alpha}$$

NB: We use the convention that an **attractive bare potential** corresponds to $U > 0$. This is the case of interest in **Fermi superfluid gases**.

[This simplified form of the interaction is no good for **high** energy processes and one has to introduce a **renormalized interaction** to get rid of these problems].

BCS theory of Fermi superfluids

$$H - \mu N = \sum_{p,\sigma} (\varepsilon_{p,\sigma} - \mu) c_{p,\sigma}^+ c_{p,\sigma} - U \sum_{p,q} c_{p\uparrow}^+ c_{-p\downarrow}^+ c_{-q\downarrow} c_{q\uparrow}$$
$$\Rightarrow \Rightarrow \approx \sum_{p,\sigma} (\varepsilon_{p,\sigma} - \mu) c_{p,\sigma}^+ c_{p,\sigma} - \sum_p (\Delta c_{p\uparrow}^+ c_{-p\downarrow}^+ + h.c.)$$

$$\Delta = U \sum_q \langle c_{-q\downarrow} c_{q\uparrow} \rangle \equiv U \phi_C$$

Cooper pairs

This is the essence of the famous **BCS-Gorkov theory** of superconductivity in an interacting Fermi gases with an **attractive** interaction - U . The **order parameter** ϕ_C describes bound states of two Fermions, which are Bose-condensed into the **same state**. **Remark:** This MFA theory **ignores** Cooper pairs **outside** of the condensate.

This **BCS mean field approximation** can be diagonalized by the Bogoliubov transformation,

$$H_{BCS} - \mu N = \sum_{p,\alpha} E_p \alpha_{p\alpha}^+ \alpha_{p\alpha} + \text{const.}$$

where the BCS **quasiparticles** have an energy given by the famous expression

$$E_q = \sqrt{(\varepsilon_q - \mu)^2 + \Delta^2}$$

$$E_p = \sqrt{(\varepsilon_p - \mu)^2 - \Delta^2}$$

One can work out the equivalent quasiparticle spectrum for Fermions **in a parabolic trap**.

More on BCS-Bogoliubov quasiparticles

One diagonalizes the BCS-Gorkov mean field Hamiltonian using the famous **Bogoliubov transformation**

$$c_{p\uparrow} = u_p \alpha_{p\uparrow} + v_{-p} \alpha_{-p\downarrow}^+$$

$$c_{p\downarrow} = u_p \alpha_{p\downarrow} - v_{-p} \alpha_{-p\uparrow}^+$$

The α, α^+ quasiparticle operators must satisfy **Fermi** anti-commutation relations, such as

$$[\alpha_{p\uparrow}, \alpha_{q\uparrow}^+]_{\pm} = \delta_{p,q}$$

As a result, the Bogoliubov amplitudes u and v are given by

$$\cdot \quad u_p^2 = \frac{1}{2} \left(1 + \frac{\varepsilon_p - \mu}{E_p} \right) \quad v_p^2 = \frac{1}{2} \left(1 - \frac{\varepsilon_p - \mu}{E_p} \right)$$

The self-consistent equations for Δ and μ

Clearly, we have **two** quantities we need to calculate using our quasiparticle solution, namely the chemical potential μ and the energy gap Δ . The **number equation** is

$$N \equiv \sum_{q,\alpha} \langle c_{q\alpha}^+ c_{q\alpha} \rangle = \sum_q \left[|u_q|^2 \langle \alpha_{q\uparrow}^+ \alpha_{q\uparrow} \rangle_{Bog} + |v_{-q}|^2 \langle \alpha_{-q\downarrow}^+ \alpha_{-q\downarrow} \rangle_{Bog} \right]$$

$$N = N_F = \sum_q \left[1 - \frac{\varepsilon_q - \mu}{E_q} + 2 \frac{\varepsilon_q - \mu}{E_q} f(E_q) \right]$$

where $f(E)$ is the **Fermi distribution function** for quasiparticles. This is the **number equation**, giving N **implicitly as a function of μ and Δ** .

The self-consistent BCS gap equation

We recall that the Cooper pair **order parameter** is defined as

$$\Delta \equiv U \sum_q \langle c_{-q\downarrow} c_{q\uparrow} \rangle \equiv U \phi_C$$

This can again be easily calculated by writing c and c^+ in terms of Bogoliubov quasiparticles,

$$\begin{aligned} \Delta &= U \sum_q u_q v_q \left[\langle \alpha_{q\uparrow}^+ \alpha_{q\uparrow} \rangle + \langle \alpha_{q\downarrow}^+ \alpha_{q\downarrow} \rangle - 1 \right] \\ &= U \sum_q \frac{\Delta}{2E_q} (2f(E_q) - 1) \end{aligned}$$

One last thing we have to do is **renormalize** the **bare** attractive interaction U to remove problems at high momentum. This is given by the usual **two-body s-wave scattering** length a_{2b} .

The **renormalized** two-body pseudopotential for s-wave scattering is given by

$$U_{2b} = \frac{U}{1 - U \sum_{q \leq q_c} \frac{1}{2\varepsilon_q}} \equiv -\frac{4\pi\hbar^2 a_{2b}}{m}$$

This **defines** the correct scattering length a_{2b} . Note that as the bare attractive U is increased, a_{2b} can go from being **negative** to **positive**. The gap equation can now be re-written in terms of U_{2b} , or equivalently a_{2b}

$$1 = -\frac{4\pi\hbar^2 a_{2b}}{m} \sum_q \left[\frac{1 - 2f(E_q)}{2E_q} - \frac{1}{2\varepsilon_q} \right]$$

The integrand in the gap equation is now **well behaved** at large momentum, as a result of using this renormalized interaction.

To summarize, standard BCS theory reduces to two coupled equations for the number of fermions N and the order parameter Δ :

$$1 = -\frac{4\pi\hbar^2 a_{2b}}{m} \sum_q \left[\frac{1 - 2f(E_q)}{2E_q} - \frac{1}{2\varepsilon_q} \right]$$

$$N = N_F = \sum_q \left[1 - \frac{\varepsilon_q - \mu}{E_q} + 2 \frac{\varepsilon_q - \mu}{E_q} f(E_q) \right]$$

In standard **weak coupling** limit (when $k_F |a_{2b}| \ll 1$) one can show there is no solution of the gap equation unless a_{2b} is **negative**. In this weak coupling BCS limit, one finds that $\mu \cong \varepsilon_F$. Also the BCS **transition temperature** is given by

$$T_{BCS} = T_F \exp\left(-\frac{\pi}{2k_F |a_{2b}|}\right) \ll T_F$$

Formally this BCS-BEC crossover can be studied by using the **number and gap equations**, considering $k_F a_{2b}$ as an adjustable parameter (Leggett, 1980) and let the solutions tell us what happens!

It turns out that the dimensional parameter $(k_F a_{2b})^{-1}$ covers the range

$$-\infty \rightarrow \frac{1}{k_F a_{2b}} \rightarrow +\infty$$

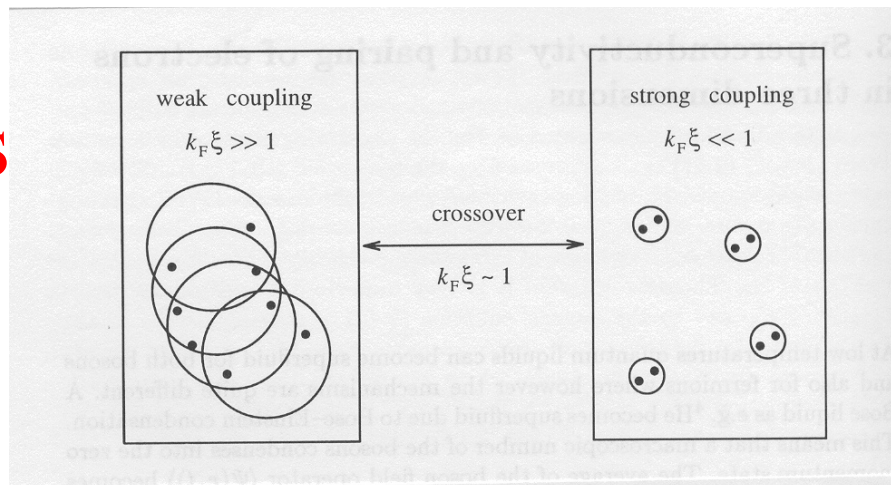
BCS **BEC**

as the bare attractive interaction is steadily increased. These original calculations **did not address how** you could vary the value of the s-wave scattering length. **Feshbach resonances** allow one to do this easily in trapped atomic Fermi gases!!
A new window on the physical world opened up in 2004.

The original BCS-BEC Crossover -1980s

As the magnitude of the **attractive interaction is increased**, the Cooper pairs become more **tightly bound** and eventually we pass over to a region described as a dilute gas of **small** Cooper pair molecules. This is the **famous BCS-BEC crossover**, first studied in **Eagles** in 1969 and in the **1980s** by **Leggett** (at $T=0$) and **Nozieres** (at T_c).

BCS



BEC

From Haussmann, 1993

A crucial bit of physics is left out of the BCS number and gap equations

When we think about it, our BCS equations **implicitly assume** that all the Cooper pairs are Bose condensed in the same $q_{\text{CM}} = 0$ state. In our number equation, we only calculated the contribution of the free Fermi atoms.

But it turns out that as the value of a_{2b} becomes > 0 , the Cooper pairs become stable two-particle states and can **occupy finite momentum states**. Thus T is increased, more and more Cooper pairs leave the condensate. In this improved theory, T_C will correspond to where the **Cooper pair condensate is depleted**, just like any other Bose gas! Nozieres (1985) was the first to calculate the **superfluid transition temperature** across the BCS-BEC crossover, as a_{2b} is varied, taking this depletion into account.

The method used by Nozieres and Schmitt-Rink (1985) replaces the BCS number equation by calculating the number of Fermions using the **thermodynamic identity**

$$N \equiv -\frac{\partial \Omega}{\partial \mu}$$

where the thermodynamic potential $\Omega(\mu, T)$ of the interacting Fermi gas is given by

$\Omega(\mu, T) =$ free energy of a **Fermi gas of atoms**

+

free energy of **fluctuations in the particle-particle channel**. These correspond to the formation of **bound states** of two atoms (Cooper pairs) with finite centre of mass momentum.

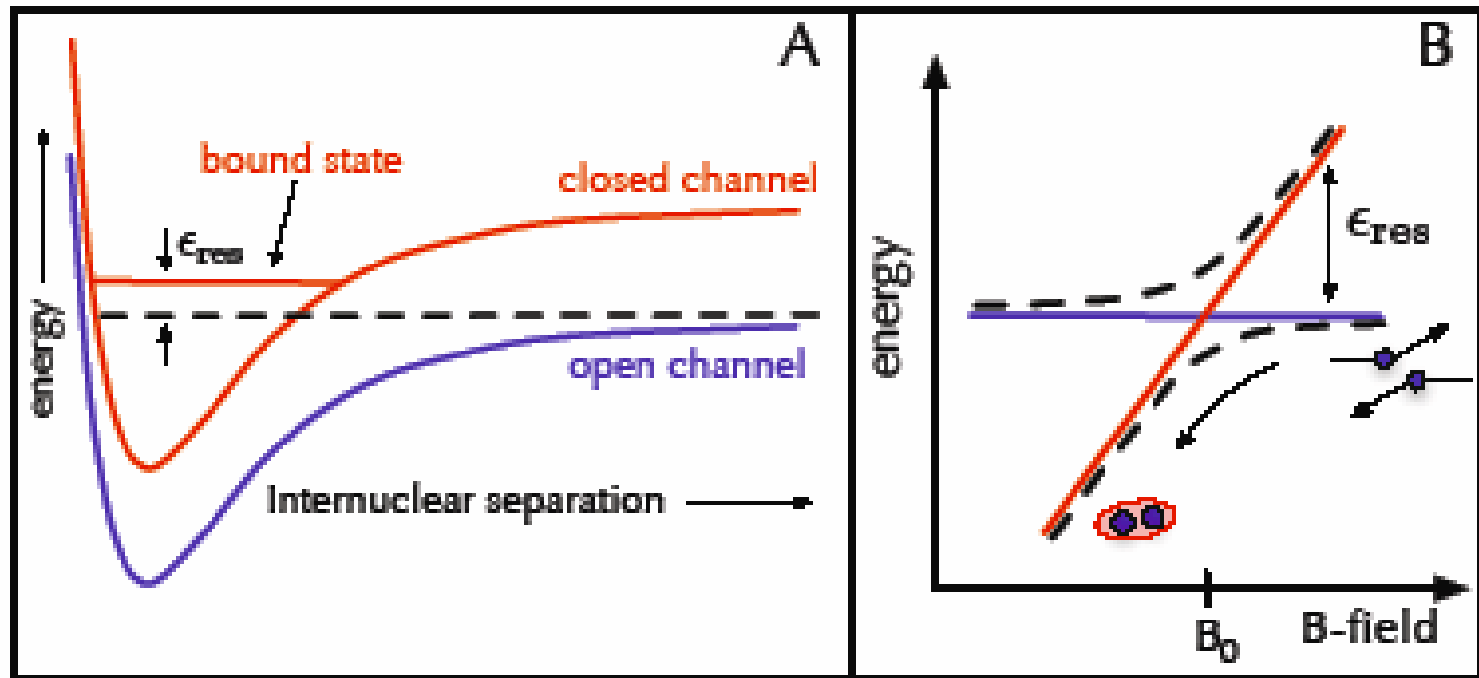
Left out in BCS \Rightarrow

Including this **Cooper pair gas** changes μ dramatically.

What are Feshbach Resonances?

- One of the most exciting recent topics in ultracold atoms has been the realization of the usefulness of **Feshbach resonances** in the **atomic scattering cross-section**. These are a **two-body** phenomenon and exist in both **Bose** and **Fermi** gases, but are most useful in Fermi gases. Allows us to get **strongly-correlated quantum gases**, even though they are very dilute.
- Such resonances arise when two colliding atoms have a **total kinetic energy** very close to the bound state energy level of a molecular potential (the so-called **closed channel**). In the literature, this **molecular bound state energy** is often denoted by **2ν** .
- The effective s-wave scattering length a_s has a resonance when **$2\nu = 0$** . The energy of the bound state molecular level can be shifted (**tuned**) by a small external magnetic field B .

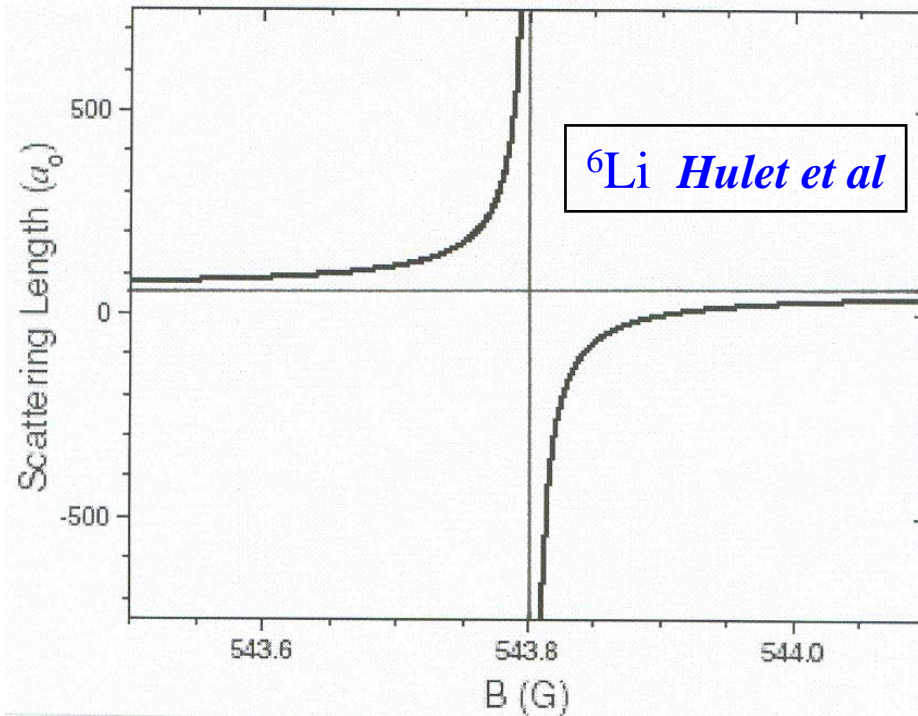
Two fermions in open channel strongly couple to a **bound state**, with energy $\epsilon_{\text{res}} = 2v$.



Two body physics of how dimer states can be created.
Process is reversible (adiabatic).

*From Jamie Williams et al,
New Journ. Phys. 6, 123 (2004)*

Feshbach resonance: two body physics



Molecules only form when $a_{2b} > 0$. This is equivalent to $2\nu < 0$ or $B < B_0$.

$$2\nu \propto B - B_0$$

$$-\frac{4\pi\hbar^2 a_{2b}}{m} \equiv U + \frac{g^2}{2\nu}$$

--->

$$a_{2b} = a_{bg} \left(1 + \frac{w}{B_0 - B} \right)$$

Model for interacting Fermi atoms and Boson molecules - putting everything together!

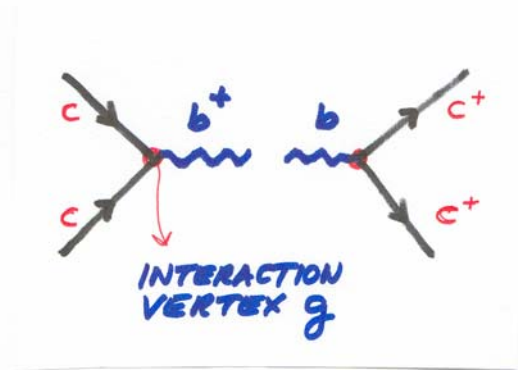
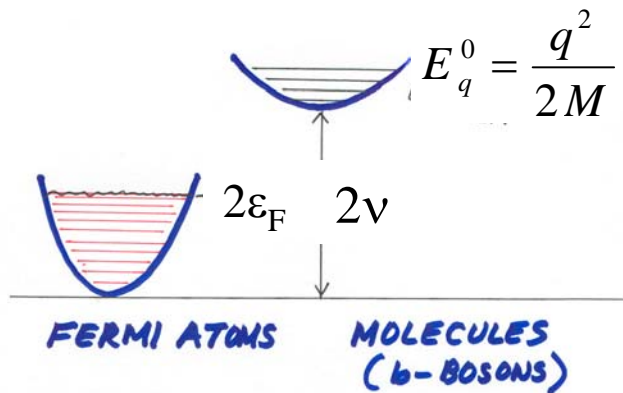
We need a microscopic model that includes:

- ✓ **Feshbach resonance in the two-body potential**
- ✓ **BCS Cooper pair formation**
- ✓ **BCS-BEC crossover as interaction increases**

The model (due to **Timmermans, Holland, Drummond** and coworkers) explicitly includes the Fermi atoms, the Bosonic molecules formed from these atoms, and the Feshbach resonance coupling term. This theory is now often called **resonance superfluidity**, a term introduced by Holland (JILA).

$$\mathcal{H} = \sum_{\mathbf{p}\sigma} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_{\mathbf{q}} (E_{\mathbf{q}}^0 + 2\nu) b_{\mathbf{q}}^\dagger b_{\mathbf{q}}$$

$$- U \sum_{\mathbf{p}, \mathbf{p}'} c_{\mathbf{p}\uparrow}^\dagger c_{-\mathbf{p}\downarrow}^\dagger c_{-\mathbf{p}'\downarrow} c_{\mathbf{p}'\uparrow} + g_{\mathbf{r}} \sum_{\mathbf{p}, \mathbf{q}} [b_{\mathbf{q}}^\dagger c_{-\mathbf{p}+\mathbf{q}/2\downarrow} c_{\mathbf{p}+\mathbf{q}/2\uparrow} + \text{h.c.}].$$



- The atom-molecule interaction is denoted by $g_{\mathbf{r}}$
- The non-resonant attractive interaction is $-U$

The molecular bound state energy 2ν can be **tuned**. Molecules (with finite lifetime) start to form when $2\nu \leq 2\varepsilon_F$ and will **not** be able to **decay** when $2\nu < 0$.

$$N = \langle \sum_{p\sigma} c_{p\sigma}^\dagger c_{p\sigma} \rangle + 2 \langle \sum_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}} \rangle$$

$$\equiv N_F + 2N_B.$$

However, since the b-molecules are **formed** from two Fermi atoms, there is thus only **one** chemical potential

$$\mathbf{H} - \mu\mathbf{N} = \mathbf{H} - \mu\mathbf{N}_F - 2\mu\mathbf{N}_B, \text{ with } \mu_B = 2\mu.$$

Our coupled Hamiltonian incorporates effect of the bare **two-body** Feshbach resonance given earlier,

$$U_{eff} = U + \frac{g^2}{2\nu}$$

Moreover, the atoms are now part of an interacting Fermi gas coupled to Bose molecules.

As before, we need to calculate the chemical potential μ and the order parameter $\tilde{\Delta}$ in a self-consistent way as function of 2ν , one has to include the **fluctuations** around the **BCS - Gorkov MFA** :

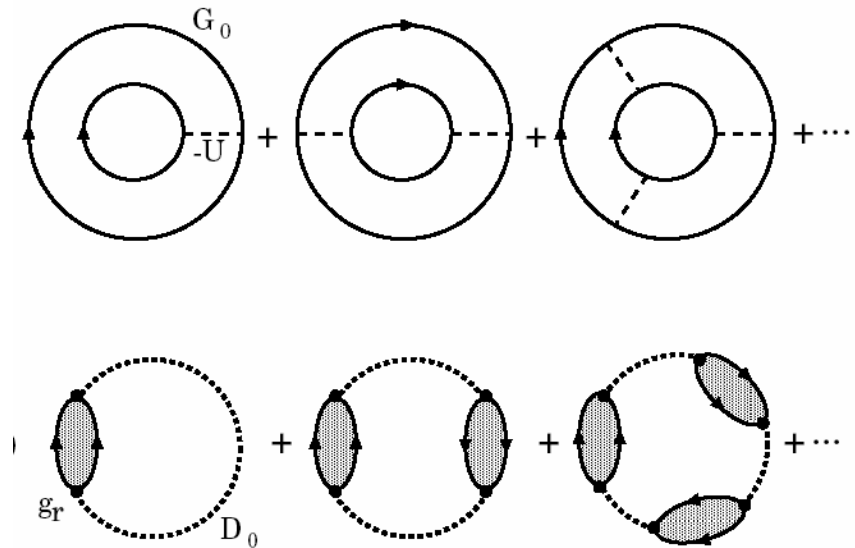
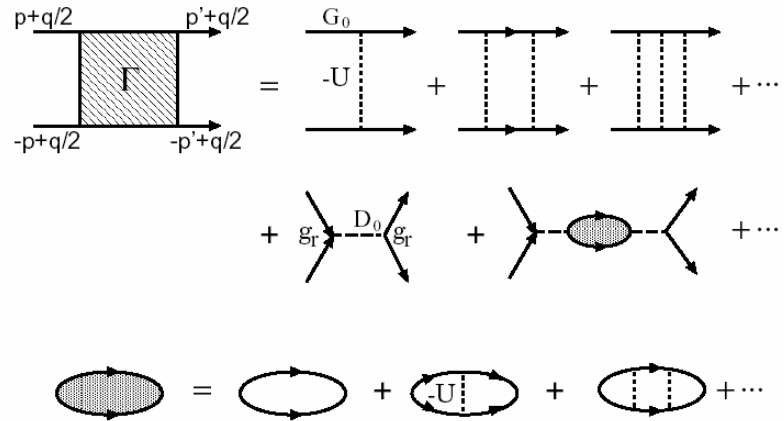
- ✓ The Cooper pairs **outside** the BCS condensate
 - Nozieres and Schmitt-Rink (1985) at T_c .
- ✓ The b-molecules **outside** the molecular condensate
 - Ohashi and Griffin (2002) at T_c .
- ✓ Both effects included **below** T_c by O&G (2003).

The **number** of b-molecules **and** Cooper pairs is self-consistently adjusted as **2ν is decreased**,

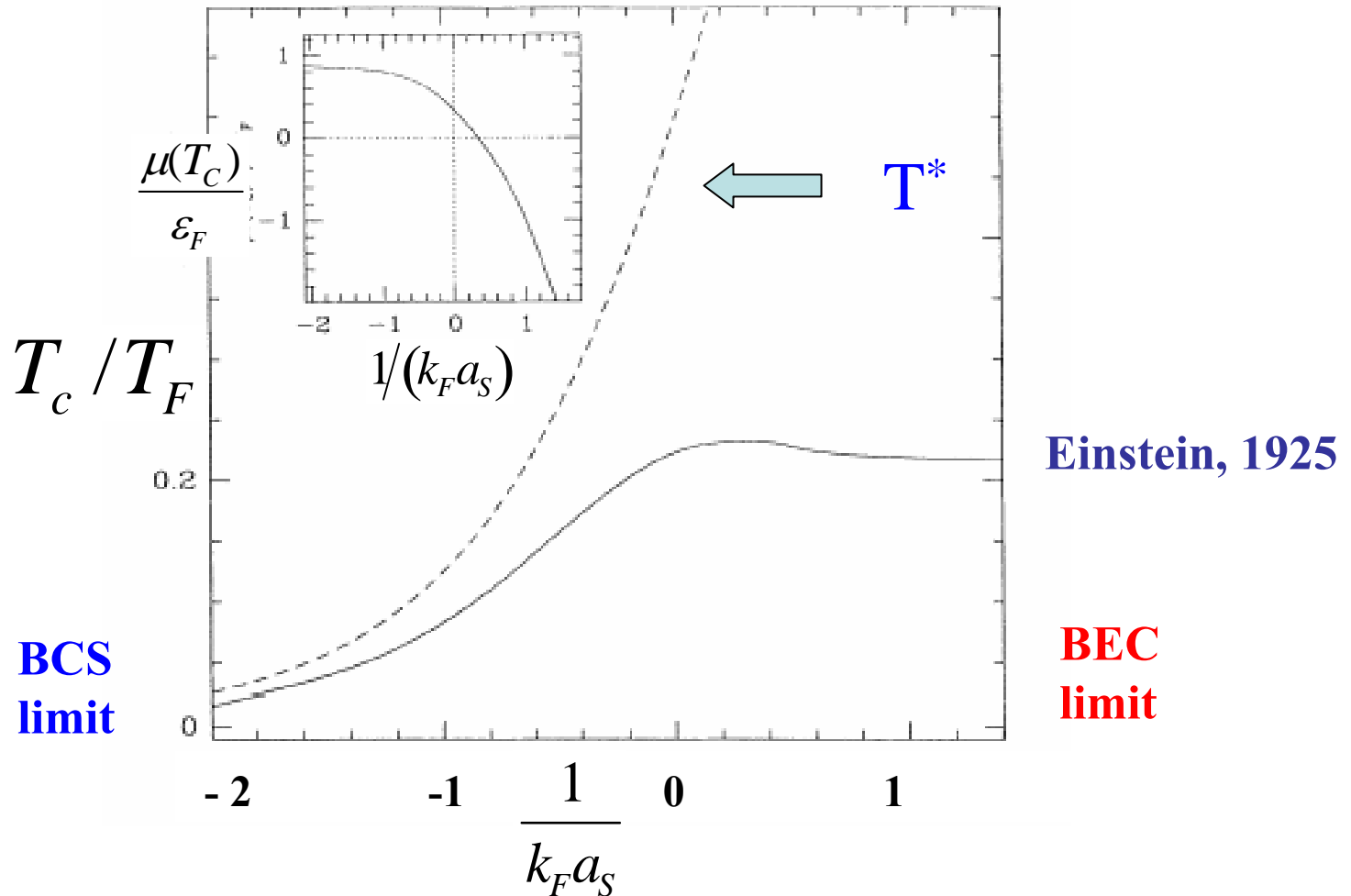
$$N_F = N_{\text{atoms}} + 2N_{\text{Cooper pairs}} + 2N_{\text{b-molecules}}$$

For experts only

Particle-particle vertex function Γ



Fluctuations in the thermodynamic potential Ω



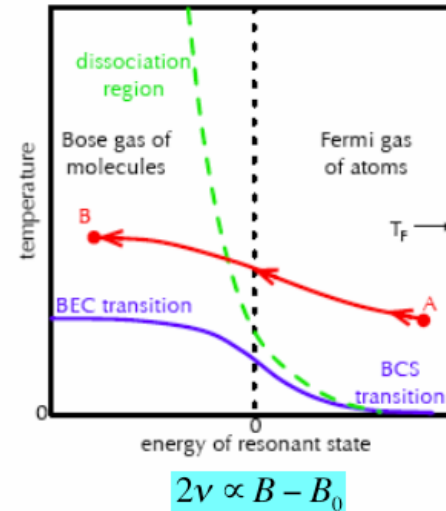
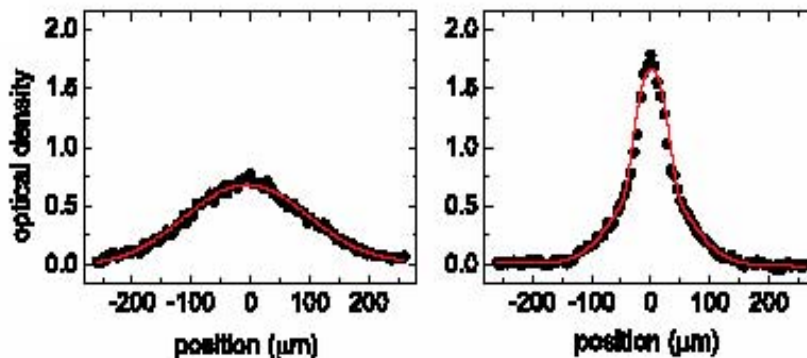
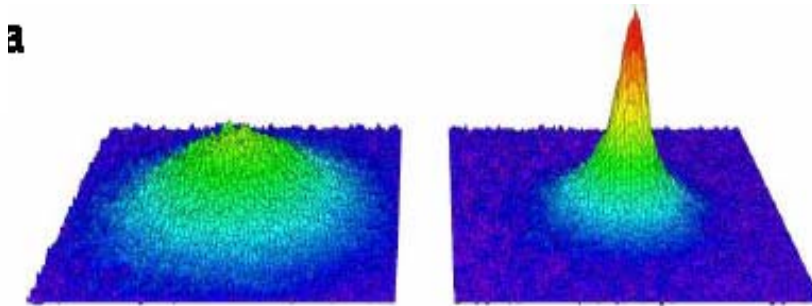
T_C is the BCS-BEC superfluid phase transition temperature. T^* shows where the bound states **breakup** or ionize. Note that the **weak coupling** T_{BEC} result corresponds to the breakup of Cooper pairs, not depletion.

Sa de Melo , Randeria and Engelbrecht, PRL, 1993

A **molecular Bose condensate** formed by **SLOWLY** ramping the magnetic field from just **above** ($a_s < 0$) to just **below** ($B - B_0 = -0.56G$) the resonance ($a_s > 0$).

$T = 0.19T_F$
 $N_C = 0\%$

$T = 0.06T_F$
 $N_C = 12\%$



© Geoffrey Wheeler
 Deborah Jin, Markus Greiner
 and Cindy Regal (left to right)

The density profile of the **free Fermi** atoms is not shown. The molecular condensate has the **same profile as an atomic BEC**, except $M = 2m$.

First thing to do is to solve our coupled FB model in a **mean field approximation**, allowing for Cooper pairs **and** a Bose condensate ($q = 0$) of b-molecules:

$$H_{FB} - \mu N = \sum_{p,\sigma} (\varepsilon_{p,\sigma} - \mu) c_{p,\sigma}^+ c_{p,\sigma} + \phi_m^2 (E_{q=0} - 2\nu - 2\mu)$$

$$-U \sum_p (\phi_C c_{p\uparrow}^+ c_{-p\downarrow}^+ + h.c.) + g \sum_p (\phi_m c_{p\uparrow}^+ c_{-p\downarrow}^+ + h.c.)$$

ϕ_C = **Cooper pair condensate**- see previous lectures

ϕ_m = **Molecular condensate** = $\langle \mathbf{b}_{q=0} \rangle$

Both condensates are **dependent** on each other.

We end up with a BCS-type theory but now with a **composite** order parameter:

$$\tilde{\Delta} = U\phi_C - g\phi_m$$

$$\tilde{\Delta} = U\phi_C - g\phi_m$$

This order parameter is the **sum** of contributions from two mechanisms:

**Pair wavefunction = scattering channel
+ molecular channel**

However, they are strongly **coupled** to each other and one determines the other:

$$\phi_m = -\frac{g}{2\nu - 2\mu} \phi_C$$

Note that μ is a strong function of molecule energy 2ν .

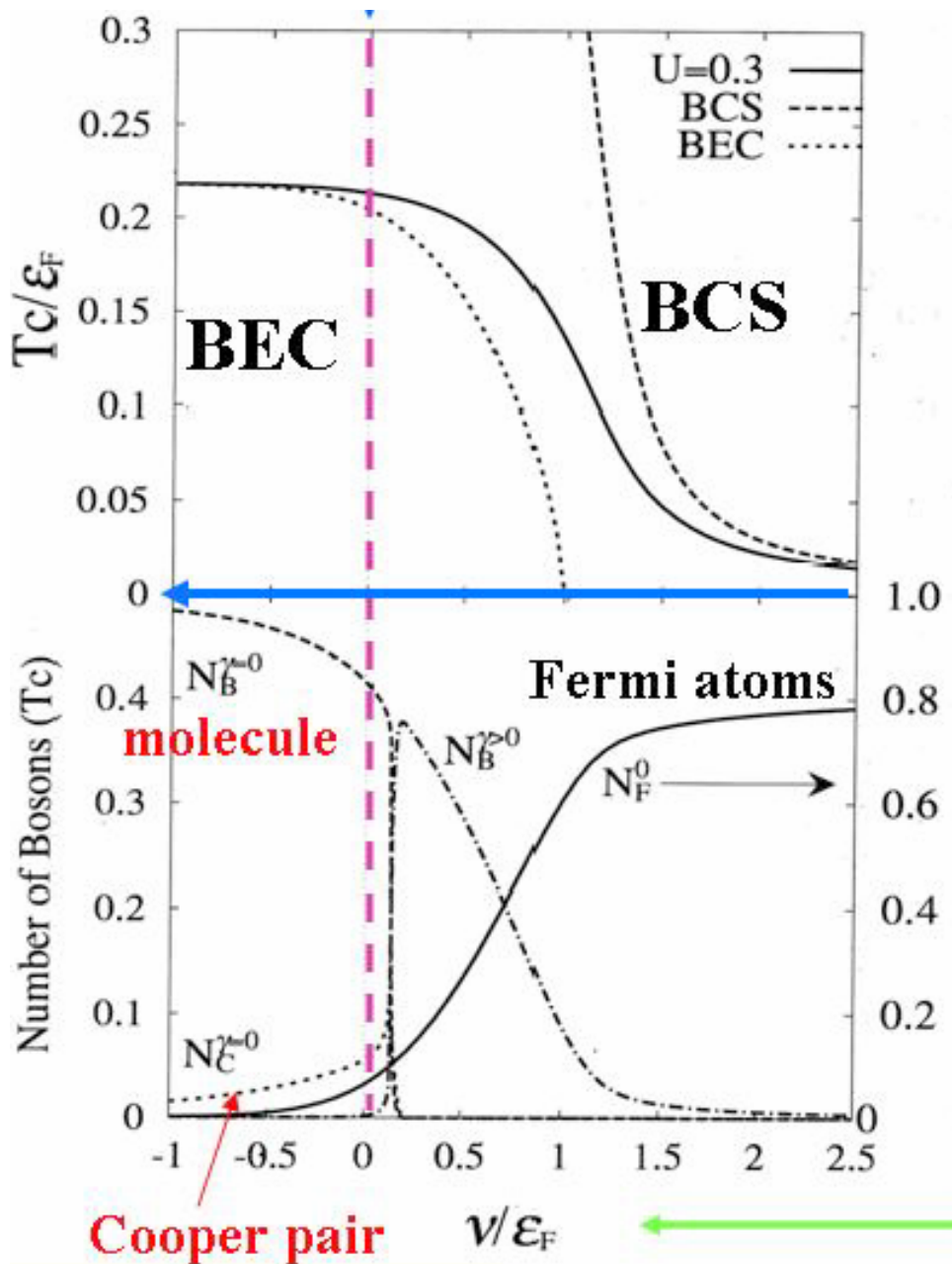
The number of Bose condensed b-molecules is given by **$N_b = |\phi_m|^2$** .

The **composite BCS order parameter** reduces to:

$$\tilde{\Delta} = U\phi_C - g\phi_m = \left(U + \frac{g^2}{2\nu - 2\mu} \right) \phi_C \equiv U_{eff} \phi_C$$

The physics is clear. The **attractive interaction** U between the Fermi atoms in the **open channel** is now renormalized to U_{eff} by the resonant coupling to the b-molecules in the **closed channel**. Calculation also shows that we always have $2\nu > 2\mu$

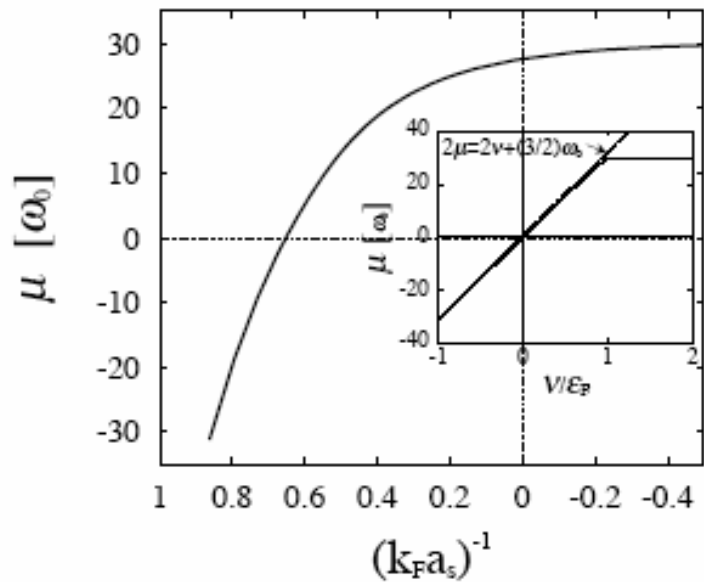
As before, we must also replace this U_{eff} by a renormalized interaction.



Uniform gas results at T_c

Similar results are obtained at $T < T_c$ and in trapped gases.

*One has long-lived **b-molecules** on the BCS side ($\nu > 0$) and stable **Cooper pairs** on the BEC side ($\nu < 0$).*



$\leq \epsilon_F$

Chemical potential μ (in units of the trap frequency ω_0) at $T = 0$.

How parameters change through crossover

BEC

$$a_s > 0$$

$$2v < 2\epsilon_F$$

$$\mu < 0$$

vs

BCS

$$a_s < 0$$

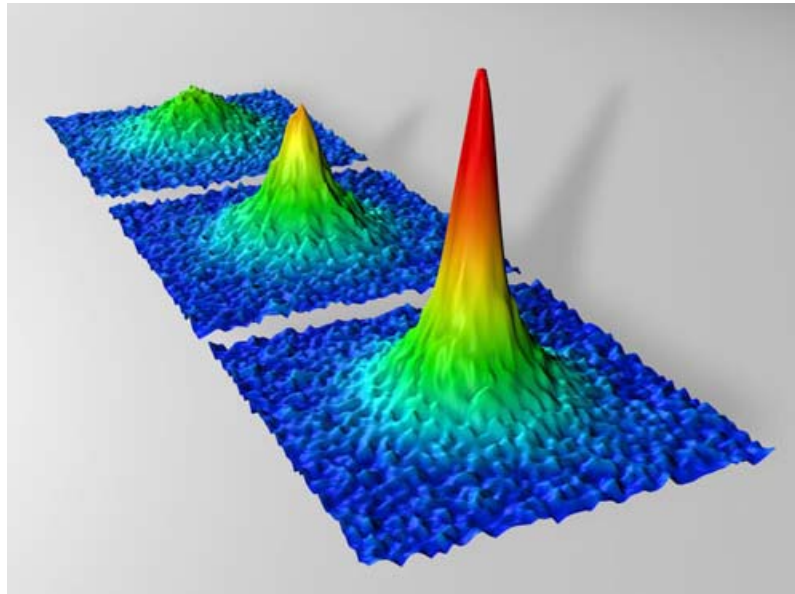
$$2v > 2\epsilon_F$$

$$\mu > 0 (\approx \epsilon_F)$$

The **holy grail** has been to find evidence for a **BCS Cooper pair condensate** above the resonance, where $a_s < 0$. The JILA experiment was done by ramping **RAPIDLY** from BCS region to BEC region so that real molecules do **not have time** to Bose-condense. The **bimodal profile** shows evidence for the appearance of a Bose condensate of **Cooper pairs**.

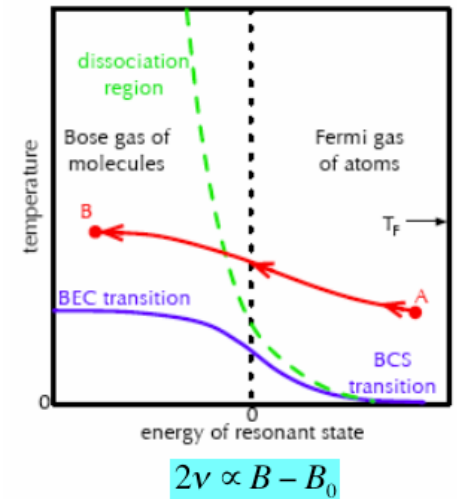
$$B - B_0 = 0.55G$$

$$N_C = 1\% \text{ ---->}$$



$$B - B_0 = 0.12G$$

$$N_C = 10\% \text{ ---->}$$



The density profile of the **free atoms** is **not** shown.

Regal, Greiner and Jin, PRL, Jan 30, 2004.

THE PICTURE THAT EMERGES

In a trapped Fermi gas, we can form two-particle bound states (dimers) which are **Bosons** and hence they can Bose-condense, forming a **Fermi superfluid**.

In the **crossover**, we go from a region where Cooper pairs dominate to one where real molecules dominate. However, the **entire region** can be described by the **same BCS type formalism**, built on a condensate of Bosonic pairs.

We now turn to discuss the **excitations** of this Fermi superfluid and how they behave as one goes through the BCS-BEC **crossover region**.

1. Single-particle Fermi excitations.

The **unbound or free Fermions** swim around in the condensate and are renormalized by the order parameter. In a trap, these **single particle BCS excitations** have an energy gap E_g and a spectrum that depends on both $\Delta(r)$ and μ in a **complicated way** (compared to the usual weak-coupling BCS theory for a uniform superfluid). I will now discuss this topic in a little more detail.

2. Collective oscillations of the condensate

These are the analogue of the well known **Anderson-Bogoliubov(1958) Goldstone phonon** in weak-coupling BCS theory of a neutral Fermi superfluid. Lots of recent work on this topic in trapped Fermi superfluids. **Needs a separate talk!**

In standard theory of a **uniform BCS superfluid**, there is a **direct relation** between the energy gap Δ of the single-particle excitations and the order parameter ϕ_C describing the superfluid

$$E_q = \left[(\varepsilon_q - \mu)^2 + \Delta^2 \right]^{1/2}$$

with

$$\Delta = U \sum_p \langle c_{-p\downarrow} c_{p\uparrow} \rangle \equiv U \phi_C$$

Since $\mu = \varepsilon_F$, the quasiparticles at the Fermi surface have an energy gap $E_g = \Delta$.

Things are **quite different** as we pass through the crossover region **into the BEC region**, where μ becomes negative and large. The energy gap is

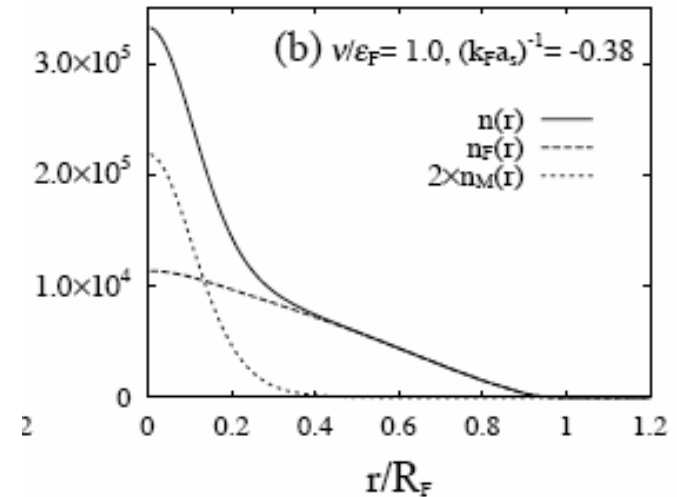
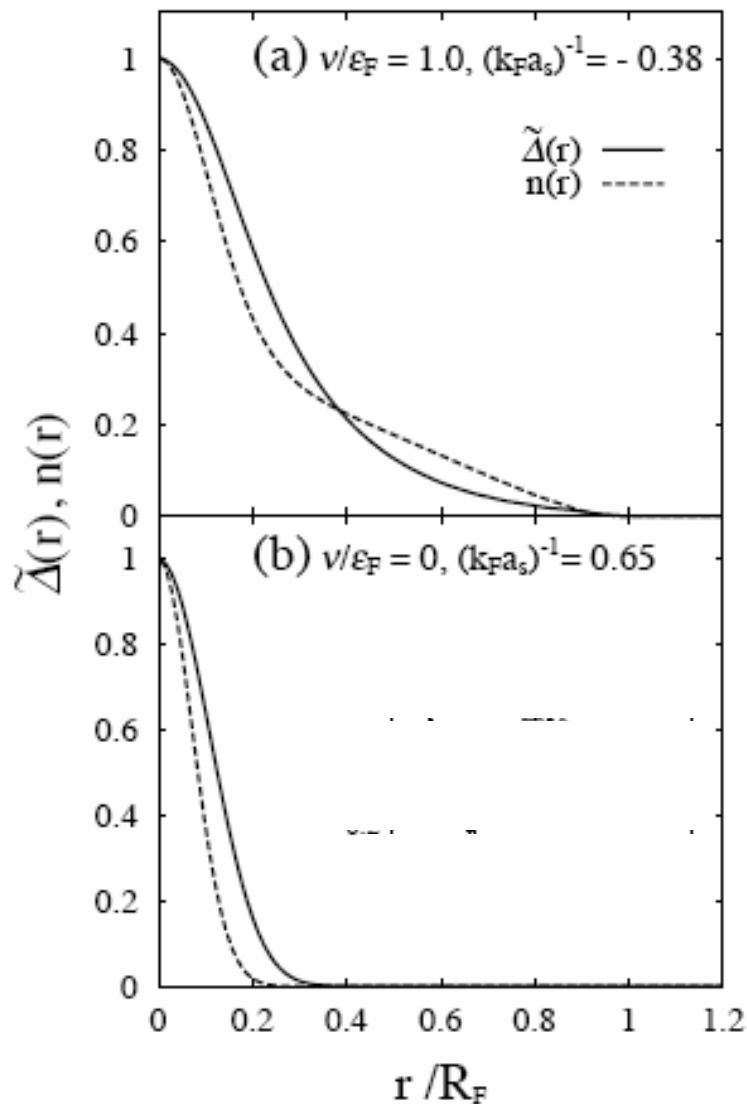
$$E_g = \left(\mu^2 + \Delta^2 \right)^{1/2} \rightarrow |\mu|$$

The **quasiparticle spectrum** at $T = 0$ of a trapped Fermi superfluid in the crossover region requires that we numerically solve the Bogoliubov-de Gennes (BdG) coupled equations for the Bogoliubov quasiparticle u and v amplitudes:

$$\begin{pmatrix} H_0 & \tilde{\Delta}(\mathbf{r}) \\ \tilde{\Delta}(\mathbf{r}) & -H_0 \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix}$$

Both the order parameter and the density are dependent on position in trap. These equations must be solved taking into the fact that some Fermi atoms are combining into molecules, going into a Bose condensate (which is treated as static).

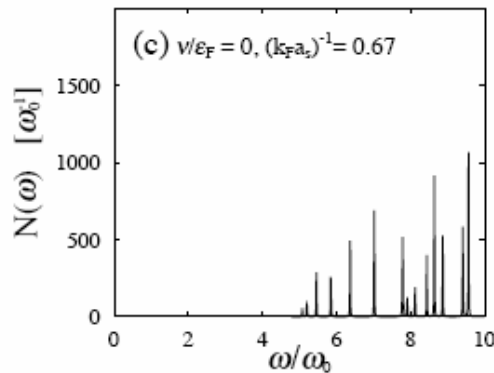
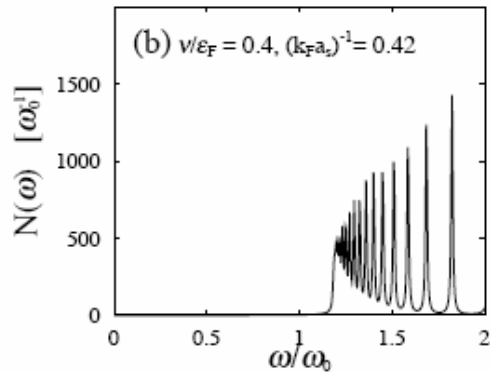
T = 0 results



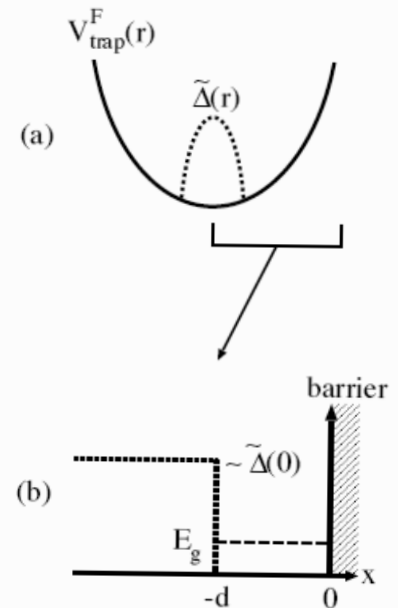
Relative number of free **Fermi** atoms and **Bosonic** bound states as a function of position.

Self-consistent solutions of the BdG equations for the local density of atoms $n(r)$ and the local order parameter $\tilde{\Delta}(r)$ in a harmonic trap. Normalized to values at **center of trap**.

New features about the spectrum of single-particle **Fermi excitations** of trapped Fermi superfluids. The energy gap is now due to low-energy states near the edge of the trap, where the density is low.



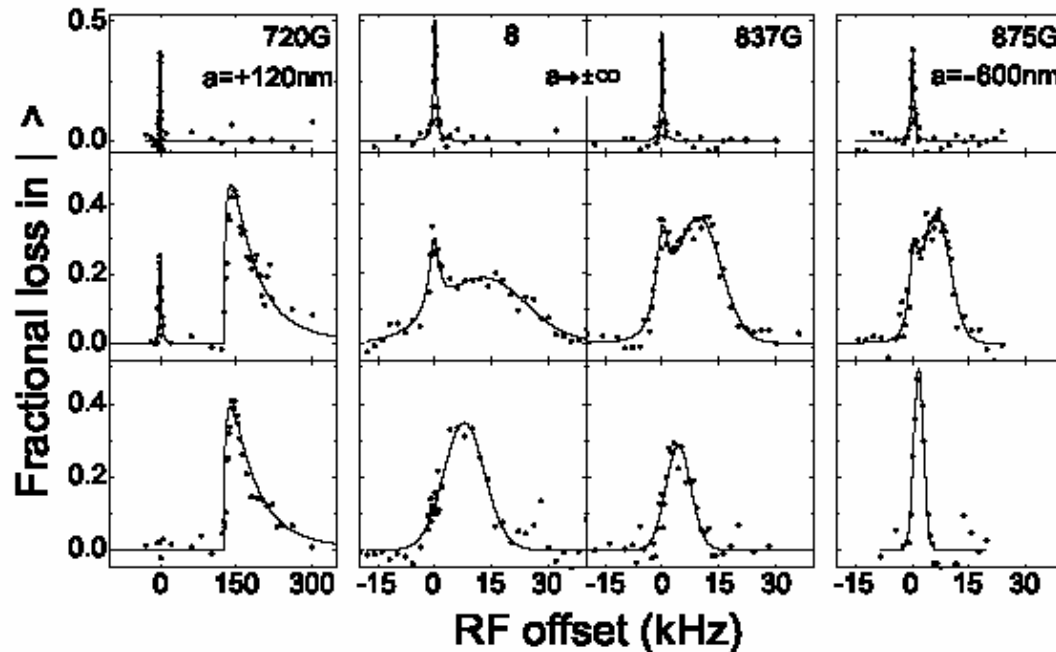
The sharp peaks at low energies come from the analogue of **Andreev states** at edge of trapped gas.



Graph of single-particle density of states $N(\omega)$ in a trap (in units of the trap frequency ω_0). The peaks are the computed BdG excitations.

Recently Grimm and coworkers at Innsbruck have used **rf-tunneling** of Fermi atoms into **another** atomic state. This type of measurement is the **analogue** of tunneling from a superconductor to a normal metal. It gives information about the **pairing gap** of the **quasiparticle excitations** of the Fermi superfluid.

New peak at finite energy is due to the finite energy gap of the excitations when we are **below** T_c



$T > T_c$

$T \ll T_c$

BEC

Crossover region

BCS

For experts only

A Golden Rule calculation of the **tunneling current** I_F of atoms in the hyperfine atomic state \uparrow making a rf-induced **transition** to a different atomic state labelled by “a” :

$$I_F(\omega) = \langle \hat{I}_F(\omega) \rangle = 2t_F^2 \text{Im} \int d\mathbf{r} d\mathbf{r}' \Pi_F(\mathbf{r}, \mathbf{r}', -\omega)$$

where the effective detuning frequency is

$$\omega \equiv \omega_L - \omega_a - \mu + \mu_a$$

and where

$$\begin{aligned} \Pi_F(\mathbf{r}, \mathbf{r}', i\nu_n) &\equiv - \int_0^\beta d\tau e^{i\nu_n \tau} \langle T_\tau \{ \Psi_a^\dagger(\mathbf{r}, \tau) \Psi_\uparrow(\mathbf{r}, \tau) \Psi_\uparrow^\dagger(\mathbf{r}') \Psi_a(\mathbf{r}') \} \rangle \\ &= \frac{1}{\beta} \sum_{i\omega_m} G_{11}(\mathbf{r}, \mathbf{r}', i\omega_m + i\nu_n) G_a(\mathbf{r}', \mathbf{r}, i\omega_m). \end{aligned}$$

This gives the atom current in terms of the **single-particle Green's functions**. Measurement of the **number of atoms making the transition** gives information about G_{11} .

Conclusions

- There is no **fundamental difference** between a molecular condensate in the **BEC limit** and a Cooper pair condensate in the **BCS limit**.
- The **single particle** Fermi excitations have the **BCS Bogoliubov spectrum** but with an energy gap. now longer simply related to the order parameter. It is due to **low energy Andreev states** localized near the edge of the trap.
- These single particle excitations can be **directly probed using rf-tunneling** into another atomic state.

■ With these **atomic superfluid Fermi gases**, we can study the effect of a pair condensate in a **direct way** compared to usual BCS superfluids, since the pairing interaction can be varied.

■ So far *s-wave* interactions have been mainly studied. However, *p-wave* and *d-wave* **atomic Fermi superfluids** are now being considered.

**WANT TO LEARN MORE ABOUT
SUPERFLUID ATOMIC GASES?**

References to recent BCS-BEC crossover literature

Creation of molecules:

Regal et al (JILA-Jin), Nature, 424, 47(2003)

Strecker et al(Rice-Hulet), PRL, 91, 0804026 (2003)

Molecular BEC:

Greiner et al(JILA-Jin), Nature, 426, 537(2003)

Jochim et al(Innsbruck-Grimm), Science, 302, 2101(2003)

BCS superfluid phase:

Regal et al(JILA-Jin),PRL, 92, 040403(2004)

Collective oscillations of condensate:

Kinast et al(Duke-Thomas), PRL, 92,150402(2004)

Bartenstein et al(Innsbruck-Grimm), PRL, 92, 203201(2004)

Single-particle Fermi excitations:

Chin et al(Innsbruck-Grimm) , Science, 305,1128(2004)

For theory references, see:

Ohashi and Griffin, cond-mat/0410220

Chen, Stajic, tan and Levin, cond-mat/0404274

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