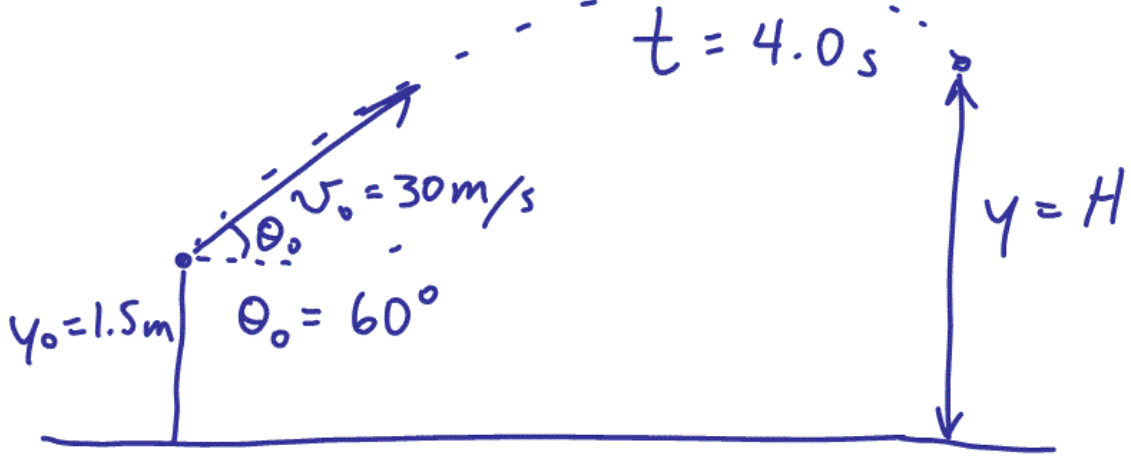
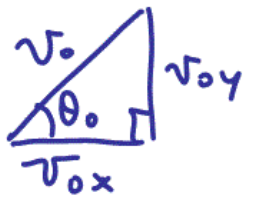
Problem 3.55

An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of 60° above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?



(a) y-component Knowns: t, v_{0y}



$v_{0y} = v_0 \sin\theta_0$

Unknown: $y = H$ ← final y-position.

Don't care about v_y final ...

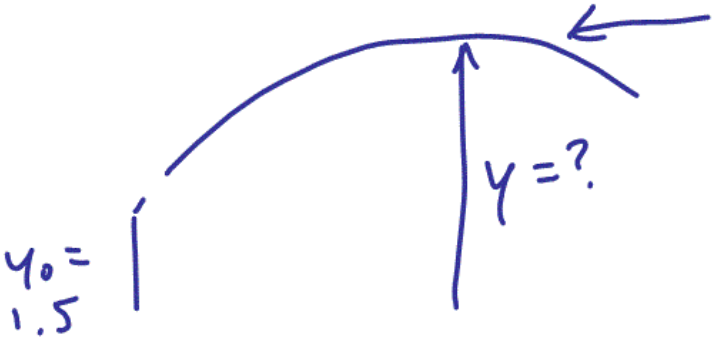
Use: $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

$H = 1.5 + (30 \sin 60)(4.0) - \frac{1}{2}(9.8)(4.0)^2$

$H = 27.023$

$\boxed{H = 27 \text{ m}}$

(b) max height during flight



Known: at max height, y-component of velocity = zero.

$v_y = 0$

$v_{oy} = v_o \sin\theta_o$

$y_o = 1.5 \text{ m}$

Don't care about t.

Use: $v_y^2 = v_{oy}^2 - 2g(y - y_o)$

$0 = v_o^2 \sin^2\theta_o - 2g(y - y_o)$

Solve for y:

$2g(y - y_o) = v_o^2 \sin^2\theta_o$

$y = y_o + \dfrac{v_o^2 \sin^2\theta_o}{2g}$

$y = 1.5 + \dfrac{30^2 (\sin 60)^2}{2(9.8)}$

$= 35.939$

$\boxed{y = 36 \text{ m}}$ ← higher than the diff, which is good.

(c) Impact speed at the cliff.

Known: $t = 4.0\ s$

$$v = \sqrt{v_y^2 + v_x^2}$$

y-comp

Known: v_{oy}, t, y_o, y

Need: v_y

... Don't care about $y - y_o$ for this part.

Use: $v_y = v_{oy} - gt$

$v_y = v_o \sin\theta_o - gt$

$= 30 \sin 60° - 9.8(4.0)$

$v_y = -13.2193\ m/s$

x-comp

$v_x = v_{xo} = v_o \cos\theta_o$

$= 30 \cos 60$

$= +15.00\ m/s$

Final v:

$v = \sqrt{v_y^2 + v_x^2}$

$= \sqrt{13.2193^2 + 15^2}$

$\boxed{v = 20\ m/s}$