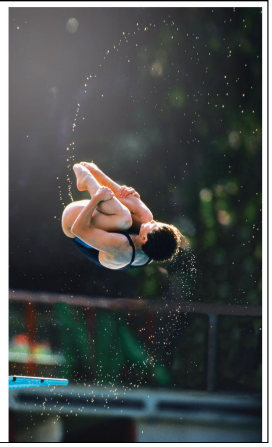
PHY131H1S - Class 17

Today:

- Rotational Motion, Rotational Kinematics (some review of Ch.4)
- Newton's 2nd Law of Rotation
- Torque
- Moment of Inertia
- Centre of Mass



Pre-class reading quiz on Chapter 12

***Moment of inertia* is**

A. the rotational equivalent of *mass*.
B. the point at which all forces appear to act.
C. the time at which inertia occurs.
D. an alternative term for *moment arm*.

Linear acceleration = force / mass:

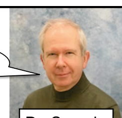
$$a_x = \frac{(F_{net})_x}{m}$$

analogous

Angular acceleration = torque / moment of inertia:

$$\alpha = \frac{\tau_{net}}{I}$$

Announcement



Dr. Savaria

- Test 2 will be on Tuesday, Nov. 22 from 8:00 to 9:30pm [note late start!] – Test 2 covers Chapters 5 through 12
- If you have a conflict at that time, and you were registered for the alternate sitting for test 1, you must send an email to April Seeley requesting an alternate sitting for test 2 by Thursday *this week* by 5:00pm.
- If you have a conflict at that time, and you were ***not*** registered for the alternate sitting for test 1, you must visit MP129 and fill out a conflict form for test 2 by Thursday *this week* by 5:00pm.
- The alternate sitting will take place on Wednesday Nov. 23 at 7:40am.

Last day I asked at the end of class:

Why is a door easier to open when the handle is far from the hinge, and more difficult to open when the handle is in the middle?

- ANSWER:
- **Torque** is the rotational analog of force:
- Force causes things to accelerate along a line.
- Torque causes things to have **angular acceleration**.
- Torque = Force × **Moment Arm**
- Moment Arm is the distance between where you apply the force and the hinge or pivot point.
- Putting the handle further from the hinge *increases your moment arm*, therefore it *increases your torque* for the same applied force: the door rotates better.

Recall from Chapters 1-4:

Linear	Rotational Analogy
s (or x or y) specifies position.	θ is angular position. The S.I. Unit is radians, where 2π radians = 360°.
Velocity: $v_x = \frac{d}{dt}(x)$ $\quad v_y = \frac{d}{dt}(y)$	Angular velocity: $\omega = \frac{d}{dt}(\theta)$
Acceleration: $a_x = \frac{d}{dt}(v_x)$ $\quad a_y = \frac{d}{dt}(v_y)$	Angular acceleration: $\alpha = \frac{d}{dt}(\omega)$

Linear / Rotational Analogy

Linear	Rotational Analogy
x	θ
v_x	ω
a_x	α
Force: F_x	Torque: τ
Mass: m	Moment of Inertia: I

Newton's Second Law:

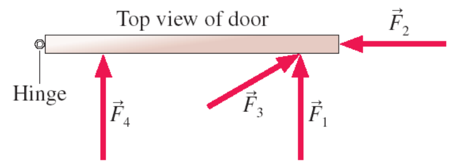
$$a_x = \frac{(F_{net})_x}{m} \qquad \alpha = \frac{\tau_{net}}{I}$$

Example

- The engine in a small airplane is specified to have a torque of 60.0 N m. This engine drives a propeller whose moment of inertia is 13.3 kg m^2. On start-up, how long does it take the propeller to reach 200 rpm?

Torque

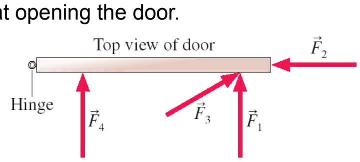
Consider the common experience of pushing open a door. Shown is a top view of a door hinged on the left. Four pushing forces are shown, all of equal strength. Which of these will be most effective at opening the door?



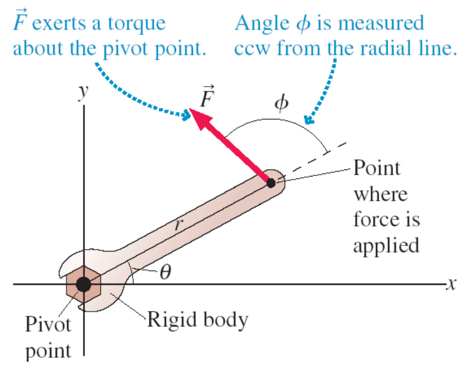
A. F_1
B. F_2
C. F_3
D. F_4

Torque

Consider the common experience of pushing open a door. Shown is a top view of a door hinged on the left. Four pushing forces are shown, all of equal strength. F_1 is most effective at opening the door.



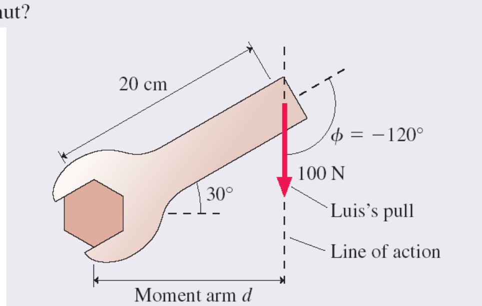
The ability of a force to cause a rotation depends on three factors:
1. the magnitude *F* of the force.
2. the distance *r* from the point of application to the pivot.
3. the angle at which the force is applied.



$$\tau = rF\sin\phi$$

Example

Luis uses a 20-cm-long wrench to turn a nut. The wrench handle is tilted 30° above the horizontal, and Luis pulls straight down on the end with a force of 100 N. How much torque does Luis exert on the nut?



Consider a body made of *N* particles, each of mass m_i, where i = 1 to *N*. Each particle is located a distance r_i from the axis of rotation. We define **moment of inertia**:

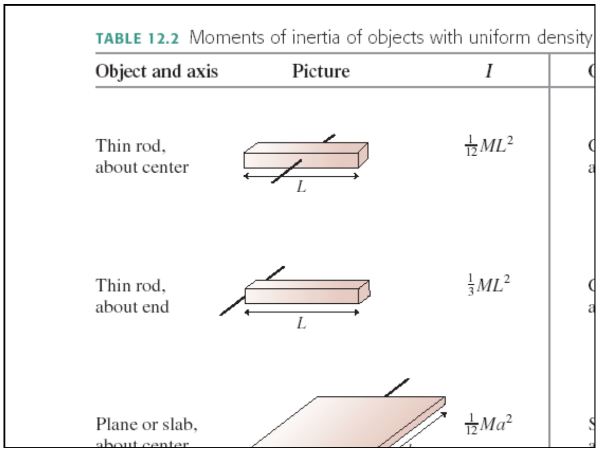
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2$$

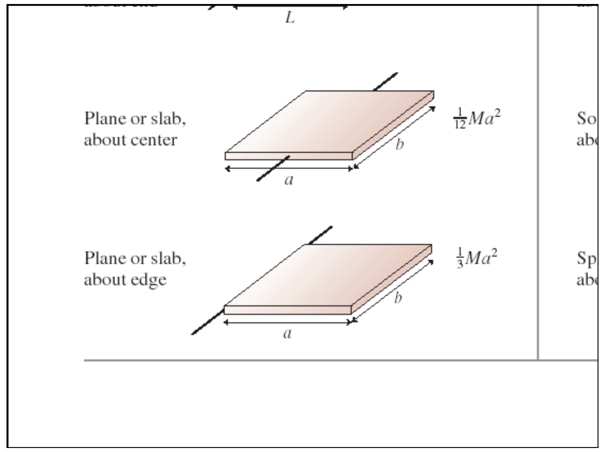
The units of moment of inertia are kg m^2. An object's moment of inertia depends on the axis of rotation.

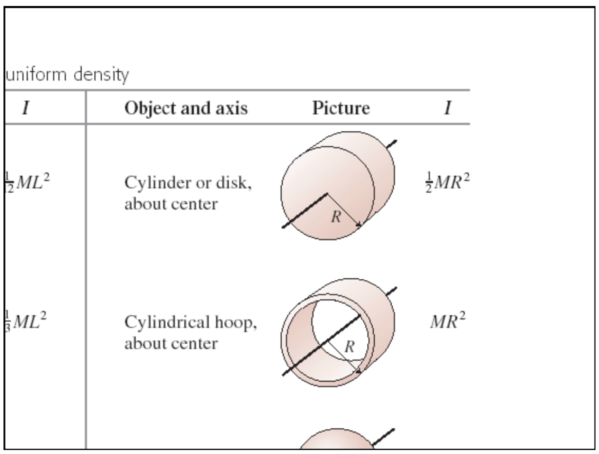
The moment of inertia

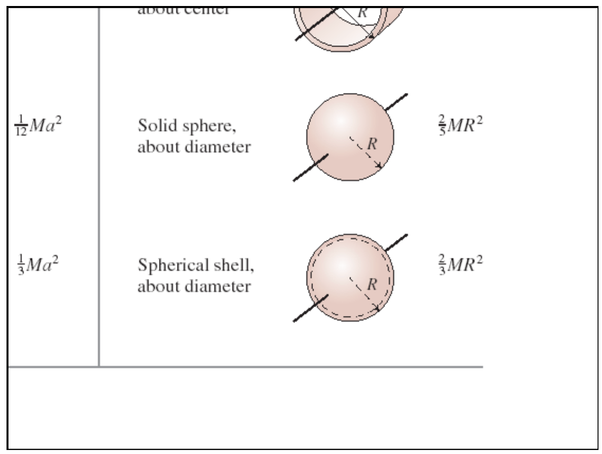
$$I = \sum_i m_i r_i^2 = \int r^2\, dm$$

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If I_{cm} is known, the I about a parallel axis distance d away is given by the **parallel-axis theorem:** $I = I_{cm} + Md^2$.





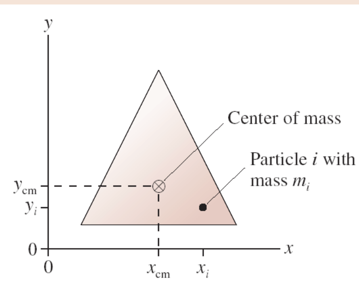




Center of Mass

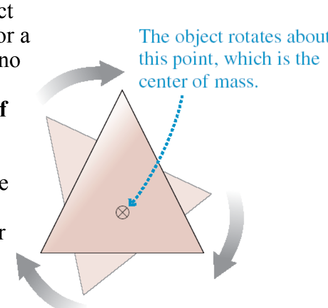
The center of mass is the mass-weighted center of the object.

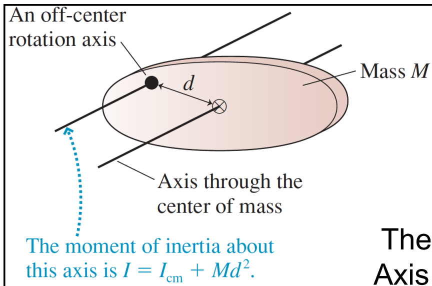
$$x_{cm} = \frac{1}{M}\int x\,dm \quad \text{and} \quad y_{cm} = \frac{1}{M}\int y\,dm$$



Rotation About the Center of Mass

- An unconstrained object (i.e., one not on an axle or a pivot) on which there is no net force rotates about a point called the **center of mass**
- The center of mass remains motionless while every other point in the object undergoes circular motion around it



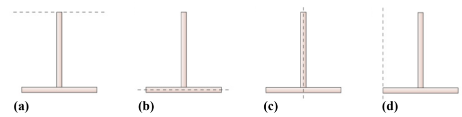


The Parallel-Axis Theorem

- Suppose you know the moment of inertia of an object when it rotates about axis its centre of mass: I_{cm}
- You can find the moment of inertia when it is rotating about any other axis which is a distance d away from the cm:

$$I = I_{cm} + Md^2$$

Four Ts are made from two identical rods of equal mass and length. Rank in order, from largest to smallest, the moments of inertia I_a to I_d for rotation about the dotted line.



A. $I_a > I_d > I_b > I_c$
B. $I_c = I_d > I_a = I_b$
C. $I_a = I_b > I_c = I_d$
D. $I_a > I_b > I_d > I_c$
E. $I_c > I_b > I_d > I_a$

Before Class 18 on Wednesday

- Please read up to and including section 12.7 of Knight Chapter 12

- Something to think about:
- In Practicals this week you will hold the string of a yo-yo fixed as you drop it. As the yo-yo falls, the string unwinds and the yo-yo rotates. Does it fall faster or slower than 9.8 m/s^2?
- The transformation of energy is U_g → kinetic; so *why* does it fall slower?