PHY131H1S - Class 21

Today:

- Energy in Simple Harmonic Motion
- Hanging Springs
- The Pendulum
- Damped Oscillations
- Driven Oscillations; Resonance

Italian opera singer Luigi Infantino tries to break a wine glass by singing top 'C' at a rehearsal.

A little pre-class reading quiz on Ch.14…

What term is used to describe an oscillator that "runs down" and eventually stops?

A. Tired oscillator
B. Out of shape oscillator
C. Damped oscillator
D. Resonant oscillator
E. Driven oscillator

Quick notes about clickers

- The in-class quiz mark counts for 2% of the total course mark
- To earn 1% for participation, you must submit some answer to at least one in-class quiz question in at least 17 classes (ie you may miss up to 6 classes with no penalty)
- To earn an additional 1% for accuracy, you must give the correct answer to at least 50% of the in-class quiz questions that you submit over the whole semester
- If a student is caught with more than one remote, both will be confiscated and both students associated with these remotes will receive an academic misconduct for impersonation

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Possible Sanctions for Academic Misconduct

- In every case a letter is sent to the Office of Student Academic Integrity (OSAI)
- A zero for the piece of work (in this case the class participation component of the course)
- A record of the sanction on the student's academic transcript for up to five years
- A reduction of the final grade in the course of 10%
- Assignment of a grade of zero for the course
- Suspension from the University for up to 12 months

Last day I asked at the end of class:

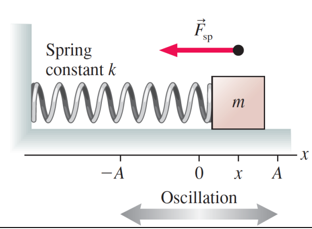
- A mass hanging from a string is swinging back and forth with a period of 2 seconds.
- What is the period if the mass is doubled?
- ANSWER:
- 2 seconds! It turns out that the mass of a pendulum is not related to its period.
- What is the period if the length of the string is doubled?
- ANSWER:
- Longer than 2 seconds (actually 2.8 s). The longer the string, the longer the period. It goes up as the square root of L.

Potential Energy in S.H.M.

- $x = 0$ is defined to be when the spring is in equilibrium

$$x = A\cos(\omega t + \phi_0)$$

- Energy stored in the spring varies with time: $U_s = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi_0)$

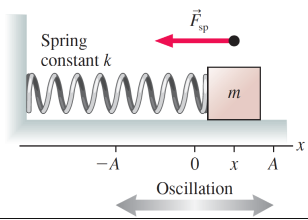


Kinetic Energy in S.H.M.

- The mass of the spring is negligible, so all the kinetic energy in the system is due to the speed of the mass:

$$v = -A\omega \sin(\omega t + \phi_0)$$

- Kinetic energy of the mass varies with time: $K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \sin^2(\omega t + \phi_0)$



Total Energy in S.H.M.

$$E = U_s + K$$

$$E = \frac{1}{2}kA^2\cos^2(\omega t + \phi_0) + \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi_0)$$

- Recall that $\omega^2 = k/m$. Rearranging:

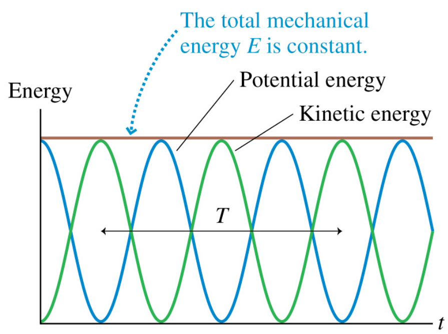
$$E = \frac{1}{2}kA^2\left[\cos^2(\omega t + \phi_0) + \sin^2(\omega t + \phi_0)\right]$$

- Recall that $\cos^2\theta + \sin^2\theta = 1$

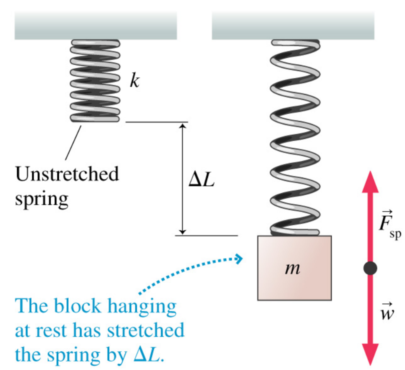
$$E = \frac{1}{2}kA^2$$

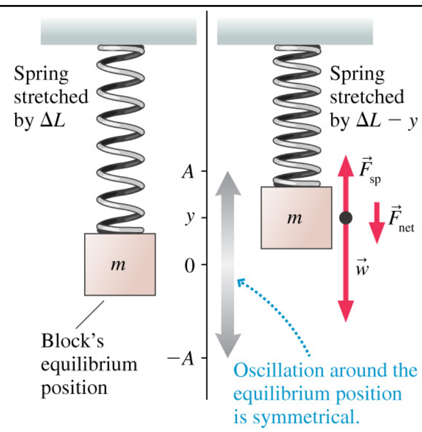
- U_s and K both vary with time, but $E = U_s + K$ is constant

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(v_{max})^2 \quad \text{(conservation of energy)}$$



The Hanging Spring: also S.H.M.!





Total energy for the system of a mass hanging on a spring is:

$$E = K + U_s + U_g$$

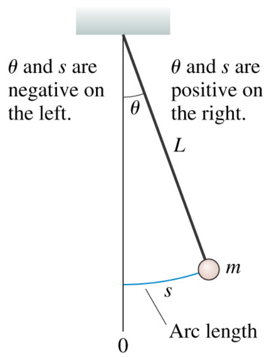
$$E = \frac{1}{2}mv^2 + \frac{1}{2}k\left(\Delta L + x\right)^2 - mgx$$

Conceptual Question:

A mass hangs motionless from a spring. When the mass is pulled down and held at rest, the ***total*** energy of the mass and spring is

A. larger than before.
B. the same as before.
C. less than before.

The Pendulum



The Pendulum

- Suppose we restrict the pendulum's oscillations to small angles (< 10°)
- Then we may use the **small angle approximation** $\sin\theta \approx \theta$, where θ is measured in radians
- In this case the net torque on the mass is:

$$\tau_{net} = mL^2\alpha = -mgL\theta$$

- The solution is Simple Harmonic Motion, with angular position: $\theta = \theta_{max}\cos(\omega t + \phi_0)$
- The oscillation frequency, in rad/s, is:

$$\omega = 2\pi f = \sqrt{\frac{g}{L}}$$

Mass on Spring versus Pendulum

	Mass on a Spring	Pendulum
Condition for S.H.M.	Small oscillations	Small angles
Angular frequency	$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{L}}$
Period	$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$

A person swings on a swing.

When the person sits still, the swing oscillates back and forth at its natural frequency.

If, instead, ***two*** people sit side-by-side on the same swing, the natural frequency of the swing is

A. greater
B. the same
C. smaller

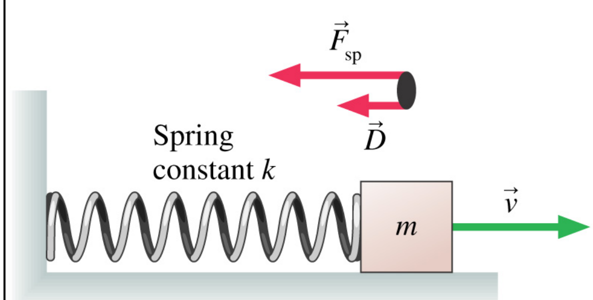
A person swings on a swing.

When the person sits still, the swing oscillates back and forth at its natural frequency.

If, instead, the person ***stands*** on the swing, the natural frequency of the swing is

A. greater
B. the same
C. smaller

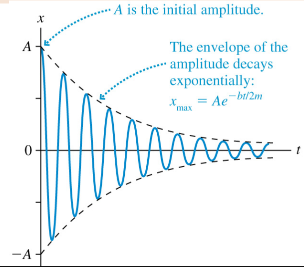
Damped Oscillations



Damped Oscillations

When a mass on a spring experiences the force of the spring as given by Hooke's Law, as well as a drag force of magnitude $|D| = bv$, the solution is

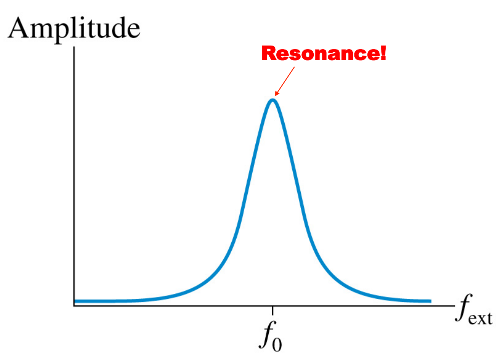
$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0) \qquad \text{(damped oscillator)}$$



Driven Oscillations and Resonance

- Consider an oscillating system that, when left to itself, oscillates at a frequency f_0. We call this the **natural frequency** of the oscillator.
- Suppose that this system is subjected to a *periodic* external force of frequency f_{ext}. This frequency is called the **driving frequency**.
- The amplitude of oscillations is generally not very high if f_{ext} differs much from f_0.
- As f_{ext} gets closer and closer to f_0, the amplitude of the oscillation rises dramatically.

14.8 Externally Driven Oscillations



Before Class 22 on Wednesday

- Something to think about: If you stand on a waterproof bathroom scale in a wading pool, so that part of your legs are immersed in the water, will your measured weight be different than normal? If so, why?