

## PHY131H1S - Class 24

Today:

- Course Review!
- The final exam, will be on Dec. 15 at 2pm.
- Note there are no Practicals this week.
- The final exam will cover Chapters 1-15, excluding Chapter 13, and Sections 6.5 and 15.6. The final exam will also cover the Error Analysis Assignment and all the material in it.
- You are allowed ONE double-sided aid-sheet for the final exam, which you must prepare yourself.

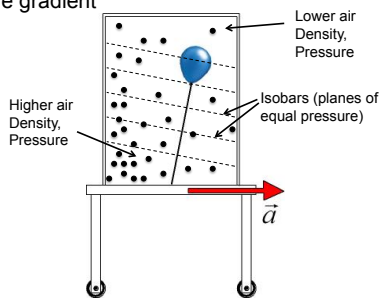
## Survey Question

- Working through the weekly MasteringPhysics homework assignments has been an effective way for me to learn course material

- A. Strongly agree
- B. agree
- C. Neutral
- D. Disagree
- E. Strongly disagree

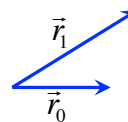
### Note about Balloon on a cart Demonstration:

As the cart is accelerating to the right, the heavier air molecules are left behind, to the left, creating a tilted pressure gradient

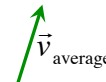


## Average Velocity

$$\vec{v}_{\text{average}} = \frac{\vec{r}_1 - \vec{r}_0}{\Delta t} = \frac{\Delta \vec{r}}{\Delta t}$$



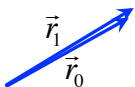
Units of  $\Delta \vec{r}$  are metres.



Units of  $\vec{v}_{\text{average}}$  are metres per second.

## Velocity (a.k.a. "instantaneous velocity")

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta \vec{r}}{\Delta t} \right) = \frac{d\vec{r}}{dt}$$



$\Delta \vec{r}$

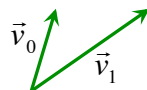
Units of  $\Delta \vec{r}$  are metres.



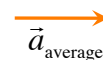
Units of  $\vec{v}$  are metres per second.

## Average Acceleration

$$\vec{a}_{\text{average}} = \frac{\vec{v}_1 - \vec{v}_0}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$




Units of  $\Delta \vec{v}$  are m/s.



Units of  $\vec{a}_{\text{average}}$  are m/s<sup>2</sup>.

### Acceleration (a.k.a. “instantaneous acceleration”)

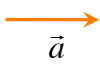
$$\vec{a} = \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta \vec{v}}{\Delta t} \right) = \frac{d\vec{v}}{dt}$$



$\vec{v}_0$   $\vec{v}_1$

$-\Delta \vec{v}$

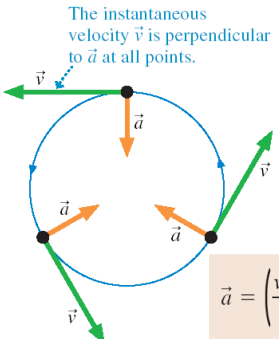
Units of  $\Delta \vec{v}$   
are m/s.



$\vec{a}$

Units of  $\vec{a}$   
are m/s<sup>2</sup>.

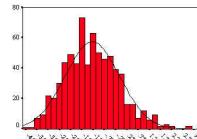
**FIGURE 4.40** For uniform circular motion, the acceleration  $\vec{a}$  always points to the center.



$$\vec{a} = \left( \frac{v^2}{r}, \text{toward center of circle} \right)$$

### ± Errors

- Why are errors so important to scientists and engineers?
- Errors **eliminate** the need to report measurements with vague terms like “approximately” or “≈”.
- Errors give a *quantitative* way of stating your confidence level in your measurement.
- Saying the answer is  $10 \pm 2$  means you are 68% sure that the actual number is between 8 and 12.
- It also implies that you are 95% confident that the actual number is between 6 and 14 (the 2-sigma range).



### Propagation of Errors

**Rule #1 (sum or difference rule):**

- If  $z = x + y$
- or  $z = x - y$
- then  $\Delta z = \sqrt{\Delta x^2 + \Delta y^2}$

**Rule #2 (product or division rule):**

- If  $z = xy$
- or  $z = x/y$
- then  $\frac{\Delta z}{z} = \sqrt{\left( \frac{\Delta x}{x} \right)^2 + \left( \frac{\Delta y}{y} \right)^2}$

### Propagation of Errors

**Rule #2.1 (multiply by exact constant rule):**

- If  $z = xy$  or  $z = x/y$
- and  $x$  is an exact number, so that  $\Delta x = 0$
- then  $\Delta z = |x|(\Delta y)$

**Rule #3 (exponent rule):**

- If  $z = x^n$
- then  $\frac{\Delta z}{z} = n \frac{\Delta x}{x}$

### The Error in the Mean

- Many individual, independent measurements are repeated  $N$  times
- Each individual measurement has the same error  $\Delta x$
- Using error propagation you can show that the error in the estimated mean is:

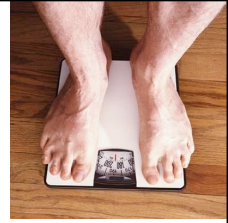
$$\Delta \bar{x}_{\text{est}} = \frac{\Delta x}{\sqrt{N}}$$

### Significant Figures

- Discussed in Section 1.9 of Knight Ch. 1
- The rules for significant figures when errors are involved are:
  - Errors should be specified to one or two significant figures.**
  - The most precise column in the number for the error should also be the most precise column in the number for the value.**
- Example: If a calculated result is (7.056 +/- 0.705) m, it is better to report (7.1 +/- 0.7) m.

### Weight ≠ Weight ???

- Physics textbooks and physics teachers do not all agree on the definition of the word “weight”!
- Sometimes “weight” means the exact same thing as “force of gravity”. That is *not* how Randall Knight uses the word. (I will follow Knight’s definitions.)
- In Knight, “weight” means the magnitude of the *upward* force being used to support an object.
- If the object is at rest or moving at a constant velocity relative to the earth, then the object is in equilibrium. The upward supporting force exactly balances the downward gravitational force, so that weight =  $mg$ .



### Knight’s Definition of weight, page 161:

The **weight** of an object is the reading of a calibrated spring scale on which the object is stationary. Weight is the result of weighing. The weight of an object with vertical acceleration  $a_y$  is

$$w = mg \left( 1 + \frac{a_y}{g} \right)$$

### “Kinetic Friction”



- Also called “sliding friction”
- When two flat surfaces are in contact and sliding relative to one another, heat is created, so it slows down the motion (kinetic energy is being converted to thermal energy).

$$f_k = \mu_k n$$

where  $n$  is the normal force.

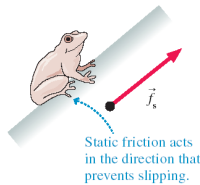


The direction of  $\vec{f}_k$  is opposite the direction of motion.

### “Static Friction”



- When two flat surfaces are in contact but are not moving relative to one another, they tend to resist slipping. They have “locked” together. This creates a force perpendicular to the normal force, called static friction.



There is no general equation for  $f_s$ .

The direction of  $f_s$  is whatever is required to prevent slipping.

### Limits to the self-adjusting forces.

- The normal force of a bridge on a truck is what holds up the truck. If the truck’s weight exceeds some maximum value, the bridge will collapse!
- The tension force of a fishing line on a fish is what pulls in the fish. If the fish is too big, the line will break!
- The static friction force is what keeps two surfaces from slipping. If the outside forces are too much, the surfaces will slip!
- In first-year physics, we do not study  $n_{\max}$  and  $T_{\max}$ . This is the Physics of Fracture.



## Maximum Static Friction

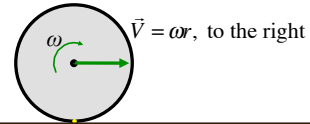
There's a limit to how big  $f_s$  can get. If you push hard enough, the object slips and starts to move. In other words, the static friction force has a *maximum* possible size  $f_{s \max}$ .

- The two surfaces don't slip against each other as long as  $f_s \leq f_{s \max}$ .
- A static friction force  $f_s > f_{s \max}$  is not physically possible. Many experiments have shown the following approximate relation usually holds:

$$f_{s \max} = \mu_s n$$

where  $n$  is the magnitude of the normal force, and the proportionality constant  $\mu_s$  is called the "coefficient of static friction".

## Rolling without slipping



The wheel rotates with angular speed  $\omega$ .

The axle moves with linear speed  $v = \omega r$ , where  $r$  is the radius of the wheel.

Since the bottom point is always at rest, it is *static friction* which acts between the ground and the wheel.

## Linear / Rotational Analogy

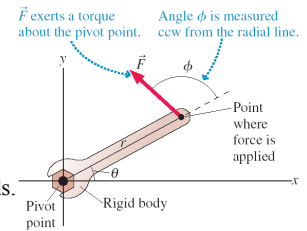
	Linear	Rotational Analogy
	▪ $\vec{s}, \vec{v}, \vec{a}$	▪ $\theta, \omega, \alpha$
	▪ Force: $\vec{F}$	▪ Torque: $\tau$
	▪ Mass: $m$	▪ Moment of Inertia: $I$
Newton's 2 <sup>nd</sup> law:	$\vec{a} = \frac{\vec{F}_{net}}{m}$	$\alpha = \frac{\tau_{net}}{I}$
Kinetic energy:	$K_{cm} = \frac{1}{2} m v^2$	$K_{rot} = \frac{1}{2} I \omega^2$
Momentum:	$\vec{p} = m \vec{v}$	$\vec{L} = I \vec{\omega}$

## Torque

Mathematically, we define torque  $\tau$  (Greek tau) as

$$\tau \equiv r F \sin \phi$$

SI units of torque are N m.  
English units are foot-pounds.



Consider a body made of  $N$  particles, each of mass  $m_i$ , where  $i = 1$  to  $N$ . Each particle is located a distance  $r_i$  from the axis of rotation. We define moment of inertia:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots = \sum_i m_i r_i^2$$

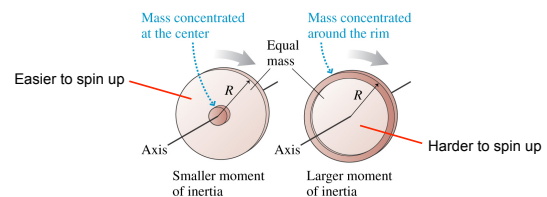
The units of moment of inertia are  $\text{kg m}^2$ . An object's moment of inertia depends on the axis of rotation.

The **moment of inertia**

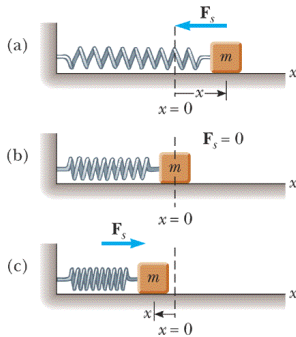
$$I = \sum_i m_i r_i^2 = \int r^2 dm$$

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis.

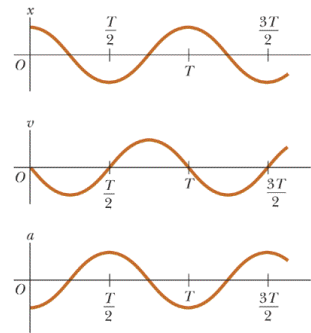
## Consequences of Moment of Inertia



Simple Harmonic Motion:  
Restoring Force provided by Hooke's Law

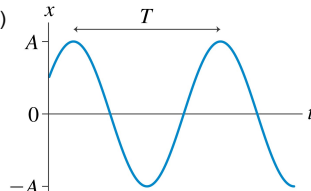


x, v, a for Simple Harmonic Motion



Simple Harmonic Motion notes...

- S.H.M. is *not* constant acceleration, or constant force – both vary with time.
- S.H.M. results when restoring force is proportional to displacement. Other types of oscillatory motion are possible, but not discussed in this course.
- Angular frequency  $\omega = 2\pi/T$ , where  $T$  = period.
- $(T = 2\pi/\omega)$
- "frequency"  $f = 1/T$  (in Hertz)



Between now and the Final Exam

- I recommend you be familiar with all Masteringphysics problem sets, the suggested End-Of-Chapter Problems, and all Practicals work.
- Please email me ( jharlow @ physics.utoronto.ca ) with any questions. Keep in touch! It's been a really fun course for me!

