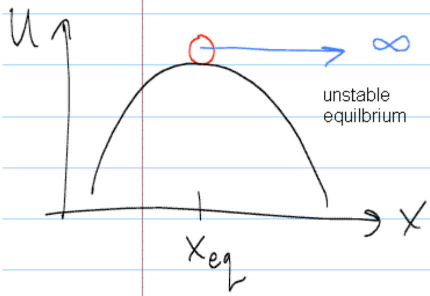
PHY131 - Class 15 - Wed. Nov. 2, 2011

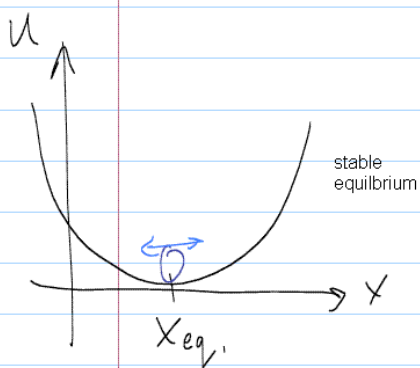


U = potential energy

x_{eq} = equilibrium position

→ small perturbation,

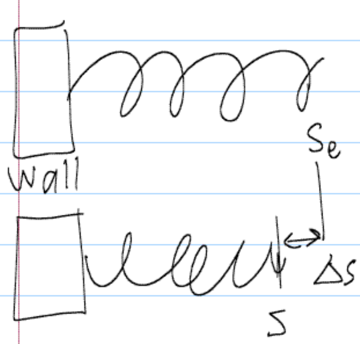
→ object accelerates away from x_{eq}



← small perturbation causes object to oscillate around x_{eq}

Robert Hooke (1660)

Hooke's Law for springs



← equilibrium position.

Force on end of spring:

$$|F_{sp}| = k\,|\Delta s|$$

k = "spring constant" [N/m]

→ Hooke's Law works only if Δs is small.

→ Not all springs obey hookes law

for stretching & compression.

Elastic Potential Energy

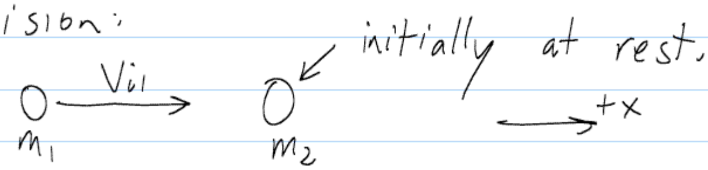
A spring that is stretched or compressed stores energy.

$$\boxed{U_s = \frac{1}{2}k(\Delta s)^2} \leftarrow \text{always positive or zero.}$$

units: [Joules]

Unit check: $\left[\frac{N}{m} \cdot m^2 = N \cdot m = J\right]$

Elastic collision:



Conservation of Momentum:

$$p_{fx} = p_{ix}$$

$$m_1 V_{f1} + m_2 V_{f2} = m_1 V_{i1} + 0 \qquad (1)$$

Conservation of Energy (Kinetic only):

$$\frac{1}{2}m_1 V_{f1}^2 + \frac{1}{2}m_2 V_{f2}^2 = \frac{1}{2}m_1 V_{i1}^2 \qquad (2)$$

2 equations, 2 unknowns: V_{f1} and V_{f2}

Use eq. (1) to solve for v_{f1}:

$$v_{f1} = v_{i1} - \frac{m_2}{m_1} v_{f2} \qquad (3)$$

Plug this into Left-side of eq. (2):

$$\frac{1}{2} m_1 \left(v_{i1} - \frac{m_2}{m_1} v_{f2} \right)^2 + \frac{1}{2} m_2 v_{f2}^2$$

$$= \frac{1}{2} m_1 \left[v_{i1}^2 - 2\frac{m_2}{m_1} v_{i1} v_{f2} + \frac{m_2^2}{m_1^2} v_{f2}^2 \right] + \frac{1}{2} m_2 v_{f2}^2$$

$$= \frac{1}{2} m_1 v_{i1}^2 - m_2 v_{i1} v_{f2} + \frac{1}{2} \frac{m_2^2}{m_1} v_{f2}^2 + \frac{1}{2} m_2 v_{f2}^2$$

from eq. (2), set L.H.S. = R.H.S.:

$$\cancel{\frac{1}{2} m_1 v_{i1}^2} - m_2 v_{i1} v_{f2} + \frac{1}{2} \frac{m_2^2}{m_1} v_{f2}^2 + \frac{1}{2} m_2 v_{f2}^2 = \cancel{\frac{1}{2} m_1 v_{i1}^2}$$

$$v_{f2} \left[-m_2 v_{i1} + \frac{1}{2} \frac{m_2^2}{m_1} v_{f2} + \frac{1}{2} m_2 v_{f2} \right] = 0$$

Two things multiplied = 0 $\Rightarrow$ One or the other = 0

$v_{f2} = 0$ is the "trivial solution" $\rightarrow$ no collision!

Assume $v_{f2} \neq 0$: $\Rightarrow$

$$\frac{1}{2} \frac{m_2^2}{m_1} v_{f2} + \frac{1}{2} m_2 v_{f2} = m_2 v_{i1}$$ × both sides by 2

$$v_{f2} \left(\frac{m_2^2}{m_1} + m_2 \right) = 2 m_2 v_{i1}$$ × both sides by $\frac{m_1}{m_2}$

$$v_{f2} (m_2 + m_1) = 2 m_2 v_{i1}$$

$$\Rightarrow \boxed{v_{f2} = \frac{2 m_1 v_{i1}}{m_1 + m_2}} \qquad (4)$$

Plug this back into eq. (3):

$$v_{f1} = v_{i1} - \frac{m_2}{m_1} \left(\frac{2 m_1 v_{i1}}{m_1 + m_2} \right)$$

$$= \frac{v_{i1}(m_1+m_2)}{m_1+m_2} - \frac{2m_2 v_{i1}}{m_1+m_2}$$

$$= \frac{m_1 v_{i1} + m_2 v_{i1} - 2m_2 v_{i1}}{m_1+m_2}$$

$$\boxed{v_{f1} = \left(\frac{m_1-m_2}{m_1+m_2}\right) v_{i1}} \qquad (5)$$

(4) & (5) are Eq. 10.43 in your text.

Very useful!!

- Demonstration and Example
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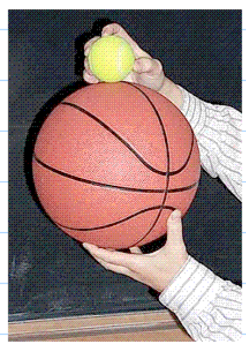
- A 0.50 kg basketball and a 0.05 kg tennis ball are stacked on top of each other, and then dropped from a height of 0.82 m above the floor.
- How high does the tennis ball bounce?
- Assume all perfectly elastic collisions.



Example from Demo. Divide motion into Segments:

Segment 1: Freefall of both balls

→ released from rest $h = 0.82$ m

above ground.

$$E_f = E_i$$

just before basketball hits ground: $h_f = 0$

$$\cancel{m_{tot}} g h_i = \frac{1}{2} \cancel{m_{tot}} v_f^2$$

$$v_f = \sqrt{2gh} = \sqrt{2(9.8)0.82}$$

$$v_f = 4.0 \text{ m/s}$$

Segment 2: Basketball collides elastically with floor, reverses its velocity.

→ New velocity is 4.0 m/s, up.

Segment 3: Tennis ball going down with 4.0 m/s collides with basketball going up at 4.0 m/s.

Basketball: $m_1 = 0.50$ kg

Tennis ball: $m_2 = 0.050$ kg

→ Switch to reference frame in which m_2 is at rest:

S' frame: $v_{2i}' = -4 - (-4) = 0$

$v_{1i}' = +4 - (-4) = +8$ m/s

Use Eqs 10.43:

$$v_{f2}' = \frac{2m_1}{m_1+m_2} v_{1i}' = \frac{2(0.5)(+8)}{0.5+0.05}$$

$$v_{f2}' = +14.55 \text{ m/s}$$

Switch back to lab frame:

$$v_{f2} = v_{f2}' - 4 = 14.55 - 4$$

$$\boxed{v_{f2} = +10.5 \text{ m/s.}}$$

Segment 4: Freefall

tennis ball is going up at 10.5 m/s, reaches a maximum height and stops.

$$E_f = E_i$$

$$mgh = \frac{1}{2}m(10.5)^2$$

$$h_f = \frac{(10.5)^2}{2g} = \boxed{5.6\text{m}}$$