PHY131 - Class 17 - Mon. Nov. 14, 2011

$\tau = 60.0\ N\cdot m$ ← assume this is net torque (ie no friction)

$I = 13.3\ kg\cdot m^2$

Need time to reach $\omega_f = 200\ \frac{rev}{min}$

Use Newton's 2nd Law for rotation:

$$\alpha = \frac{\tau_{net}}{I} = \frac{60}{13.3} = 4.5\ \frac{rad}{s^2}$$

Units check: $\frac{60\ N\cdot m}{13.3\ kg\, m^2} = \left[\frac{\cancel{kg}\cdot \cancel{m}\, s^{-2}\cdot \cancel{m}}{\cancel{kg}\cdot \cancel{m^2}}\right] = [s^{-2}]$ okay.

(radians are magic unit which can come or go)

Use eq. of kinematics from Ch. 4.

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} \qquad \omega_i = 0$$

Solve for Δt: $\Delta t = \frac{\omega_f - \omega_i}{\alpha}$ Need ω_f in SI units.

$$\omega_f = 200\ \frac{\cancel{rev}}{\cancel{min}}\left[\frac{2\pi\ rad}{\cancel{rev}}\right]\left[\frac{1\ \cancel{min}}{60\ sec}\right] = 20.94\ \frac{rad}{s}\ ✓$$

$$\Delta t = \frac{20.94 - 0}{4.5} = \boxed{4.64\ s}$$

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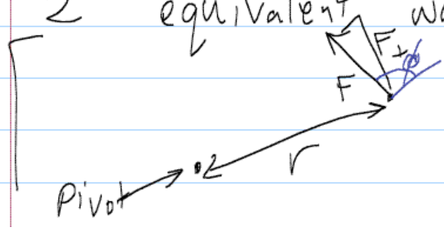
2 equivalent ways of computing torque.

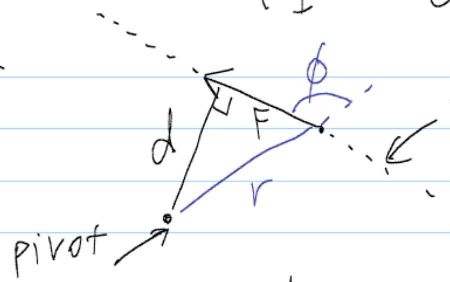
(1)



"$\perp$" = "perpendicular"

$$\tau = r F_\perp = r(F \sin\phi)$$

$F_\perp$ = component of F $\perp$ to $\vec{r}$



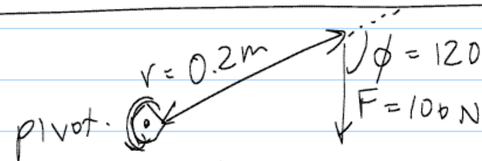
(2)

"line of action" — parallel to force and passes through point of force application

d = moment arm = distance from pivot to line of action

$$\tau = F d = F (r \sin\phi)$$

Example.



$$|\tau| = |r F \sin\phi|$$
$$= (0.2)(100)(\sin 120^\circ)$$
$$|\tau| = 17.3 \text{ N}\cdot\text{m}$$

NOTE: Sign Convention: τ is + for counterclockwise torque

τ is − for clockwise

$$\boxed{\tau = -17.3 \text{ N}\cdot\text{m}}$$

Moment of Inertia I.

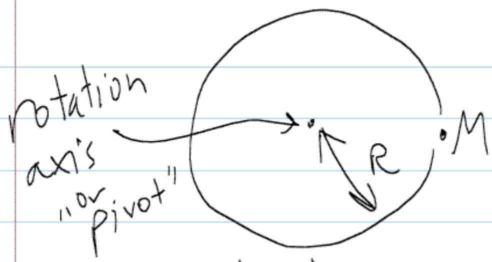
→ I depends on MASS and on distribution of this mass around a rotion axis.

I has SI units : $kg \cdot m^2$

→ The moment of inertia of a point particle that is rotating about its centre is zero.

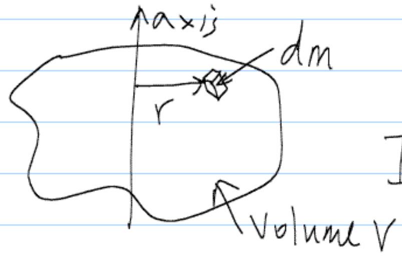
→ Also, $I = 0$ for an infinitely thin

line of mass, rotating about itself.



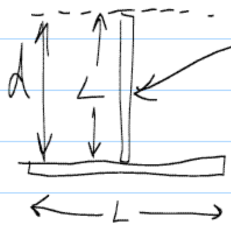
I for a point particle mass M constrained to move on a circular path of radius R.

$$I = MR^2$$



$$I = \int_V r^2\, dm$$

(a)



$I_b = \frac{1}{3} ML^2$

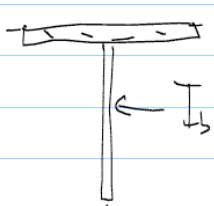
$I_t = 0 + M(d^2)$ ← parallel axis theorem.

thin rod rotated about its length has $I = 0$

$I_t = ML^2$

$$I_{tot} = \frac{1}{3} ML^2 + ML^2 = \frac{4}{3} ML^2$$

(b)

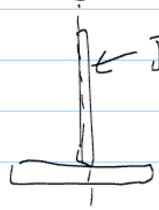


$I_t = 0$

$I_b = \frac{1}{3} ML^2$

$$I_{tot} = \frac{1}{3} ML^2$$

(c)

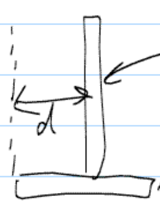


$I_b = 0$

$I_t = \frac{1}{12} ML^2$

$$I_{tot} = \frac{1}{12} ML^2$$

(d)



$I_b = 0 + Md^2$ ← parallel axis theorem.

$d = \frac{L}{2}$ $\quad I_b = M\left(\frac{L}{2}\right)^2 = \frac{1}{4} ML^2$

$I_t = \frac{1}{3} ML^2$

$$I = \left(\frac{1}{3} + \frac{1}{4}\right) ML^2 = \frac{7}{12} ML^2$$