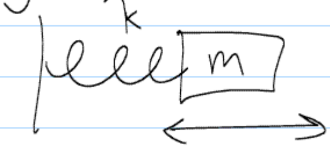
Energy for mass + spring system.



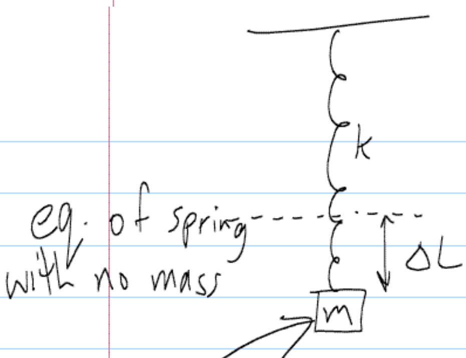
$$E = U_s + K = \frac{1}{2}kA^2\cos^2(\omega t+\phi_0) + \frac{1}{2}mA^2\omega^2\sin^2(\omega t+\phi_0)$$

$\omega^2 = k/m$

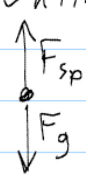
$$E = \frac{1}{2}A^2\left[k\cos^2(\omega t+\phi_0) + \cancel{m}\frac{k}{\cancel{m}}\sin^2(\omega t+\phi_0)\right]$$

$$= \frac{1}{2}kA^2\underbrace{\left[\cos^2(\omega t+\phi_0) + \sin^2(\omega t+\phi_0)\right]}_{=1}$$

$$E = \frac{1}{2}kA^2 \quad \leftarrow \text{constant}$$



f.b.d. of mass when it is not moving or accelerating:



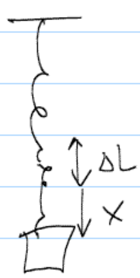
$\vec{F}_s$ is up.

define $x=0$ to be position of mass in equilibrium.

Set $+x$ to be down

$$|F_{sp}| = k(\Delta L) = mg$$

$$\delta L = \frac{mg}{k}$$



$$F_s = -k(\Delta L + x)$$

$$F_{net} = F_s + F_g$$
$$= -k(\Delta L + x) + mg$$

$$F_{Net} = -\cancel{m}g - kx + \cancel{m}g$$

$$F_{Net} = -kx = ma_x$$

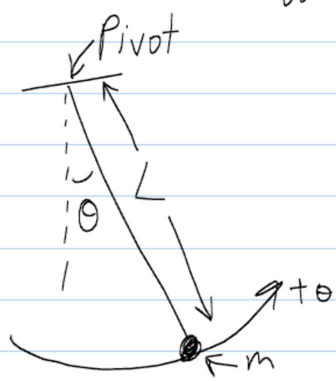
$$\boxed{a_x = -\frac{k}{m}x}$$

* S.H.M.

$$x = A\cos(\omega t + \phi_0)$$

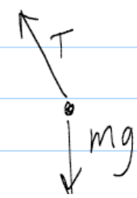
$$\omega = \sqrt{k/m}$$

Simple Pendulum



Define $\theta = 0°$ when mass is hanging below pivot

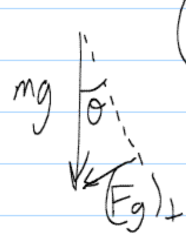
f.b.d. of mass



Torque around pivot: $\tau_{Net} = \tau_{Tension} + \tau_g$

Tension pulls toward pivot

$\Rightarrow$ moment arm = 0, $\tau_{tension} = 0$



$$(F_g)_\perp = mg\sin\theta$$

$$|\tau_g| = L(F_g)_\perp$$

$$= Lmg\sin\theta$$

$\rightarrow$ negative because it goes clockwise.

$$\tau_{Net} = I\alpha = -Lmg\sin\theta$$

I for point mass is $I = mL^2$

$$\tau_{Net} = -Lmg\sin\theta = mL^2\alpha$$

$\alpha = -\frac{g}{L} \sin\theta$ ← Not S.H.M.

But if θ is small, and measured in radians: $\boxed{\theta \approx \sin\theta}$ ← small angle approximation.

$$\alpha = \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\,\theta} \quad * \text{SHM}$$

$$\theta = A\cos(\omega t + \phi_0)$$

$$\omega = \sqrt{\frac{g}{L}}$$

In real life, dissipative friction forces create thermal energy, and oscillations slow down.

A reasonable model of air resistane is:

drag force $= \vec{D} = -b\vec{v}$

b = "damping constant" $\left[\frac{kg}{s}\right]$

$\vec{v}$ = velocity

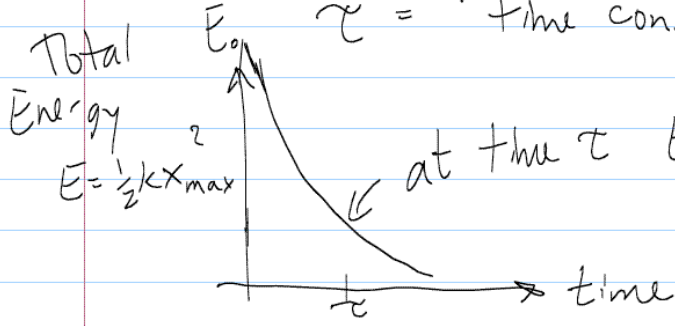
Solution to mass on spring with damping:

$$x = \underbrace{A e^{-bt/2m}}_{\text{exponentially decaying amplitude}} \cos(\omega t + \phi_0).$$

We define the "envelope" function (dashed lines).

$$x_{max} = A e^{-bt/2m} = A e^{-t/\tau}$$

τ = "time constant", in seconds.



at time τ $E = \frac{1}{e}$ of its initial value.