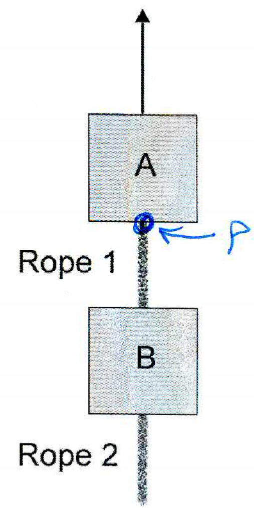
PHY131H1F **Term Test 2 —version 1**

Tuesday, November 22, 2011 Duration: 80 minutes

Solutions by Jason Harlow

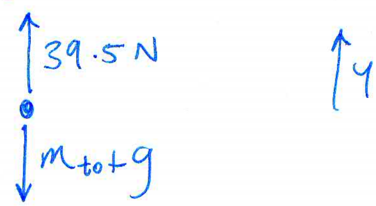
Question 1 from version 1

The figure shows two 1.00 kg blocks connected by a rope. A second rope hangs beneath the lower block. Both ropes have a mass of 250 g. The entire assembly is accelerated upwards (near the surface of the Earth) by a force of magnitude 39.5 N. The tension, in N, at the top end of Rope 1 is closest to:



A. 9.00
B. 13.9
C. 23.7
D. 33.5
E. 39.5

Free-body diagram of entire assembly:



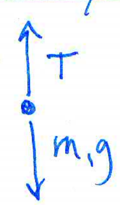
$m_{tot} = (1.00 + 1.00 + 0.250 + 0.250)$ kg

$= 2.50$ kg

$(F_{Net})_y = m_{tot}a_y = 39.5 - (2.50 \times 9.80) = +15$ N

$a_y = \dfrac{15 \text{ N}}{2.5 \text{ kg}} = +6 \text{ m/s}^2$

Free-body diagram of everything below point P:



T = tension at the top of rope 1

m_1 = mass below point P

$m_1 = 0.250 + 1.00 + 0.250$

$m_1 = 1.50$ kg

$(F_{Net})_y = m_1 a_y = T - m_1 g$

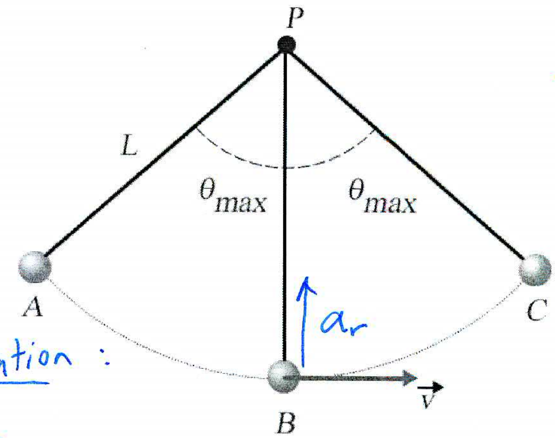
$T = m_1 a_y + m_1 g = m_1(g + a_y)$

$T = 1.50(6 + 9.8)$

$T = 23.7$ N

Question 2 from version 1

You are supported by a string, swinging from A to B to C, and then back towards B. The length of the string from the support point P to you is L. Your mass is m and the acceleration due to gravity is g. At point B your speed is v. What is the tension in the string at point B?



A. mg

B. $mg + mv^2 / L$

C. $mg - mv^2 / L$

D. $\sqrt{(mg)^2 + (mv^2 / L)^2}$

E. 0

Centripetal acceleration:

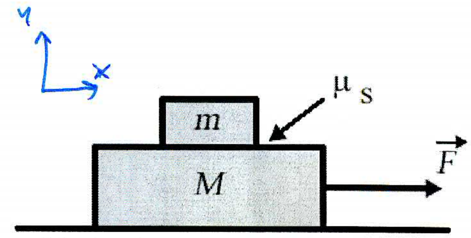
$$a_r = \frac{v^2}{r}, \text{ up.}$$

$$(F_{Net})_r = m a_r = \frac{mv^2}{r} = T - mg$$

$$T = \frac{mv^2}{r} + mg$$, where $r = L$, length of string.

So $T = mg + mv^2/L$

Question 3 from version 1

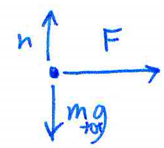


frictionless

A block of mass M sits on a frictionless surface. Sitting on top of M is a block of mass m. A force $\vec{F}$ is pulling M to the right. The coefficient of static friction between blocks M and m is μ_s. If block m is not to slip, the maximum magnitude of the force, F, is

A. $m\mu_s g$
B. $M\mu_s g$
C. $(m + M)\mu_s g$
D. $\left(\frac{M}{m}\right)(m+M)\mu_s g$
E. 0, it will always slip

Free-body diagram of system of both blocks:

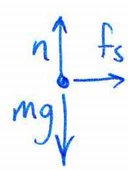


$(F_{Net})_y = 0 = n - mg_{tot}$

$\Rightarrow n = mg_{tot}$.

$(F_{Net})_x = m_{tot} a_x = F \Rightarrow \boxed{a_x = \frac{F}{m_{tot}} = \frac{F}{m+M}}$

Free-body diagram of top-block only.



$(F_{Net})_y = 0$

$\Rightarrow n = mg$

$(F_{Net})_x = ma_x = f_s$

At maximum acceleration, $f_s = f_{s,max} = \mu_s n = \mu_s mg$

$\Rightarrow \mu_s mg = ma_x$

$\mu_s g = a_x = \frac{F}{m+M}$

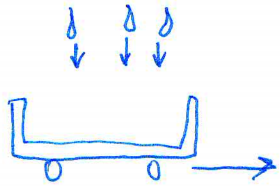
$F = \mu_s g(m + M)$ ← same as C

Question 4 from version 1

A big wagon with a mass of 250 kg is initially rolling without friction at 3.00 m/s. It starts to rain, and the rain accumulates in the wagon at a constant rate of 10.0 kg/hr. The rain is falling straight down. The speed of the wagon, in m/s, after 2.00 hours is closest to:

A. 1.50
B. 2.78
C. 3.00
D. 3.24
E. 37.5

initial:



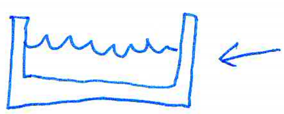
$$(p_x)_i = m_{wag} \cdot v_i = (250\ \text{kg})\ 3.00\ \text{m/s}$$

$$(p_x)_i = 750\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Frictionless $\Rightarrow$ no x-force on wagon.

$$\Rightarrow (p_x)_f = (p_x)_i$$

final:



$$m_f = m_{wag} + (\text{rate} \times \text{time})$$

$$= \cancel{m}\ 250 + 10\ \frac{\text{kg}}{\cancel{\text{hr}}} \cdot 2.0\ \cancel{\text{hr}}$$

$$= 250 + 20$$

$$m_f = 270\ \text{kg}$$

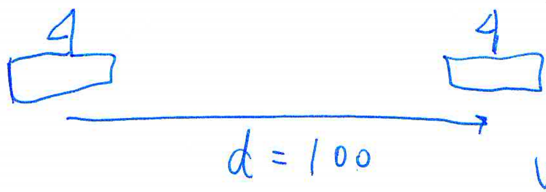
$$m_{wag} \cdot v_i = m_f\, v_f$$

$$v_f = \frac{(p_x)_i}{m_f} = \frac{750}{270} = 2.78\ \text{m/s}$$

Question 5 from version 1

Two sailboats have identical size and shape, but sailboat A has a significantly larger mass than sailboat B. They are both subject to the same wind and the same constant net force as they start from rest and move the same distance of 100.0 m. After traveling 100.0 m,

A. they both have the same momentum and kinetic energy.
B. they both have the same kinetic energy but sailboat A has larger momentum than sailboat B.
C. they both have the same kinetic energy but sailboat B has larger momentum than sailboat A.
D. they both have the same momentum but sailboat A has larger kinetic energy than sailboat B.
E. they both have the same momentum but sailboat B has larger kinetic energy than sailboat A.



Work = ΔK = $F \cdot d$

same for both.

Impulse = Δp = $F \cdot \Delta t$.

$\Delta t_A > \Delta t_B$, since boat A is more massive, and so accelerates slower under the same force.

$\Rightarrow \Delta p_A > \Delta p_B$

Question 6 from version 1

A thin uniform metal hoop of mass m and radius R rolls without slipping along a flat horizontal surface. The coefficient of static friction between the hoop and the surface is μ_s. As the hoop rolls, what is the force of static friction on the hoop?

A. $mg\mu_s$

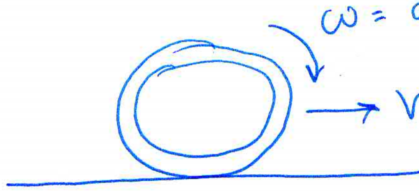
B. mgR^2

C. $\frac{mg\mu_s}{2}$

D. $\frac{mg}{2}$

E. 0

ω = constant $\Rightarrow \alpha = 0$



v = constant $\Rightarrow a_x = 0$

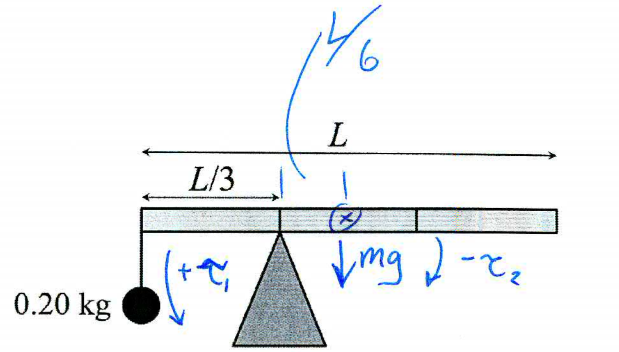
$(F_{Net})_x = 0$, $\tau_{Net} = 0$

No static friction $f_s = 0$.

Question 7 from version 1

A uniform metre stick of length $L = 1.0$ m has a 0.20 kg mass suspended by a string from its left side, and rests on a pivot that is $L/3$ from the left side. If the system consisting of the metre stick and the hanging mass is in equilibrium, the mass of the metre stick is closest to:

A. 0.10 kg
B. 0.20 kg
C. 0.29 kg
D. 0.33 kg
E. 0.40 kg



τ_1 = torque due to hanging mass

τ_2 = torque due to metre stick.

$$\tau_{net} = 0 = \tau_1 - \tau_2$$

$$0 = \frac{(0.2)9.8\,L}{3} - \frac{mg\,L}{6}$$

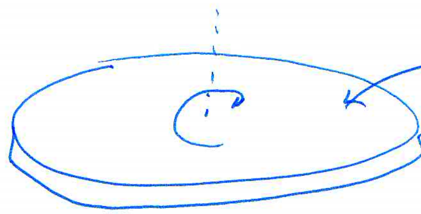
$$\frac{0.2}{3} = \frac{m}{6}$$

$$m = \frac{6}{3} \cdot 0.2 = 0.40 \text{ kg.}$$

Question 8 from version 1

A vinyl record can be modeled as a uniform disk with a mass of 0.200 kg and a radius of 15.2 cm. When this record is spinning about its centre at 3.49 rad/s, the magnitude of its angular momentum in kg m^2 s^{-1} is closest to:

A. 8.06×10^{-3}
B. 0.0161
C. 0.0281
D. 0.0530
E. 0.106



$I_{disk} = \frac{1}{2} M R^2$

$$L = I\omega$$
$$= \frac{1}{2}(0.200)(0.152)^2 \cdot 3.49$$
$$= 0.00806 \ \frac{kg\,m^2}{s}$$
$$= 8.06 \times 10^{-3} \leftarrow A$$

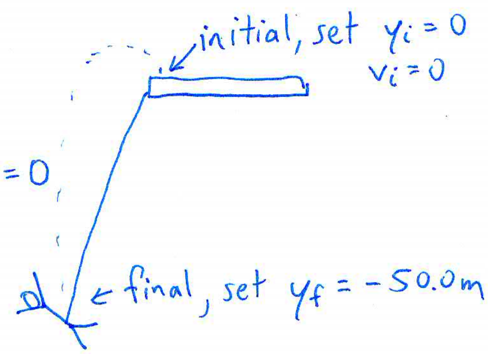
LONG ANSWER (44 marks total)
There are three parts to the Long-Answer Problem. Clearly show your reasoning as some part marks may be awarded. Write your final answers in the provided boxes.

The following situation applies to parts A, B and C:

An 80.0 kg bungee jumper drops from rest off a bridge that is 200.0 m above a river. As he falls, air resistance is negligible, and the unstretched bungee cord attached to him uncoils. The mass of the bungee cord is negligible. When the jumper is 50.0 m below the bridge, the bungee cord is completely uncoiled and forms a straight line. Below that point, the bungee cord acts as a spring, obeying Hooke's law as it is stretched beyond its equilibrium length of 50.0 m, with a spring constant of $k = 15$ N/m.

PART A (10 marks)
What is the speed of the jumper at the moment when he is 50.0 m below the bridge? Please express your answer in m/s to 3 significant figures, and write it clearly in the box below.

Bungee is unstretched $\Rightarrow U_{si} = U_{sf} = 0$

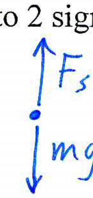


$$E_f = E_i$$
$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$
$$\frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f = 0 + 0$$
$$\frac{v_f^2}{2} = -gy_f$$
$$v_f = \pm\sqrt{-2gy_f}$$
$$|v_f| = \sqrt{-2(9.8)(-50)} = 31.305$$

$$\boxed{31.3 \text{ m/s}}$$

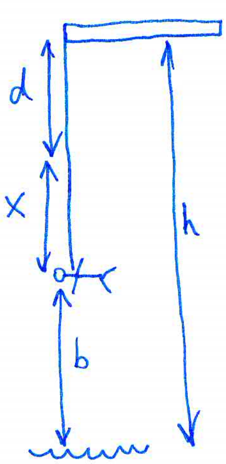
PART B (10 marks)
As the bungee cord stretches, there is a moment when the net force on the jumper is zero. How far above the river will the jumper be at this time? Please express your final answer in m to 2 significant figures, and write it clearly in the box below.

f.b.d. for jumper:



$d = 50$ m
$h = 200$ m

Bungee is stretched by x beyond equilibrium



$$F_{net} = 0$$
$$F_s - mg = 0$$
$$F_s = mg = kx$$
$$x = \frac{mg}{k}$$

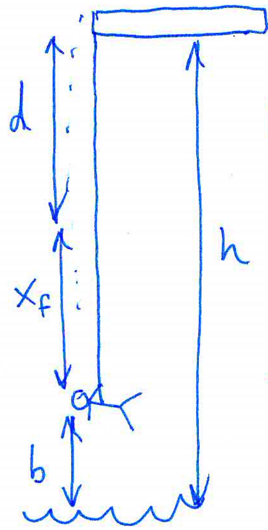
$$b = h - d - x$$

$$b = h - d - \frac{mg}{k}$$
$$= 200 - 50 - \left(\frac{80 \cdot 9.8}{15}\right)$$
$$b = 97.73$$

$$\boxed{98 \text{ m}}$$

PART C (24 marks)
How far above the river will the jumper be at the lowest point in his motion, just before he springs back upwards again? Please express your answer in m to 2 significant figures, and write it clearly in the box below.

$d = 50\,m$



initial: $y_i = 0$
$v_i = 0$
$x_i = 0$
bungee is unstretched

final: $v_f = 0$
$\boxed{y_f = -d - x_f}$

$$E_f = E_i$$

$$\tfrac{1}{2}mv_f^2 + mgy_f + \tfrac{1}{2}kx_f^2 = \tfrac{1}{2}mv_i^2 + mgy_i + \tfrac{1}{2}kx_i^2$$

$$0 + mgy_f + \tfrac{1}{2}kx_f^2 = 0 + 0 + 0$$

$$\tfrac{1}{2}kx_f^2 + mg(-d - x_f) = 0$$

$$\tfrac{1}{2}kx_f^2 - mgx_f - mgd = 0$$

Quadratic equation in the form
$ax^2 + bx + c = 0$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{mg \pm \sqrt{m^2g^2 + 4\frac{k}{2}mgd}}{k}$$

$$x_f = \frac{mg}{k} \pm \sqrt{\left(\frac{mg}{k}\right)^2 + 2\left(\frac{mg}{k}\right)d}$$

where $\frac{mg}{k} = \frac{80(9.8)}{15}$
$= 52.267$
and $d = 50$

$$x_f = 52.267 \pm \sqrt{52.267^2 + 2(52.267)(50)}$$

$= 52.267 \pm 89.210$ ← choose positive solution.

$x_f = 141.48\ m$

$b = h - d - x_f = 200 - 50 - 141.48$

$b = 8.523$

$\boxed{8.5\ m}$