- Position, Velocity, Acceleration
- Significant Figures, Measurements, Errors
- Vectors, Relative Motion
- Studying Tips
- Equilibrium and Non-equilibrium Problems
- Circular Motion, Centripetal Force

1

The Particle Model

- Often motion of the object *as a whole* is not influenced by details of the object's size and shape
- We only need to keep track of a single point on the object
- So we can treat the object *as if* all its mass were concentrated into a single point
- A mass at a single point in space is called a **particle**
- Particles have no size, no shape and no top, bottom, front or back
- Below us a motion diagram of a car stopping, using the **particle model**

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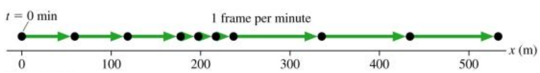
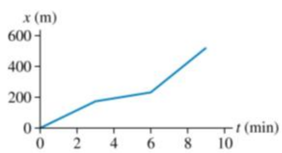
Position-versus-Time Graphs

- Below is a motion diagram, made at 1 frame per minute, of a student walking to school.



- A motion diagram is one way to represent the student's motion.
- Another way is to make a graph of x versus t for the student:



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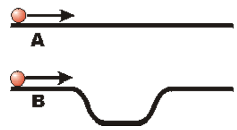
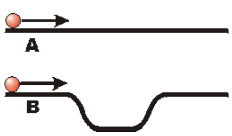


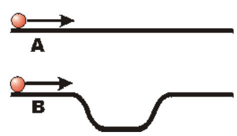
Two balls are launched along a pair of tracks with equal velocities, as shown. Both balls reach the end of the track. *Predict*: Which ball will reach the end of the track first?

- A
- B
- C: They will reach the end of the track at the same time



Demo: Two balls were launched along a pair of tracks with equal velocities. Both balls reached the end of the track. *Observe*: Which ball reached the end of the track first?

- A
- B
- C: They reached the end of the track at the same time



Explanation: *Why* does ball B reach the end of the track first?

A. Ball B is always traveling faster than ball A, so it reaches the end of the track first.
B. Balls A and B start and end with the same speed. But while ball B is on the lower part, it is going faster than ball A because gravity has sped it up. Its *average* speed is greater, so it gets there first.
C. Ball B travels a shorter distance than ball A.
D. Ball B travels a longer distance, but is pulled faster by an extra force we cannot know about.
E. The observation is flawed – ball B should not reach the end first.

Acceleration

- Sometimes an object's velocity is constant as it moves
- More often, an object's velocity changes as it moves
- Acceleration describes a *change* in velocity

- Consider an object whose velocity changes from $\vec{v}_1$ to $\vec{v}_2$ during the time interval Δt
- The quantity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ is the change in velocity
- The *rate of change of velocity* is called the **average acceleration:**

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

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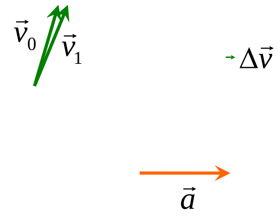
Acceleration (a.k.a. "instantaneous acceleration")

$$\vec{a} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{v}}{\Delta t}\right) = \frac{d\vec{v}}{dt}$$



Units of $\Delta\vec{v}$ are m/s.

Units of $\vec{a}$ are m/s^2.

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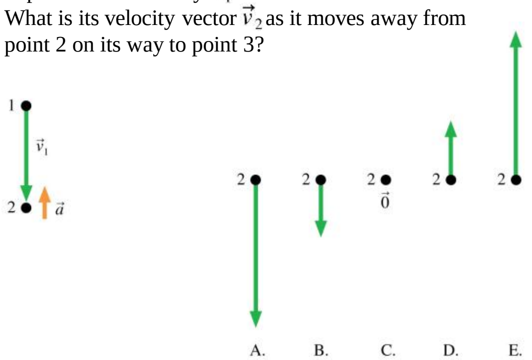
A particle has velocity $\vec{v}_1$ as it accelerates from 1 to 2. What is its velocity vector $\vec{v}_2$ as it moves away from point 2 on its way to point 3?



Unit Conversions

1 in = 2.54 cm
1 mi = 1.609 km
1 mph = 0.447 m/s
1 m = 39.37 in
1 km = 0.621 mi
1 m/s = 2.24 mph

- It is important to be able to convert back and forth between SI units and other units
- One effective method is to write the conversion factor as a ratio equal to one

- Because multiplying by 1 does not change a value, these ratios are easily used for unit conversions
- For example, to convert the length 2.00 feet to meters, use the ratio:

$$\frac{2.54\ \text{cm}}{1\ \text{in}} = 1$$

- So that:

$$2.00\ \cancel{\text{ft}} \times \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} \times \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}} \times \frac{10^{-2}\ \text{m}}{1\ \cancel{\text{cm}}} = 0.610\ \text{m}$$

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Suggested Problem Solving Strategy

- **MODEL** Think about and simplify the situation, guess at what the right answer might be.
- **VISUALIZE** Draw a diagram. It doesn't have to be artistic: stick figures and blobs are okay!
- **SOLVE** Set up the equations, solve for what you want to find. (This takes time..)
- **ASSESS** Check your units, significant figures, do a "sanity check": does my answer make sense?

This is just a suggested strategy. Whatever method works for ***you*** is fine, as long as you don't make a mistake, and you show how you got to the correct answer, it's 100%!

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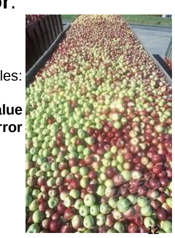
Error Analysis

- Almost every time you make a measurement, the result will not be an exact number, but it will be a *range* of possible values.
- The range of values associated with a measurement is described by the uncertainty, or **error**.

Exactly 3 apples (no error)

1600 ± 100 apples:

1600 is the **value**
100 is the **error**



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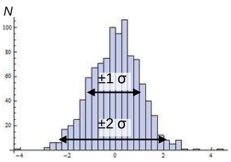
Errors

- Errors **eliminate** the need to report measurements with vague terms like “approximately” or “≈”.
- Errors give a *quantitative* way of stating your confidence level in your measurement.
- Saying the answer is 10 ± 2 means you are 68% confident that the actual number is between 8 and 12.
- It also implies that and 14 (the 2-σ range).

A histogram of many, many measurements of the same thing:



Estimating the Mean from a Sample

- Suppose you make N measurements of a quantity x, and you expect these measurements to be normally distributed
- Each measurement, or trial, you label with a number i, where i = 1, 2, 3, etc
- You do not know what the true mean of the distribution is, and you cannot know this
- However, you can estimate the mean by adding up all the individual measurements and dividing by N:

$$\bar{x}_{\text{est}} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Estimating the Standard Deviation from a Sample

- Suppose you make N measurements of a quantity x, and you expect these measurements to be normally distributed
- It is impossible to know the true standard deviation of the distribution
- The best estimate of the standard deviation is:

$$\sigma_{\text{est}} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x}_{\text{est}})^2}$$

- The quantity $N - 1$ is called the **number of degrees of freedom**
- In this case, it is the number of measurements minus one because you used one number from a previous calculation (mean) in order to find the standard deviation.

Reading Error (Digital)

- For a measurement with an instrument with a digital readout, the reading error is usually “± one-half of the last digit.”
- This means one-half of the power of ten represented in the last digit.
- With the digital thermometer shown, the last digit represents values of a tenth of a degree, so the reading error is ½ × 0.1 = 0.05°C

- You should write the temperature as 12.80 ± 0.05 °C.

Propagation of Errors

- Rule #1 (sum or difference rule):
- If $z = x + y$
- or $z = x - y$
- then $\Delta z = \sqrt{\Delta x^2 + \Delta y^2}$

- Rule #2 (product or division rule):
- If $z = xy$
- or $z = x/y$
- then $\dfrac{\Delta z}{z} = \sqrt{\left(\dfrac{\Delta x}{x}\right)^2 + \left(\dfrac{\Delta y}{y}\right)^2}$

Propagation of Errors

- Rule #2.1 (multiply by exact constant rule):
- If $z = xy$ or $z = x/y$
- and x is an exact number, so that Δx=0
- then $\Delta z = |x|(\Delta y)$

- Rule #3 (exponent rule):
- If $z = x^n$
- then $\dfrac{\Delta z}{z} = n\dfrac{\Delta x}{x}$

The Error in the Mean

- Many individual, independent measurements are repeated N times
- Each individual measurement has the same error Δx
- Using error propagation you can show that the error in the estimated mean is:

$$\Delta\bar{x}_{\text{est}} = \frac{\Delta x}{\sqrt{N}}$$

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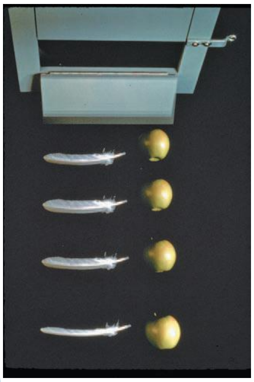
Free Fall

- The motion of an object moving under the influence of gravity only, and no other forces, is called **free fall**
- Two objects dropped from the same height will, if air resistance can be neglected, hit the ground at the same time and with the same speed
- Consequently, any two objects in free fall, regardless of their mass, have the same acceleration:

$$\vec{a}_{\text{free fall}} = (9.80\ \text{m/s}^2,\ \text{vertically downward})$$



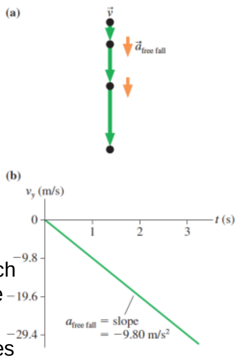
The apple and feather seen here are falling in a vacuum.

Free Fall

- Figure (a) shows the motion diagram of an object that was released from rest and falls freely
- Figure (b) shows the object's velocity graph
- The velocity graph is a straight line with a slope:

$$a_y = a_{\text{free fall}} = -g$$

- where g is a positive number which is equal to 9.80 m/s^2 on the surface of the earth
- Other planets have different values of g



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Two-Dimensional Kinematics

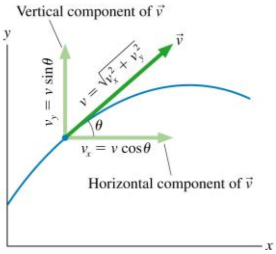
- If the velocity vector's angle θ is measured from the positive x-direction, the velocity components are

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

where the particle's *speed* is

$$v = \sqrt{v_x^2 + v_y^2}$$



- Conversely, if we know the velocity components, we can determine the direction of motion:

$$\tan\theta = \frac{v_y}{v_x}$$

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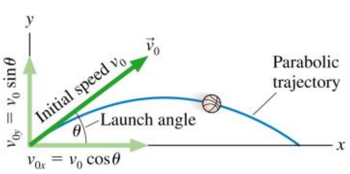
Projectile Motion

- The start of a projectile's motion is called the *launch*
- The angle θ of the initial velocity v_0 above the x-axis is called the **launch angle**



- The initial velocity vector can be broken into components

$$v_{0x} = v_0\cos\theta$$

$$v_{0y} = v_0\sin\theta$$

where v_0 is the initial speed

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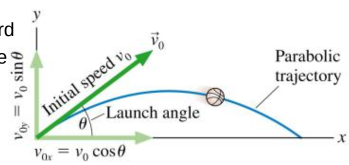
Projectile Motion

- Gravity acts downward
- Therefore, a projectile has no horizontal acceleration
- Thus



$$a_x = 0 \qquad \text{(projectile motion)}$$
$$a_y = -g$$

- The vertical component of acceleration a_y is $-g$ of free fall
- The horizontal component of a_x is zero
- Projectiles are in free fall

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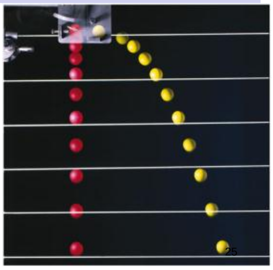
Reasoning About Projectile Motion

A heavy ball is launched exactly horizontally at height *h* above a horizontal field. At the exact instant that the ball is launched, a second ball is simply dropped from height *h*. Which ball hits the ground first?

- If air resistance is neglected, the balls hit the ground *simultaneously*
- The initial horizontal velocity of the first ball has *no* influence over its vertical motion
- Neither ball has any initial vertical motion, so both fall distance *h* in the same amount of time



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Range of a Projectile

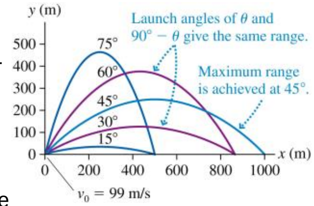
A projectile with initial speed v_0 has a launch angle of θ above the horizontal. How far does it travel over level ground before it returns to the same elevation from which it was launched?

- This distance is sometimes called the *range* of a projectile
- Example 4.5 from your textbook shows:

$$\text{distance} = \frac{v_0^2 \sin(2\theta)}{g}$$

- The maximum distance occurs for $\theta = 45°$

Trajectories of a projectile launched at different angles with a speed of 99 m/s.



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Relative Velocity

- Relative velocities are found as the time derivative of the relative positions.
- $\vec{v}_{CA}$ is the velocity of C relative to A.
- $\vec{v}_{CB}$ is the velocity of C relative to B.
- $\vec{v}_{AB}$ is the velocity of reference frame A relative to reference frame B.

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$$

- This is known as the **Galilean transformation of velocity.**

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Slide 4-69

Relative Motion

- Note the "cancellation"
- $\vec{v}_{TG}$ = velocity of the **T**rain relative to the **G**round
- $\vec{v}_{PT}$ = velocity of the **P**assenger relative to the **T**rain
- $\vec{v}_{PG}$ = velocity of the **P**assenger relative to the **G**round

$$\vec{v}_{PG} = \vec{v}_{PT} + \vec{v}_{TG}$$

Inner subscripts disappear

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What is a force?

- A force is a *push* or a *pull*
- A force acts on an object
- Pushes and pulls are applied *to* something
- From the object's perspective, it has a force *exerted* on it
- The S.I. unit of force is the Newton (N)
- 1 N = 1 kg m s^{-2}

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Tactics: Drawing force vectors

TACTICS BOX 5.1 Drawing force vectors

1. Represent the object as a particle.
2. Place the *tail* of the force vector on the particle.
3. Draw the force vector as an arrow pointing in the proper direction and with a length proportional to the size of the force.
4. Give the vector an appropriate label.

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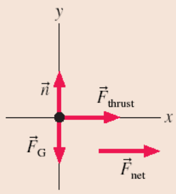
Free-Body Diagrams

A free-body diagram represents the object as a particle at the origin of a coordinate system. Force vectors are drawn with their tails on the particle. The net force vector is drawn beside the diagram.



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Equilibrium

- An object on which the net force is zero is in *equilibrium*
- If the object is at rest, it is in *static equilibrium*
- If the object is moving along a straight line with a constant velocity it is in *dynamic equilibrium*

- The requirement for either type of equilibrium is:

$$(F_{\text{net}})_x = \sum_i (F_i)_x = 0$$

$$(F_{\text{net}})_y = \sum_i (F_i)_y = 0$$

The concept of equilibrium is essential for the engineering analysis of stationary objects such as bridges.

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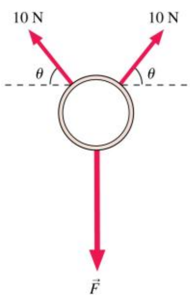
A ring, seen from above, is pulled on by three forces. The ring is not moving. How big is the force F?

A. 20 N
B. $10\cos\theta$ N
C. $10\sin\theta$ N
D. $20\cos\theta$ N
E. $20\sin\theta$ N



Why are you at University?

A. To get a pretty degree with a red sticker, which I can frame and hang on my wall
B. My parents said I "have to"
C. You can't get a good job without a university education
D. I'm just here to learn interesting stuff
E. …other

Time Management

- Having a daily and weekly schedule and sticking to it will improve your marks.
- Organize your time so that "all-nighters" never happen!
- Most nights you should get an **adequate amount of sleep**. An adequate amount of sleep is such that you do not feel sleepy during the rest of the day.

Food and Exercise

- There are four food groups:
  - Vegetables and Fruit
  - Grain Products
  - Dairy
  - Meat and Alternatives (like nuts, tofu, eggs)

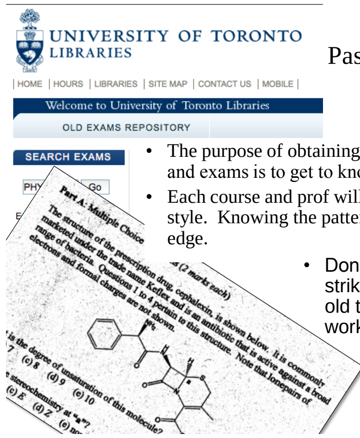
- Physical activity not only improves health but it improves circulation of blood to the brain.
- Try to get 35-40 minutes of brisk physical activity, 5 or 6 times per week. (I ♥ DDR!)

Study Groups – working with Peers

- Find student (students) in class that you work well with on MasteringPhysics, end-of-chapter suggested problems, and past tests.

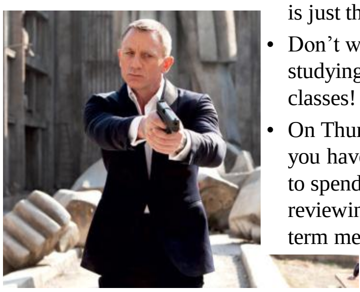
- ***The best way to learn is to teach!*** If you can't explain to someone else what you have done, you haven't really understood it! (This is harder than you think!)

Past Tests and Exams



- The purpose of obtaining and going through old tests and exams is to get to know "the system".
- Each course and prof will have a certain pattern and style. Knowing the pattern in advance gives you an edge.
- Don't count on lightning to strike twice – memorizing old test questions rarely works!

Wed. Dec. 12 evening: Go see a movie!



- The evening before a test is NOT the best time to study (it is just the most popular)
- Don't worry – you have been studying since the 1st week of classes!
- On Thursday before 2:00, if you have time, it can be good to spend some extra time reviewing (utilizing short term memory)

During the Exam

- Skim over the entire exam from front to back **before** you begin. Look for problems that you have confidence to solve first.
- If you start a problem but can't finish it, leave it, make a mark on the edge of the paper beside it, and come back to it after you have solved all the easy problems.
- When you are in a hurry and your hand is not steady, you can make little mistakes; if there is time, do the calculation twice and obtain agreement.
- Bring a snack or drink.
- *Don't leave a test early!*

Non-Equilibrium

- Suppose the x- and y-components of acceleration are *independent* of each other
- That is, a_x does not depend on y or v_y, and a_y does not depend on x or v_x
- Your problem-solving strategy is to:

1. Draw a free-body diagram
2. Use Newton's second law in component form:

$$(F_{\text{net}})_x = \sum F_x = ma_x \qquad \text{and} \qquad (F_{\text{net}})_y = \sum F_y = ma_y$$

The force components (including proper signs) are found from the free-body diagram

Gravity: $F_G = mg$ is just a short form!

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$$

and

$$\vec{F}_G = (mg, \text{straight down})$$

are the same equation, with different notation!

The only difference is that in the second equation we have assumed that $m_2 = M$ (mass of the earth) and $r \approx R$ (radius of the earth).

Weight: A Measurement

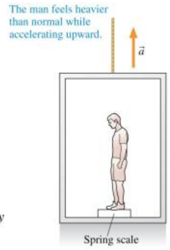
- The figure shows a man weighing himself in an accelerating elevator
- Looking at the free-body diagram, the *y*-component of Newton's second law is:

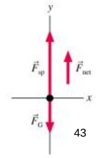
$$(F_{\text{net}})_y = (F_{\text{sp}})_y + (F_G)_y = F_{\text{sp}} - mg = ma_y$$

- The man's weight as he accelerates vertically is:

$$w = \text{scale reading } F_{\text{sp}} = mg + ma_y = mg\left(1 + \frac{a_y}{g}\right)$$

- You weigh *more* as an elevator accelerates upward!



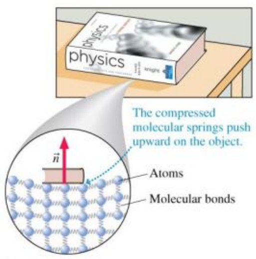


An astronaut takes her bathroom scales to the moon, where $g = 1.6 \text{ m/s}^2$. On the moon, compared to at home on earth:

A. Her weight is the same and her mass is less.
B. Her weight is less and her mass is less.
C. Her weight is less and her mass is the same.
D. Her weight is the same and her mass is the same.
E. Her weight is zero and her mass is the same.

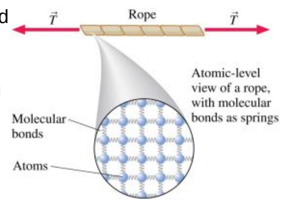
Normal Force

- When an object sits on a table, the table surface exerts an upward contact force on the object
- This pushing force is directed *perpendicular* to the surface, and thus is called the **normal force**
- A table is made of *atoms* joined together by *molecular bonds* which can be modeled as springs
- Normal force is a result of many molecular springs being compressed ever so slightly



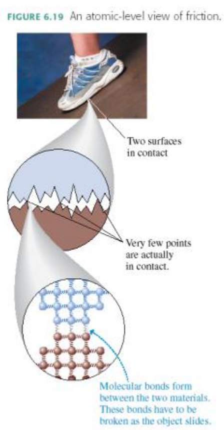
Tension Force

- When a string or rope or wire pulls on an object, it exerts a contact force called the **tension force**
- The tension force is in the direction of the string or rope
- A rope is made of *atoms* joined together by *molecular bonds*
- Molecular bonds can be modeled as tiny *springs* holding the atoms together
- Tension is a result of many molecular springs stretching ever so slightly



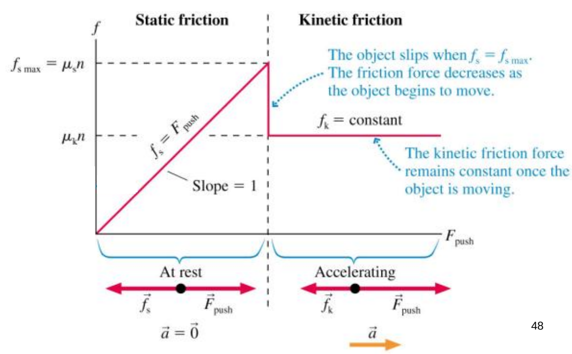
Why does friction exist?

Because at the microscopic level, ***nothing is smooth!***



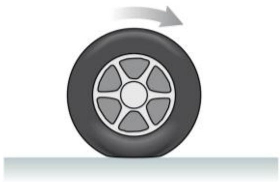
A Model of Friction

The friction force response to an increasing applied force.



Rolling Motion

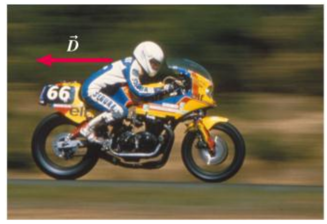
- If you slam on the brakes so hard that the car tires slide against the road surface, this is kinetic friction
- Under normal driving conditions, the portion of the rolling wheel that contacts the surface is *stationary*, not sliding



- If your car is accelerating or decelerating or turning, it is *static friction* of the road on the wheels that provides the net force which accelerates the car

Drag

- The air exerts a drag force on objects as they move through the air
- Faster objects experience a greater drag force than slower objects
- The drag force on a high-speed motorcyclist is significant
- The drag force direction is opposite the object's velocity



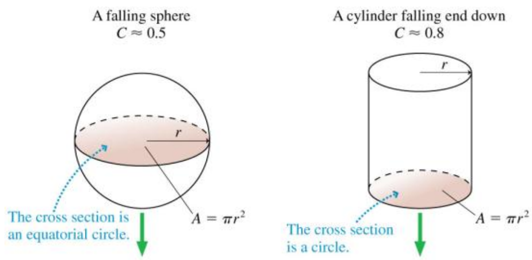
Drag

- For normal sized objects on earth traveling at a speed v which is less than a few hundred meters per second, air resistance can be modeled as:

$$\vec{D} = (\tfrac{1}{2} C\rho A v^2, \text{direction opposite the motion})$$

- A is the *cross-section area* of the object
- ρ is the density of the air, which is about 1.2 kg/m^3
- C is the drag coefficient, which is a dimensionless number that depends on the shape of the object

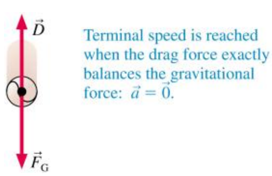
Cross Sectional Area depends on size, shape, and direction of motion.



…Consider the forces on a falling piece of paper, crumpled and not crumpled.

Terminal Speed

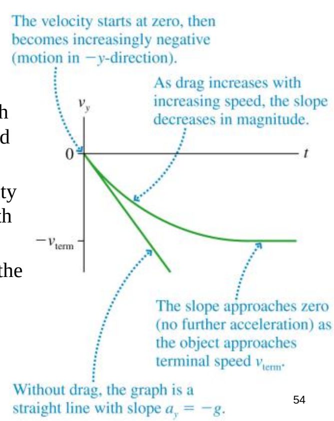
- The drag force from the air increases as an object falls and gains speed
- If the object falls far enough, it will eventually reach a speed at which $D = F_G$
- At this speed, the net force is zero, so the object falls at a *constant* speed, called the terminal speed v_{term}



$$v_{term} = \sqrt{\frac{2mg}{C\rho A}}$$

Terminal Speed

- The figure shows the velocity-versus-time graph of a falling object with and without drag
- Without drag, the velocity graph is a straight line with $a_y = -g$
- When drag is included, the vertical component of the velocity asymptotically approaches $-v_{term}$



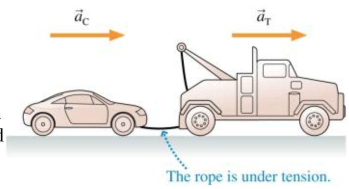
Propulsion

- If you try to walk across a frictionless floor, your foot slips and slides *backward*
- In order to walk, your foot must *stick* to the floor as you straighten your leg, moving your body forward
- The force that prevents slipping is *static friction*
- The static friction force points in the *forward* direction
- It is static friction that propels you forward!

What force causes this sprinter to accelerate?

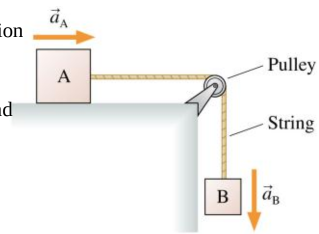
Acceleration Constraints

- If two objects A and B move together, their accelerations are *constrained* to be equal: $a_A = a_B$
- This equation is called an **acceleration constraint**
- Consider a car being towed by a truck
- In this case, the acceleration constraint is $a_{Cx} = a_{Tx} = a_x$
- Because the accelerations of both objects are equal, we can drop the subscripts C and T and call both of them a_x



Acceleration Constraints

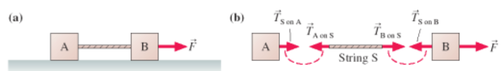
- Sometimes the acceleration of A and B may have different signs
- Consider the blocks A and B in the figure
- The string constrains the two objects to accelerate together



- But, as A moves to the right in the $+x$ direction, B moves down in the $-y$ direction
- In this case, the acceleration constraint is $a_{Ax} = -a_{By}$

The Massless String Approximation

FIGURE 7.22 The string's tension pulls forward on block A, backward on block B.



Often in physics problems the mass of the string or rope is much less than the masses of the objects that it connects. In such cases, we can adopt the following **massless string approximation**:

$$T_{B \text{ on } S} = T_{A \text{ on } S} \qquad \text{(massless string approximation)}$$

A car is parked on a flat surface.

The car has a mass, m, and a downward force of gravity on it of magnitude $F_G = mg$.

Why is the normal force equal to mg?

A. Because that is the equation for normal force: $n = mg$

B. Because acceleration is zero, so the forces must balance

C. Because of Newton's 3[rd] Law: the two forces must be equal and opposite