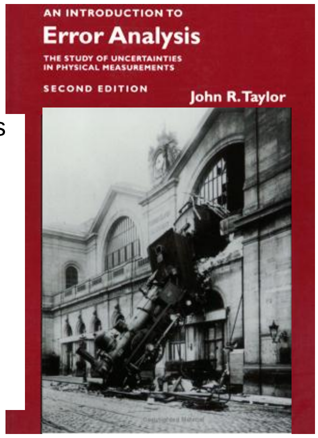
PHY131H1F
Class 3



Today: Error Analysis
- Significant Figures
- Unit Conversion
- Normal Distribution
- Standard Deviation
- Reading Error
- Propagation of Errors
- Error in the Mean

"Error"

- In this course, the word "error" means:

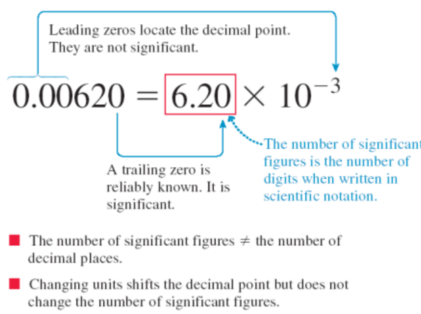
A. Difference between a measurement and the accepted value.
B. Mistake or blunder.
C. The width of a Gaussian distribution.
D. Uncertainty in a measurement.
E. A misplay by a fielder in baseball.

Last Wednesday I asked at the end of class:

- If your height is 150 cm, is there an error in that number?
- ANSWER: YES! Almost every measured number has an error, even if it is not stated. If you told me your height was 150 cm, I would guess the error is probably between 1 and 5 cm. [But there is no way to know this, unless you investigate how the 150 was measured.]

From Knight Chapter 1:

FIGURE 1.25 Determining significant figures.



In Class Question:
Rank in order, from the most to the least, the number of significant figures in the following numbers.

a. 8200 b. 0.0052 c. 0.430 d. 4.321×10^{-10}

A. a = b = d > c
B. b = d > c > a
C. d > c > b = a
D. d > c > a > b
E. b > a = c = d

When do I round?

- The final answer of a problem should be displayed to the correct number of significant figures
- Numbers in intermediate calculations should ***not*** be rounded off
- It's best to keep lots of digits in the calculations to avoid round-off error, which can compound if there are several steps

An important skill: Unit Conversion

- Example
- Knight Ch. 1 Problem 24d (page 30).
- Convert 14 in^2 to SI units.
- Known:
  - 1 in = 2.54 cm
  - 100 cm = 1 m
  - The SI unit of area is m^2.

- Final answer: 0.0090 m^2

Demo and Example: What is the Period of a Swinging Ball? (Pendulum)

- Procedure: Measure the time for 5 oscillations, t_5.
- The period is calculated as $T = t_5 / 5$.

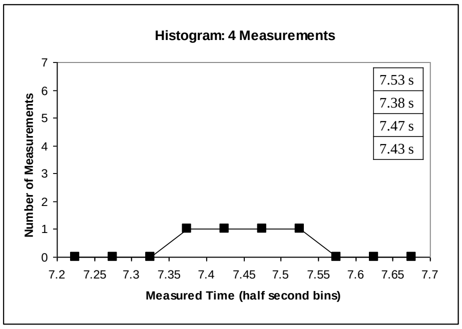
t_5 data: 7.53s
7.38s

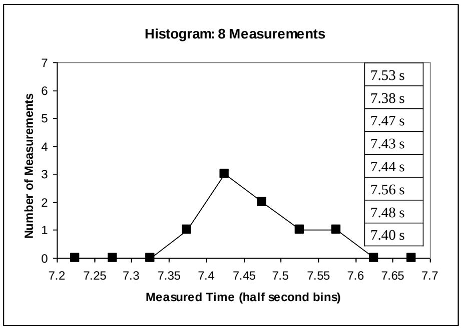
Here were Harlow's measurements of t_5:

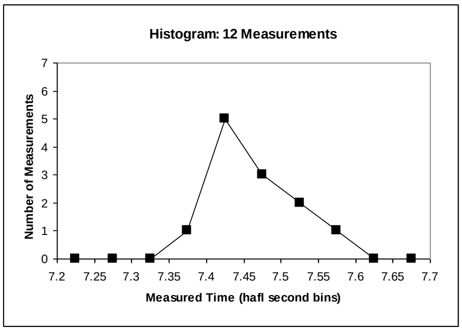
7.53 s
7.38 s
7.47 s
7.43 s

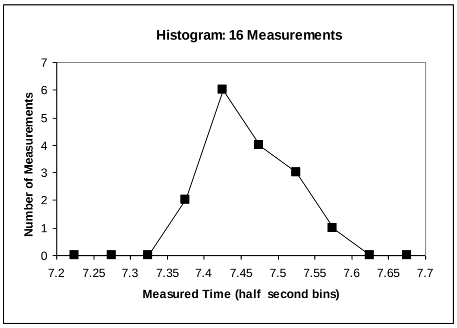
Which of the following might be a good estimate for the error in Harlow's ***first*** measurement of 7.53 seconds?

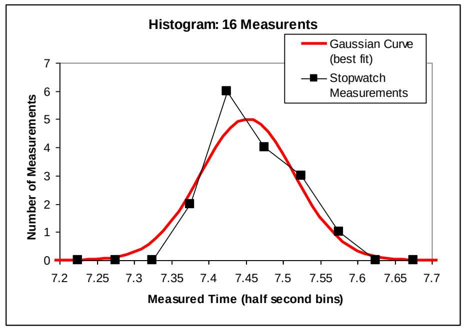
A. 0.005 s
B. 0.05 s
C. 0.5 s
D. 5 s
E. Impossible to determine







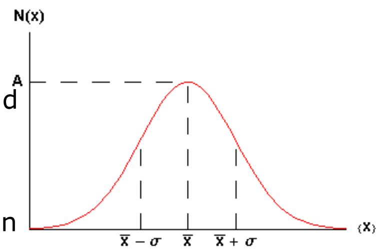




The Gaussian:

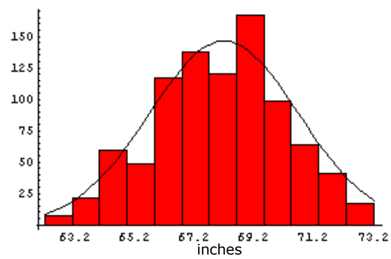
$$N(x) = Ae^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$$

When you integrate this curve, you find that 68% of area lies between the dotted lines on the graph.

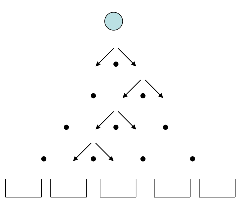


Gaussian Distributions turn up everywhere!

Heights of some People (London, 1886)



Random Walk



Where does an object end up, if it takes N steps randomly left or right?

The final distribution is described by a Gaussian function!

Pre-Class Quizzes

- As you can see under My Grades on the portal, I have posted the first MasteringPhysics Pre-Class quiz mark
- If you did the pre-class quiz on time before 8am last Wednesday, and I was satisfied with your effort, then you got 1/1. Portal lists your official mark.
- If you see you got 0/1, and you know you did the pre-class quiz on time, there could be a problem with your Student ID on MasteringPhysics. It must be identical to your UTORid in order to obtain the credit on portal.
- Note, we will do 18 preclass quizzes through the semester and we will count the top 16 for the pre-class quiz mark worth 3% of the course. So you can miss up to 2 pre-class quizzes on MP with no penalty.

In-class Clicker Quizzes

- Keep an eye on My Grades on portal – some time in the next week I will be posting the first few clicker scores - - you should see the number of classes that you successfully participated in clicker questions.
- The i-clicker system is quite forgiving; even if there is a problem with your registration now, if you fix it within the next couple of weeks you will still get marks for all previous classes
- If you're getting the green light when you vote, then there is proof in my computer that your clicker was used in class today.

The t_5 data

7.53 s $\pm$ 0.06 s
7.38 s $\pm$ 0.06 s
7.47 s $\pm$ 0.06 s
7.43 s $\pm$ 0.06 s

Numerically:
$\bar{t}_{5,est}$ = 7.45250 s
σ_{est} = 0.0634429 s

σ_{est} = 0.06 s

Propagation of Errors

- Rule #1 (sum or difference rule):
- If $z = x + y$
- or $z = x - y$
- then $\Delta z = \sqrt{\Delta x^2 + \Delta y^2}$

- Rule #2 (product or division rule):
- If $z = xy$
- or $z = x/y$
- then $\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$

Propagation of Errors

- Rule #2.1 (multiply by exact constant rule):
- If $z = xy$
- and x is an exact number, so that Δx=0
- then $\Delta z = |x|(\Delta y)$

- Rule #3 (exponent rule):
- If $z = x^n$
- then $\frac{\Delta z}{z} = n\frac{\Delta x}{x}$

The Error in the Mean

- Many individual, independent measurements are repeated N times
- Each individual measurement has the same error Δx
- Using error propagation you can show that the error in the estimated mean is:

$$\Delta \bar{x}_{est} = \frac{\Delta x}{\sqrt{N}}$$

- You wish to know the time it takes to travel from Finch station to Yonge/Bloor by subway. You ask 10 people to take a stopwatch and time the trip. After analyzing all the data you find that it takes an average of 26 minutes and 40 seconds, with an error in this average of ± 100 seconds.
- If you expand your survey and ask 1000 people to time the trip, when you analyze the data, what would you expect to be the error in the average time?

A. 100 seconds
B. 50 seconds
C. 10 seconds
D. 1 second
E. 0.05 seconds

Significant Figures

- Discussed in Section 1.8 of Knight Ch.1
- The rules for significant figures when errors are involved are:

1. **Errors should be specified to one or two significant figures.**
2. **The most precise column in the number for the error should also be the most precise column in the number for the value.**

- Example: If a calculated result is (7.056 +/- 0.705) m, it is better to report (7.1 +/- 0.7) m.

Before Class 4 on Wednesday

- Please read Chapter 2 of Knight (or at least watch the pre-class video for Class 4)
- Please do the pre-class quiz before Wednesday at 8:00am.
- Don’t forget to start the Problem Set due Friday – there’s no time limit, just a deadline
- Something to think about: Does constant velocity imply constant acceleration? Does constant acceleration imply constant velocity?