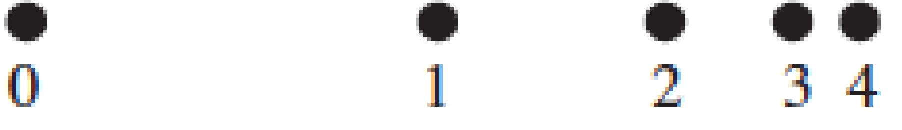
PHY131H1F
Introduction to Physics I
Review of the first half
Chapters 1-8 + Error Analysis

- Position, Velocity, Acceleration
- Significant Figures, Measurements, Errors
- Equations of constant acceleration
- Vectors, Relative Motion
- Forces and Newton’s 3 Laws
- Free Body Diagrams
- Equilibrium and Non-equilibrium Problems
- Circular Motion, Centripetal Force

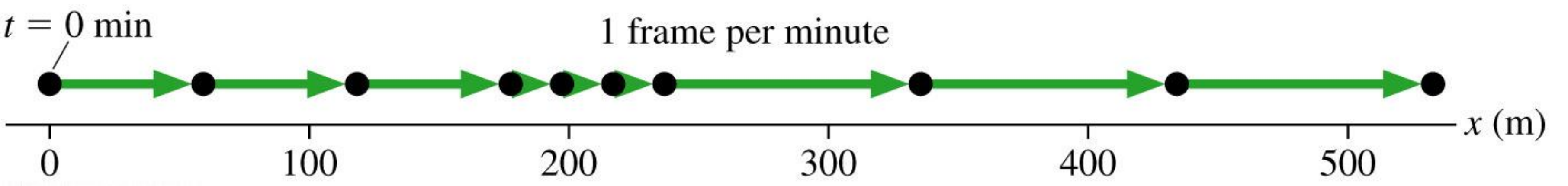
The Particle Model

- Often motion of the object *as a whole* is not influenced by details of the object's size and shape
- We only need to keep track of a single point on the object
- So we can treat the object *as if* all its mass were concentrated into a single point
- A mass at a single point in space is called a **particle**
- Particles have no size, no shape and no top, bottom, front or back
- Below us a motion diagram of a car stopping, using the **particle model**

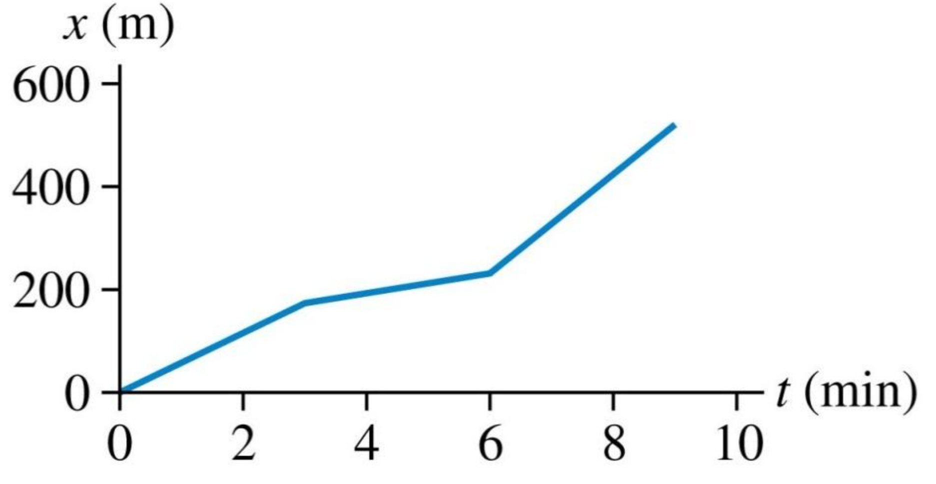


Position-versus-Time Graphs

- Below is a motion diagram, made at 1 frame per minute, of a student walking to school.



- A motion diagram is one way to represent the student's motion.
- Another way is to make a graph of *x* versus *t* for the student:



Acceleration

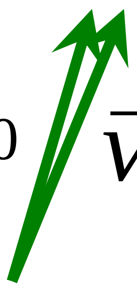
- Sometimes an object's velocity is constant as it moves
- More often, an object's velocity changes as it moves
- Acceleration describes a *change* in velocity

- Consider an object whose velocity changes from $\vec{v}_1$ to $\vec{v}_2$ during the time interval Δt
- The quantity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ is the change in velocity
- The *rate of change of velocity* is called the **average acceleration:**

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

Acceleration (a.k.a. “instantaneous acceleration”)

$$\vec{a} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{v}}{\Delta t}\right) = \frac{d\vec{v}}{dt}$$



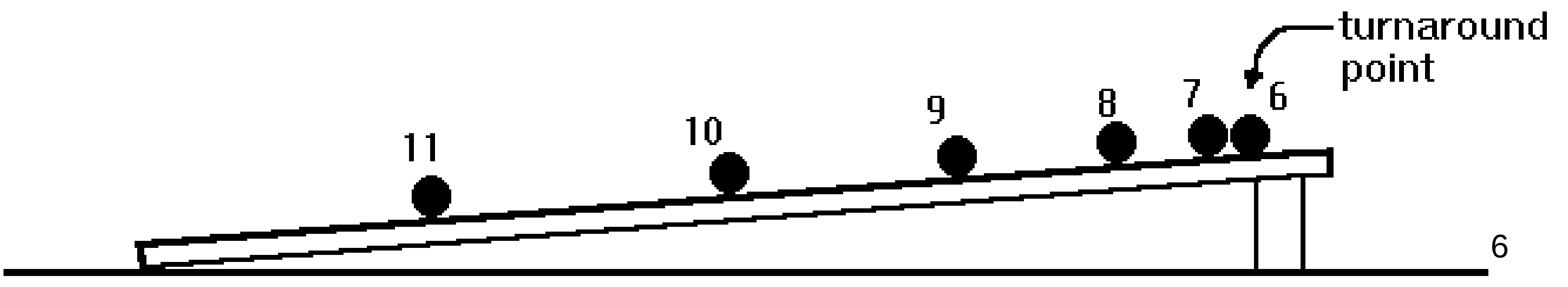
Units of $\Delta\vec{v}$ are m/s.

Units of $\vec{a}$ are m/s^2.

A ball rolls up a ramp, and then down the ramp. We keep track of the position of the ball at 6 instants as it climbs up the ramp. At instant 6, it stops momentarily as it turns around. Then it rolls back down. Shown below is the motion diagram for the final 6 instants as it rolls down the ramp.

At which instant is the **speed** of the ball the greatest?

A. 6
B. 9
C. 11
D. The speed is zero at point 6, but the same at points 7 to 11
E. The speed is the same at points 6 through 11



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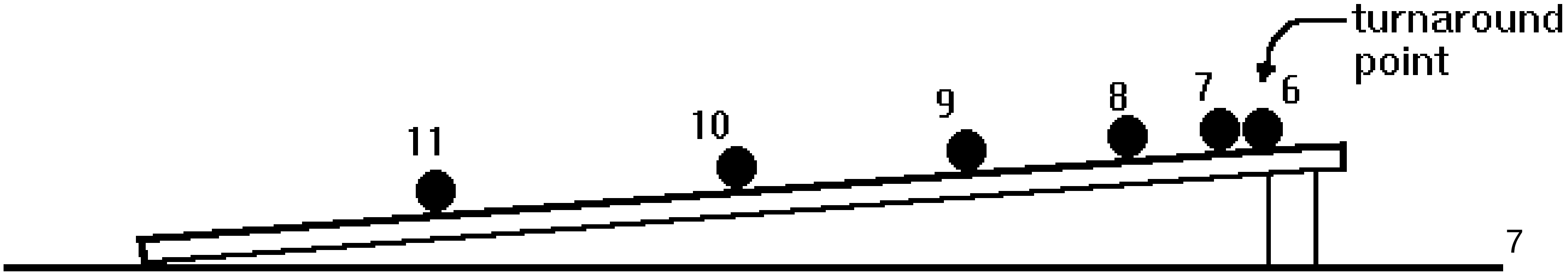

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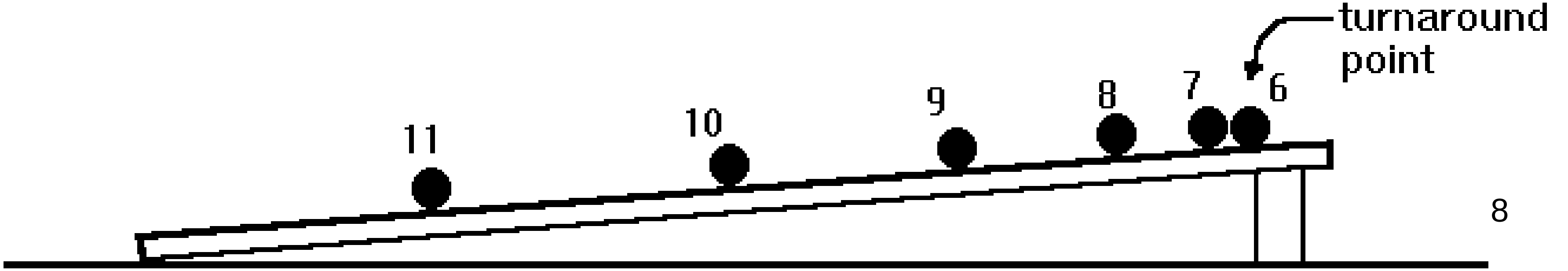
C. 11
D. The acceleration is zero at point 6, but about the same at points 7 to 11
E. The acceleration is about the same at points 6 through 11



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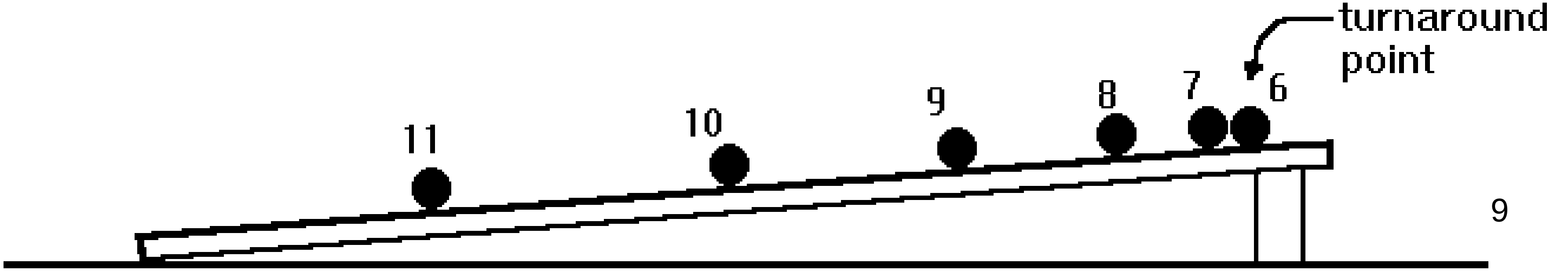


Unit Conversions

$$1\ \text{in} = 2.54\ \text{cm}$$
$$1\ \text{mi} = 1.609\ \text{km}$$
$$1\ \text{mph} = 0.447\ \text{m/s}$$
$$1\ \text{m} = 39.37\ \text{in}$$
$$1\ \text{km} = 0.621\ \text{mi}$$
$$1\ \text{m/s} = 2.24\ \text{mph}$$

- It is important to be able to convert back and forth between SI units and other units
- One effective method is to write the conversion factor as a ratio equal to one
- Because multiplying by 1 does not change a value, these ratios are easily used for unit conversions
- For example, to convert the length 2.00 feet to meters, use the ratio:

$$\frac{2.54\ \text{cm}}{1\ \text{in}} = 1$$

- So that:

$$2.00\ \text{ft} \times \frac{12\ \text{in}}{1\ \text{ft}} \times \frac{2.54\ \text{cm}}{1\ \text{in}} \times \frac{10^{-2}\ \text{m}}{1\ \text{cm}} = 0.610\ \text{m}$$

Significant Figures

- It’s important in science and engineering to state clearly what you know about a situation – no less, and no more
- For example, if you report a length as 6.2 m, you imply that the actual value is between 6.15 m and 6.25 m and has been rounded to 6.2
- The number 6.2 has *two* significant figures
- More precise measurement could give more significant figures
- The appropriate number of significant figures is determined by the data provided
- Calculations follow the “weakest link” rule: the input value with the smallest number of significant figures determines the number of significant figures to use in reporting the output value

Leading zeros locate the decimal point. They are not significant.

$$0.00620 = 6.20 \times 10^{-3}$$

A trailing zero after the decimal place is reliably known. It is significant.

The number of significant figures is the number of digits when written in scientific notation.

- The number of significant figures $\neq$ the number of decimal places.
- In whole numbers, trailing zeros are not significant. 320 is 3.2×10^2 and has 2 significant figures, not 3.
- Changing units shifts the decimal point but does not change the number of significant figures.

When do I round?

- The final answer of a problem should be displayed to the correct number of significant figures
- Numbers in intermediate calculations should ***not*** be rounded off
- It's best to keep lots of digits in the calculations to avoid round-off error, which can compound if there are several steps

Suggested Problem Solving Strategy

- MODEL Think about and simplify the situation, guess at what the right answer might be.
- VISUALIZE Draw a diagram. It doesn't have to be artistic: stick figures and blobs are okay!
- SOLVE Set up the equations, solve for what you want to find. (This takes time..)
- ASSESS Check your units, significant figures, do a "sanity check": does my answer make sense?

This is just a suggested strategy. Whatever method works for *you* is fine, as long as you don't make a mistake, and you show how you got to the correct answer, it's 100%!

Error Analysis

- Almost every time you make a measurement, the result will not be an exact number, but it will be a *range* of possible values.
- The range of values associated with a measurement is described by the uncertainty, or **error**.

Exactly 3 apples (no error)

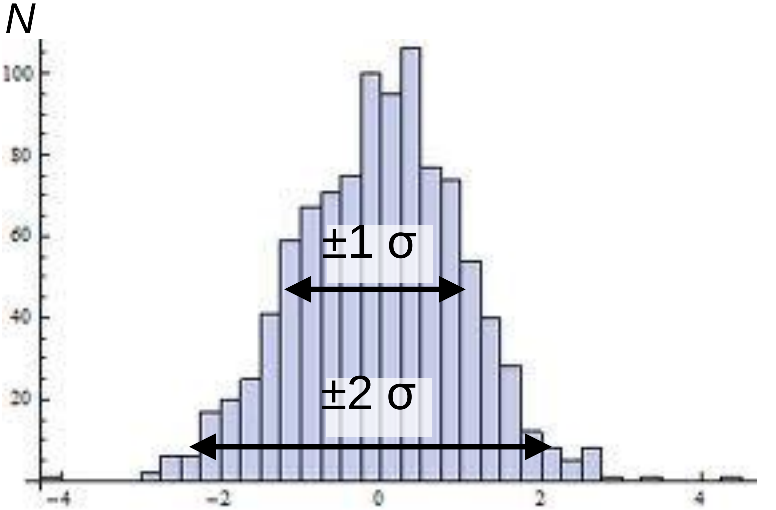
1600 ± 100 apples:

1600 is the **value**
100 is the **error**

Errors

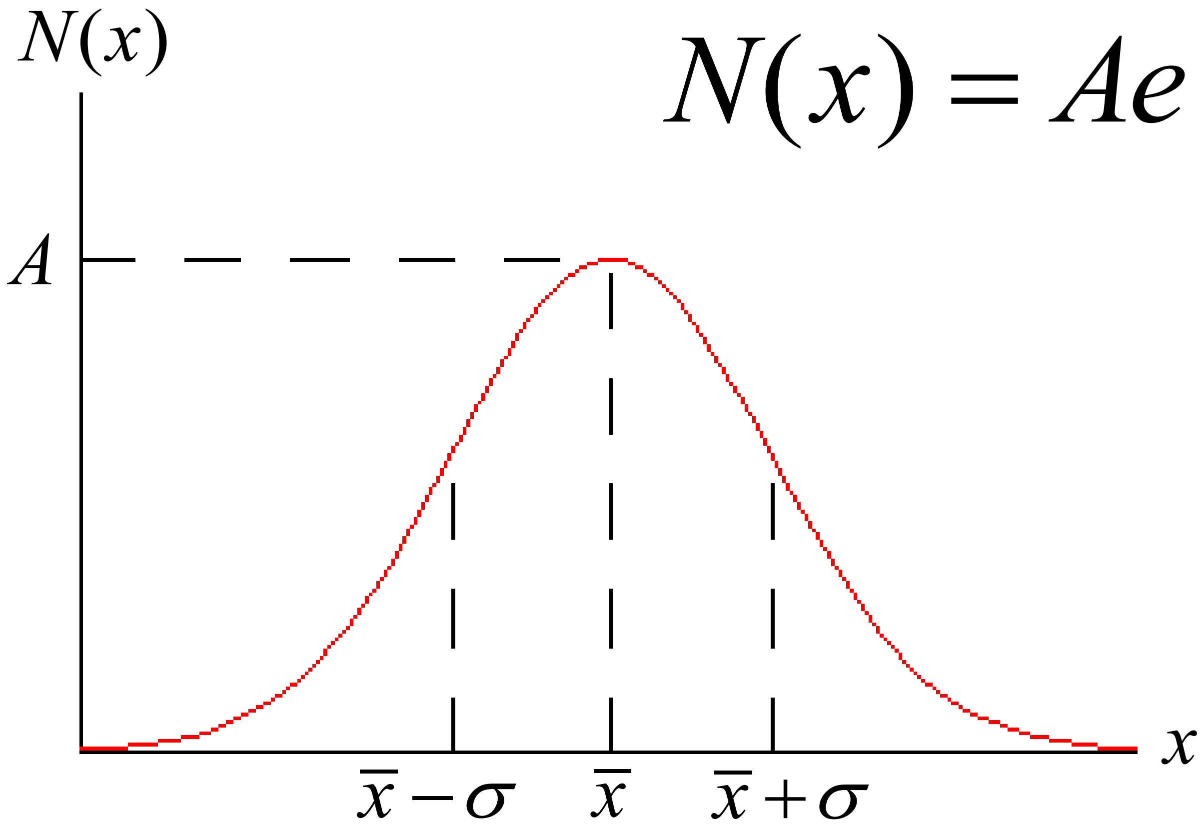
- Errors **eliminate** the need to report measurements with vague terms like "approximately" or "≈".
- Errors give a *quantitative* way of stating your confidence level in your measurement.
- Saying the answer is 10 ± 2 means you are 68% confident that the actual number is between 8 and 12.
- It also implies that and 14 (the 2-σ range).

A histogram of many, many measurements of the same thing:



The Gaussian:

$$N(x) = Ae^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$$



- A is the *maximum amplitude*.
- $\bar{x}$ is the *mean* or *average*.
- σ is the *standard deviation* of the distribution.

Normal Distribution

- σ is the **standard deviation** of the distribution
- Statisticians often call the square of the standard deviation, σ^2, the **variance**
- σ is a measure of the width of the curve: a larger σ means a wider curve
- 68% of the area under the curve of a Gaussian lies between the mean minus the standard deviation and the mean plus the standard deviation
- 95% of the area under the curve is between the mean minus twice the standard deviation and the mean plus twice the standard deviation

Estimating the Mean from a Sample

- Suppose you make *N* measurements of a quantity *x*, and you expect these measurements to be normally distributed
- Each measurement, or trial, you label with a number *i*, where *i* = 1, 2, 3, etc
- You do not know what the true mean of the distribution is, and you cannot know this
- However, you can estimate the mean by adding up all the individual measurements and dividing by *N*:

$$\bar{x}_{\text{est}} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Estimating the Standard Deviation from a Sample

- Suppose you make N measurements of a quantity x, and you expect these measurements to be normally distributed
- It is impossible to know the true standard deviation of the distribution
- The best estimate of the standard deviation is:

$$\sigma_{\text{est}} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x}_{\text{est}})^2}$$

- The quantity $N - 1$ is called the **number of degrees of freedom**
- In this case, it is the number of measurements minus one because you used one number from a previous calculation (mean) in order to find the standard deviation.

Reading Error (Digital)

- For a measurement with an instrument with a digital readout, the reading error is usually "± one-half of the last digit."
- This means one-half of the power of ten represented in the last digit.
- With the digital thermometer shown, the last digit represents values of a tenth of a degree, so the reading error is ½ × 0.1 = 0.05°C

- You should write the temperature as 12.80 ± 0.05 °C.

Significant Figures when Error are Involved

- There are two general rules for significant figures used in experimental sciences:

1. Errors should be specified to one, or at most two, significant figures.
2. The most precise column in the number for the error should also be the most precise column in the number for the value.

- So if the error is specified to the 1/100th column, the quantity itself should also be specified to the 1/100th column.

Propagation of Errors of Precision

- When you have two or more quantities with known errors you may sometimes want to combine them to compute a derived number
- You can use the rules of Error Propagation to infer the error in the derived quantity
- We assume that the two directly measured quantities are x and y, with errors Δx and Δy respectively
- The measurements x and y must be independent of each other.
- The fractional error is the value of the error divided by the value of the quantity: $\Delta x / x$
- To use these rules for quantities which cannot be negative, the fractional error should be much less than one

Propagation of Errors

- Rule #1 (sum or difference rule):
- If $z = x + y$
- or $z = x - y$
- then $\Delta z = \sqrt{\Delta x^2 + \Delta y^2}$

- Rule #2 (product or division rule):
- If $z = xy$
- or $z = x/y$
- then $\dfrac{\Delta z}{z} = \sqrt{\left(\dfrac{\Delta x}{x}\right)^2 + \left(\dfrac{\Delta y}{y}\right)^2}$

Propagation of Errors

- Rule #2.1 (multiply by exact constant rule):
- If $z = xy$ or $z = x/y$
- and x is an exact number, so that $\Delta x = 0$
- then $\Delta z = |x|(\Delta y)$

- Rule #3 (exponent rule):
- If $z = x^n$
- then $\dfrac{\Delta z}{z} = n \dfrac{\Delta x}{x}$

The Error in the Mean

- Many individual, independent measurements are repeated N times
- Each individual measurement has the same error Δx
- Using error propagation you can show that the error in the estimated mean is:

$$\Delta \bar{x}_{\text{est}} = \frac{\Delta x}{\sqrt{N}}$$

The 4 Equations of Constant Acceleration:

1. $$v_f = v_i + a\Delta t$$

Does not contain position!

2. $$s_f = s_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

Does not contain v_f !

3. $${v_f}^2 = {v_i}^2 + 2a(s_f - s_i)$$

Does not contain Δt !

4. $$s_f = s_i + \left(\frac{v_i + v_f}{2}\right)\Delta t$$

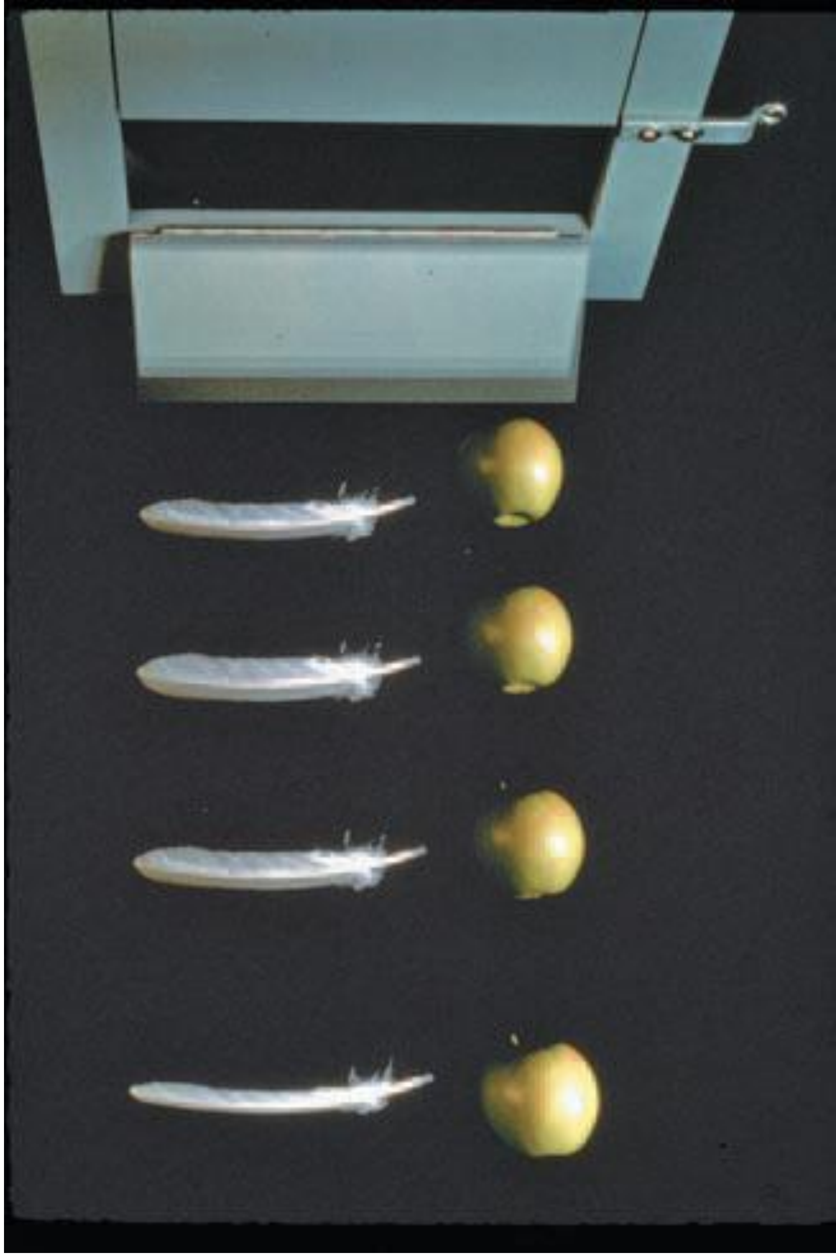
Does not contain a ! (but you know it's constant)

Strategy: When a = constant, you can use one of these equations. Figure out which variable you don't know and don't care about, and use the equation which doesn't contain it.

Free Fall

- The motion of an object moving under the influence of gravity only, and no other forces, is called **free fall**
- Two objects dropped from the same height will, if air resistance can be neglected, hit the ground at the same time and with the same speed
- Consequently, any two objects in free fall, regardless of their mass, have the same acceleration:

$$\vec{a}_{\text{free fall}} = (9.80\ \text{m/s}^2,\ \text{vertically downward})$$



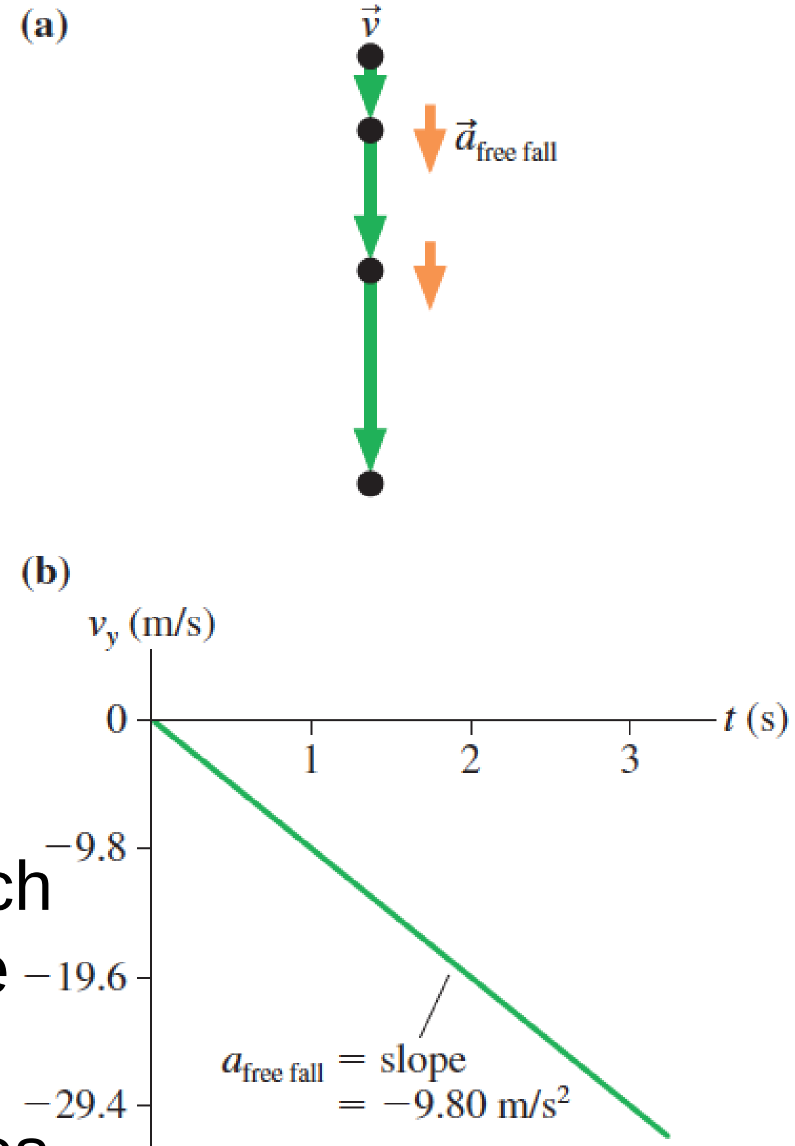
The apple and feather seen here are falling in a vacuum.

Free Fall

- Figure (a) shows the motion diagram of an object that was released from rest and falls freely
- Figure (b) shows the object's velocity graph
- The velocity graph is a straight line with a slope:

$$a_y = a_{\text{free fall}} = -g$$

- where g is a positive number which is equal to 9.80 m/s^2 on the surface of the earth
- Other planets have different values of g



Two-Dimensional Kinematics

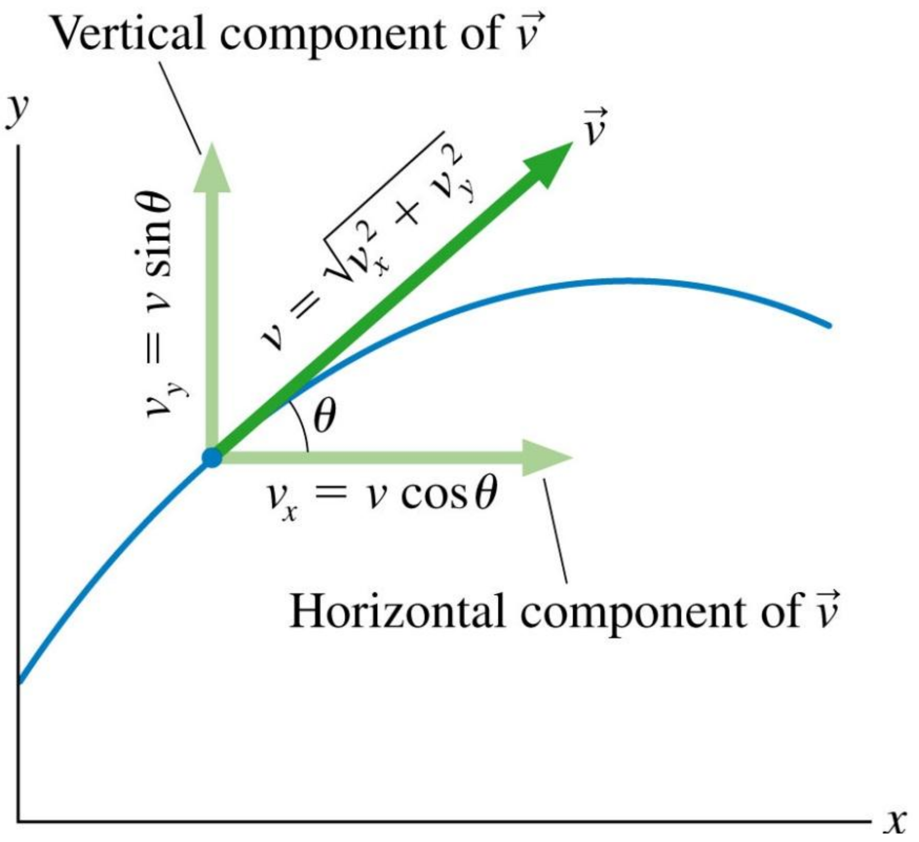
- If the velocity vector's angle *θ* is measured from the positive *x*-direction, the velocity components are

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

where the particle's *speed* is

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

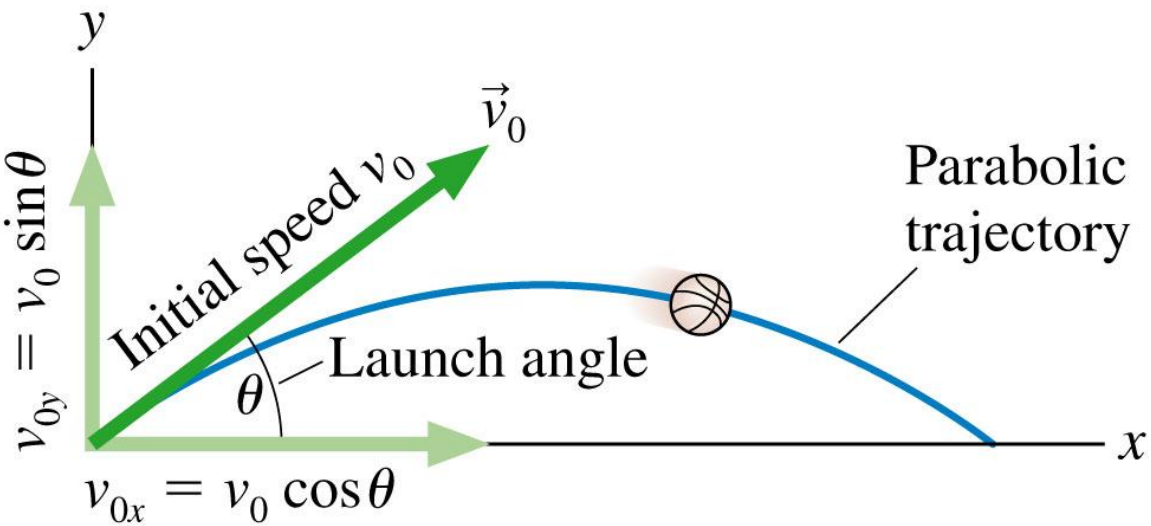


- Conversely, if we know the velocity components, we can determine the direction of motion:

$$\tan\theta = \frac{v_y}{v_x}$$

Projectile Motion

- The start of a projectile's motion is called the *launch*
- The angle θ of the initial velocity v_0 above the x-axis is called the **launch angle**



- The initial velocity vector can be broken into components

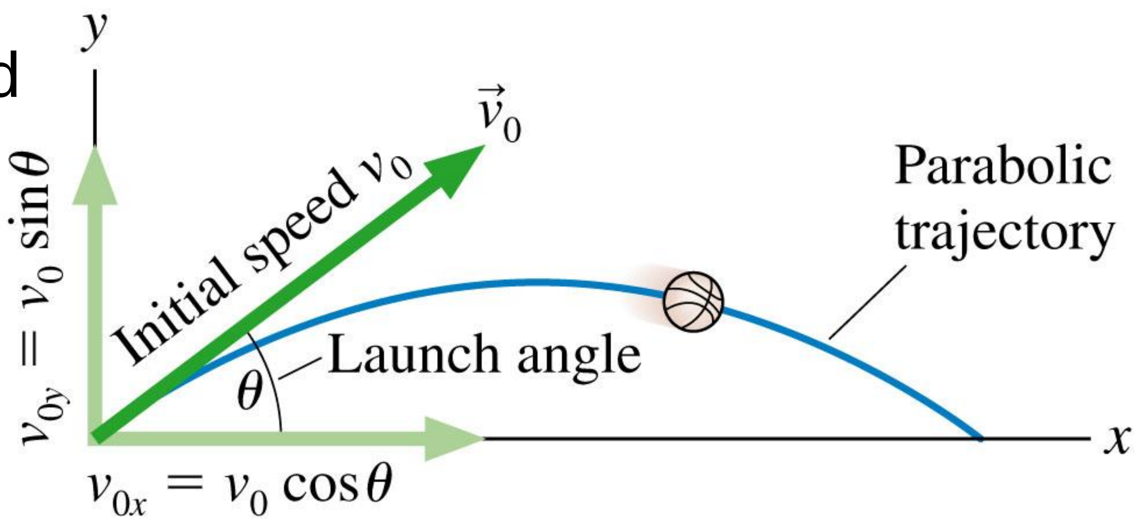
$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$

where v_0 is the initial speed

Projectile Motion

- Gravity acts downward
- Therefore, a projectile has no horizontal acceleration
- Thus



$$a_x = 0$$

(projectile motion)

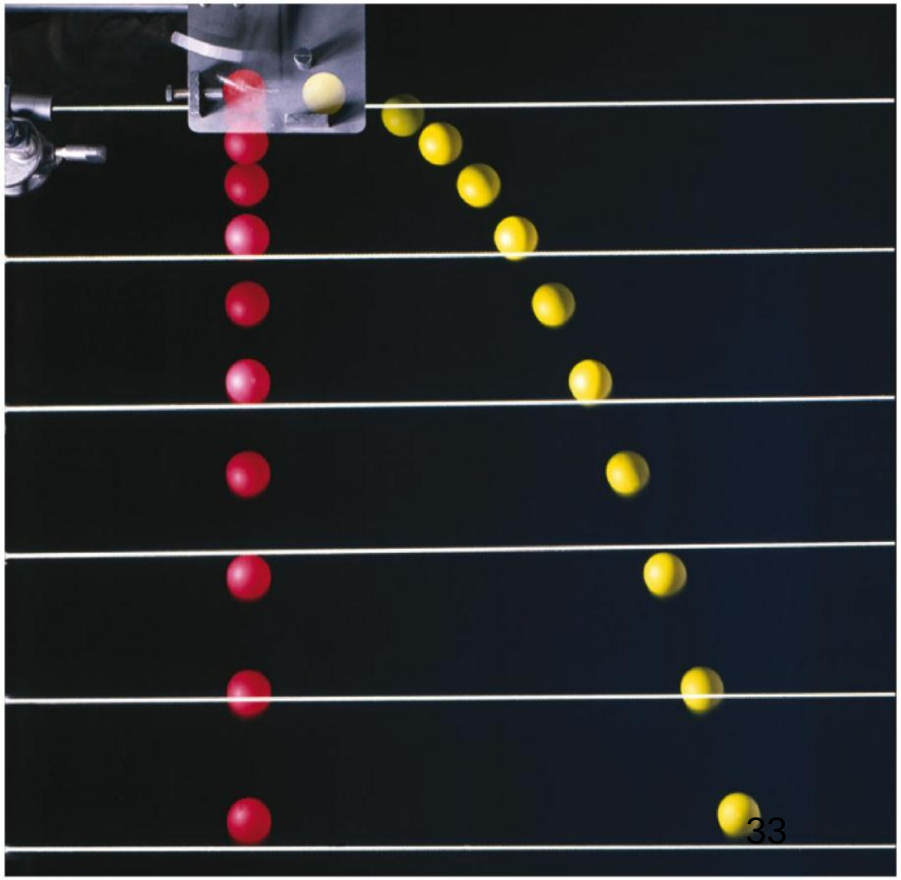
$$a_y = -g$$

- The vertical component of acceleration a_y is $-g$ of free fall
- The horizontal component of a_x is zero
- Projectiles are in free fall

Reasoning About Projectile Motion

A heavy ball is launched exactly horizontally at height *h* above a horizontal field. At the exact instant that the ball is launched, a second ball is simply dropped from height *h*. Which ball hits the ground first?

- If air resistance is neglected, the balls hit the ground *simultaneously*
- The initial horizontal velocity of the first ball has *no* influence over its vertical motion
- Neither ball has any initial vertical motion, so both fall distance *h* in the same amount of time



Range of a Projectile

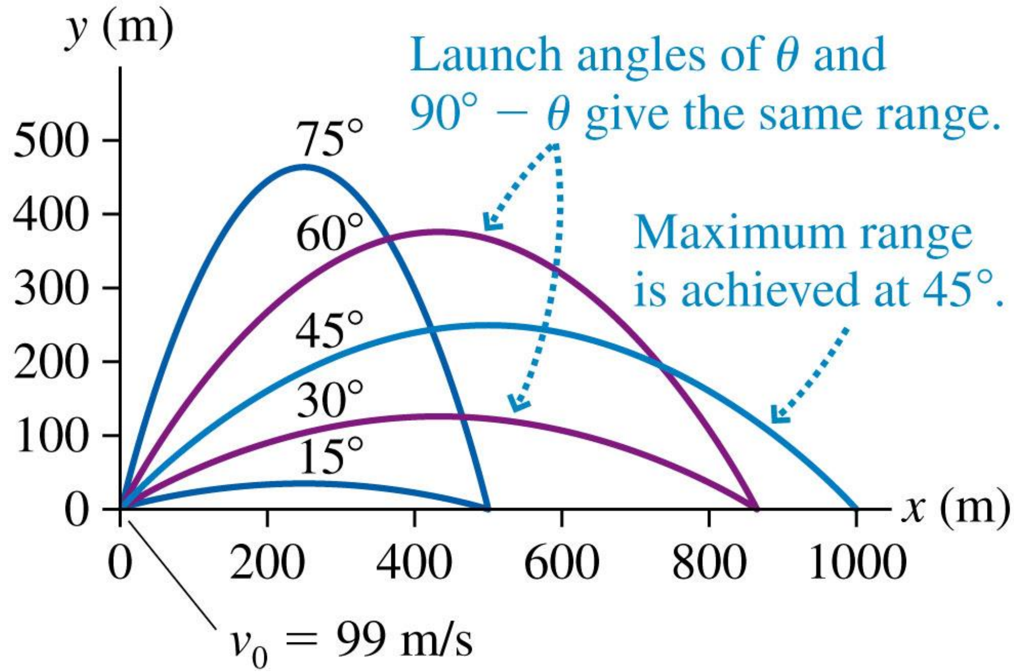
A projectile with initial speed v_0 has a launch angle of θ above the horizontal. How far does it travel over level ground before it returns to the same elevation from which it was launched?

- This distance is sometimes called the *range* of a projectile
- Example 4.5 from your textbook shows:

$$\text{distance} = \frac{{v_0}^2 \sin(2\theta)}{g}$$

- The maximum distance occurs for $\theta = 45°$

Trajectories of a projectile launched at different angles with a speed of 99 m/s.



Relative Velocity

- Relative velocities are found as the time derivative of the relative positions.
- $\vec{v}_{CA}$ is the velocity of C relative to A.
- $\vec{v}_{CB}$ is the velocity of C relative to B.
- $\vec{v}_{AB}$ is the velocity of reference frame A relative to reference frame B.

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$$

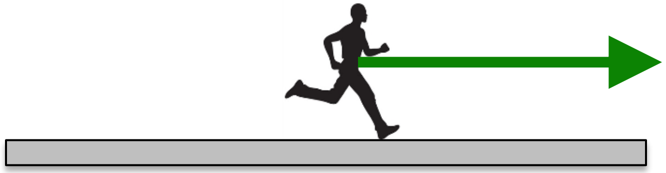
- This is known as the **Galilean transformation of velocity.**

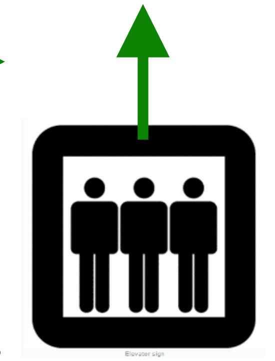
Relative Motion

- Note the "cancellation"
- $\vec{v}_{TG}$ = velocity of the **T**rain relative to the **G**round
- $\vec{v}_{PT}$ = velocity of the **P**assenger relative to the **T**rain
- $\vec{v}_{PG}$ = velocity of the **P**assenger relative to the **G**round

$$\vec{v}_{PG} = \vec{v}_{PT} + \vec{v}_{TG}$$

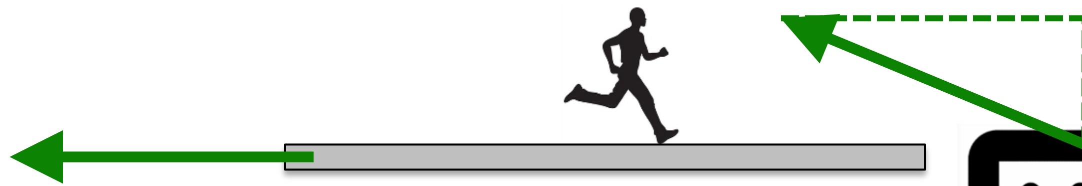
Inner subscripts disappear





You are running toward the right at 5 m/s toward an elevator that is moving up at 2 m/s. Relative to you, the direction and magnitude of the elevator's velocity are

A. down and to the right, less than 2 m/s.
B. up and to the left, less than 2 m/s.
C. up and to the left, more than 2 m/s.
D. up and to the right, less than 2 m/s.
E. up and to the right, more than 2 m/s.



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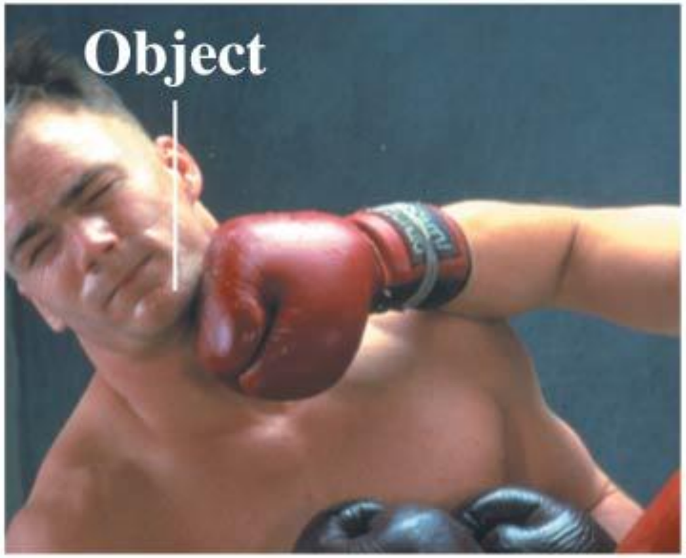
C. up and to the left, more than 2 m/s.
D. up and to the right, less than 2 m/s.
E. up and to the right, more than 2 m/s.

What is a force?

- A force is a *push* or a *pull*
- A force acts on an object
- Pushes and pulls are applied *to* something
- From the object's perspective, it has a force *exerted* on it

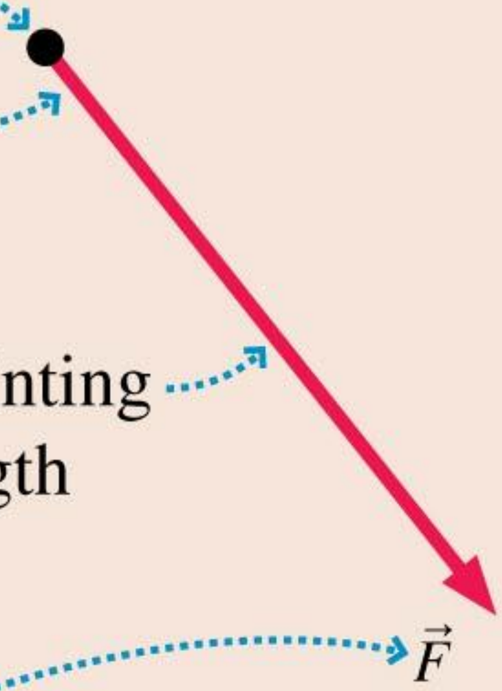
- The S.I. unit of force is the Newton (N)
- $1\ N = 1\ kg\ m\ s^{-2}$



Tactics: Drawing force vectors

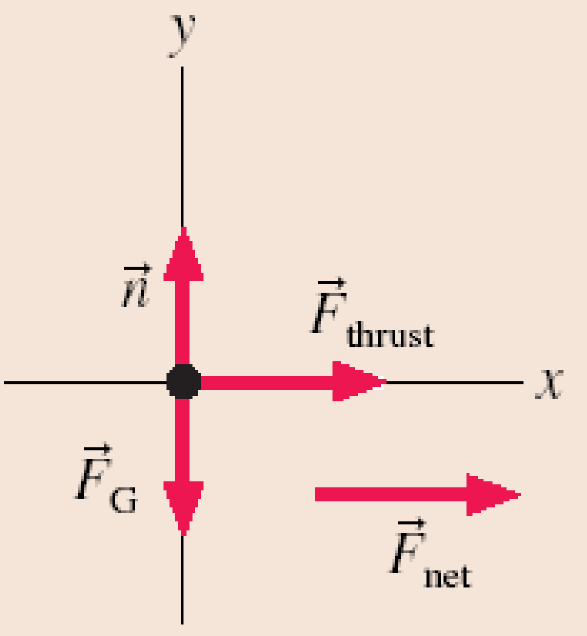
TACTICS BOX 5.1 **Drawing force vectors**

1. Represent the object as a particle.
2. Place the *tail* of the force vector on the particle.
3. Draw the force vector as an arrow pointing in the proper direction and with a length proportional to the size of the force.
4. Give the vector an appropriate label.



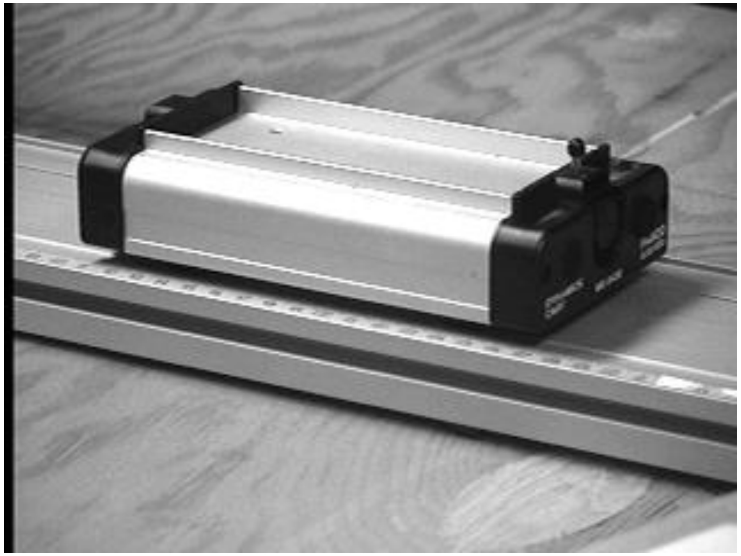
Free-Body Diagrams

A free-body diagram represents the object as a particle at the origin of a coordinate system. Force vectors are drawn with their tails on the particle. The net force vector is drawn beside the diagram.



N1 Newton's First Law

The natural state of an object with no net external force on it is to either remain at rest or continue to move in a straight line with a constant velocity.



Inertial Reference Frames

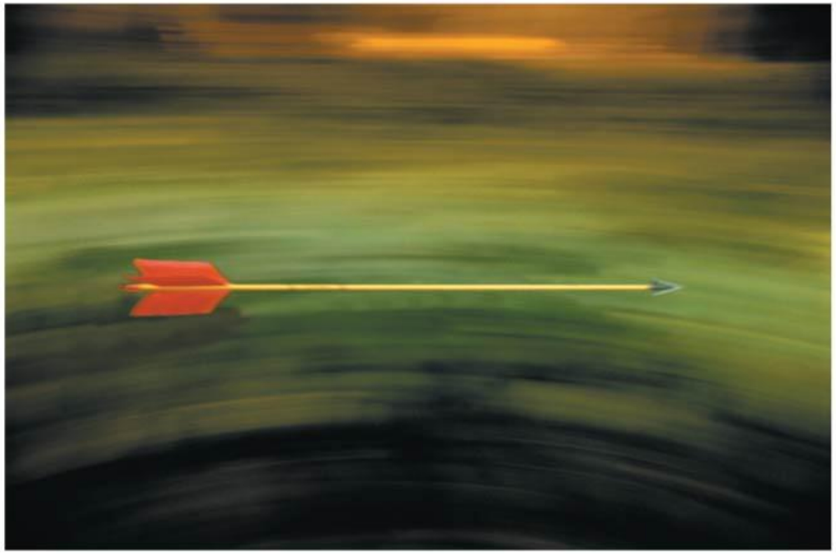
- If a car stops suddenly, you may be "thrown" forward
- You do have a forward acceleration *relative to the car*
- However, there is no force pushing you forward

This guy thinks there's a force hurling him into the windshield. What a dummy!

- We define an **inertial reference frame** as one in which Newton's laws are valid
- The interior of a crashing car is *not* an inertial reference frame!

Thinking About Force

- Every force has an agent which causes the force
- Forces exist at the point of contact between the agent and the object (except for the few special cases of long-range forces)
- Forces exist due to interactions happening *now*, not due to what happened in the past
- Consider a flying arrow
- A pushing force was required to accelerate the *arrow as it was shot*
- However, *no force* is needed to keep the arrow moving forward as it flies



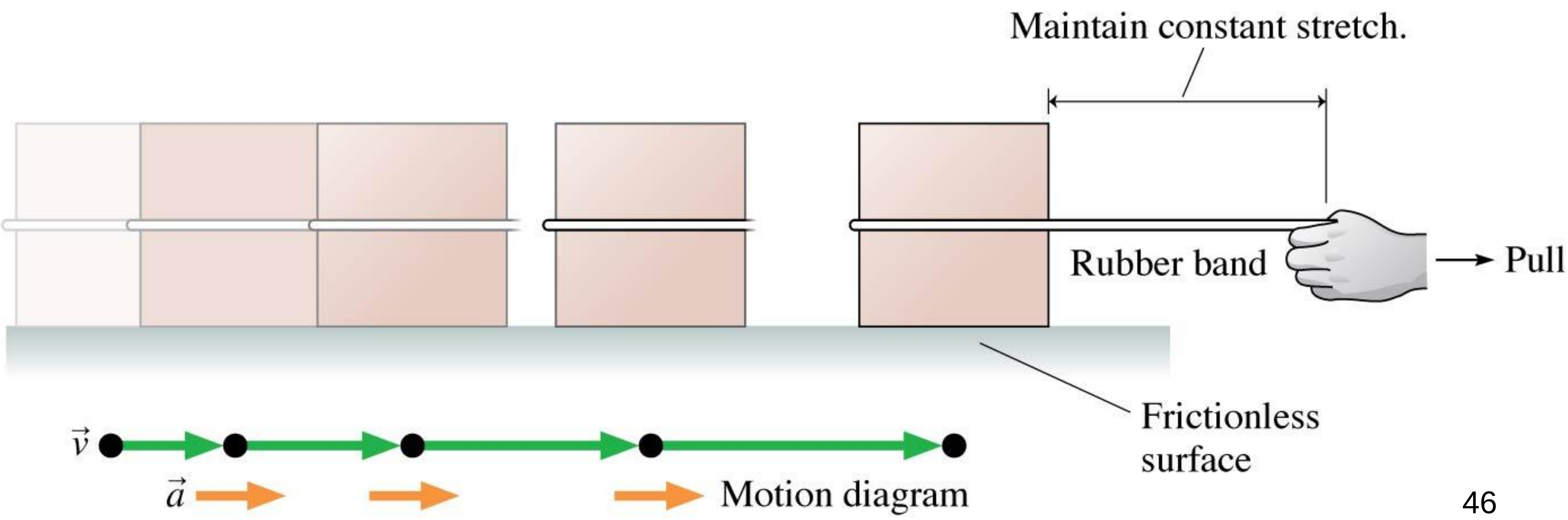
- It continues to move because of inertia

What is Mass?

- Mass is a scalar quantity that describes an object's inertia.
- It describes the amount of matter in an object.
- **Mass is an intrinsic property of an object.**
- It tells us something about the object, regardless of where the object is, what it's doing, or whatever forces may be acting on it.

What Do Forces Do? A Virtual Experiment

- Attach a stretched rubber band to a 1 kg block
- Use the rubber band to pull the block across a horizontal, frictionless table
- Keep the rubber band stretched by a fixed amount
- We find that the block moves with a **constant acceleration**



N2 Newton's Second Law

The acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to its mass.

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

Equilibrium

- An object on which the net force is zero is in *equilibrium*
- If the object is at rest, it is in *static equilibrium*
- If the object is moving along a straight line with a constant velocity it is in *dynamic equilibrium*

- The requirement for either type of equilibrium is:

$$(F_{\text{net}})_x = \sum_i (F_i)_x = 0$$

$$(F_{\text{net}})_y = \sum_i (F_i)_y = 0$$

The concept of equilibrium is essential for the engineering analysis of stationary objects such as bridges.

Non-Equilibrium

- Suppose the x- and y-components of acceleration are *independent* of each other
- That is, a_x does not depend on y or v_y, and a_y does not depend on x or v_x
- Your problem-solving strategy is to:

1. Draw a free-body diagram
2. Use Newton's second law in component form:

$$(F_{\text{net}})_x = \sum F_x = ma_x \qquad \text{and} \qquad (F_{\text{net}})_y = \sum F_y = ma_y$$

The force components (including proper signs) are found from the free-body diagram

Universal Law of Gravitation

Gravity is an attractive, long-range force between *any* two objects.

When two objects with masses m_1 and m_2 are separated by distance r, each object pulls on the other with a force given by Newton's law of gravity, as follows:

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2} \qquad \text{(Newton's law of gravity)}$$

(Sometimes called "Newton's 4th Law", or "Newton's Law of Universal Gravitation")

Gravity for Earthlings

If you happen to live on the surface of a large planet with radius *R* and mass *M*, you can write the gravitational force even more simply as

$$\vec{F}_G = (mg, \text{straight down}) \qquad \text{(gravitational force)}$$

where the quantity *g* is defined to be:

$$g = \frac{GM}{R^2}$$

At sea level, $g = 9.83 \text{ m/s}^2$.
At 39 km altitude, $g = 9.71 \text{ m/s}^2$.

Gravity: $F_G = mg$ is just a short form!

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$$

and

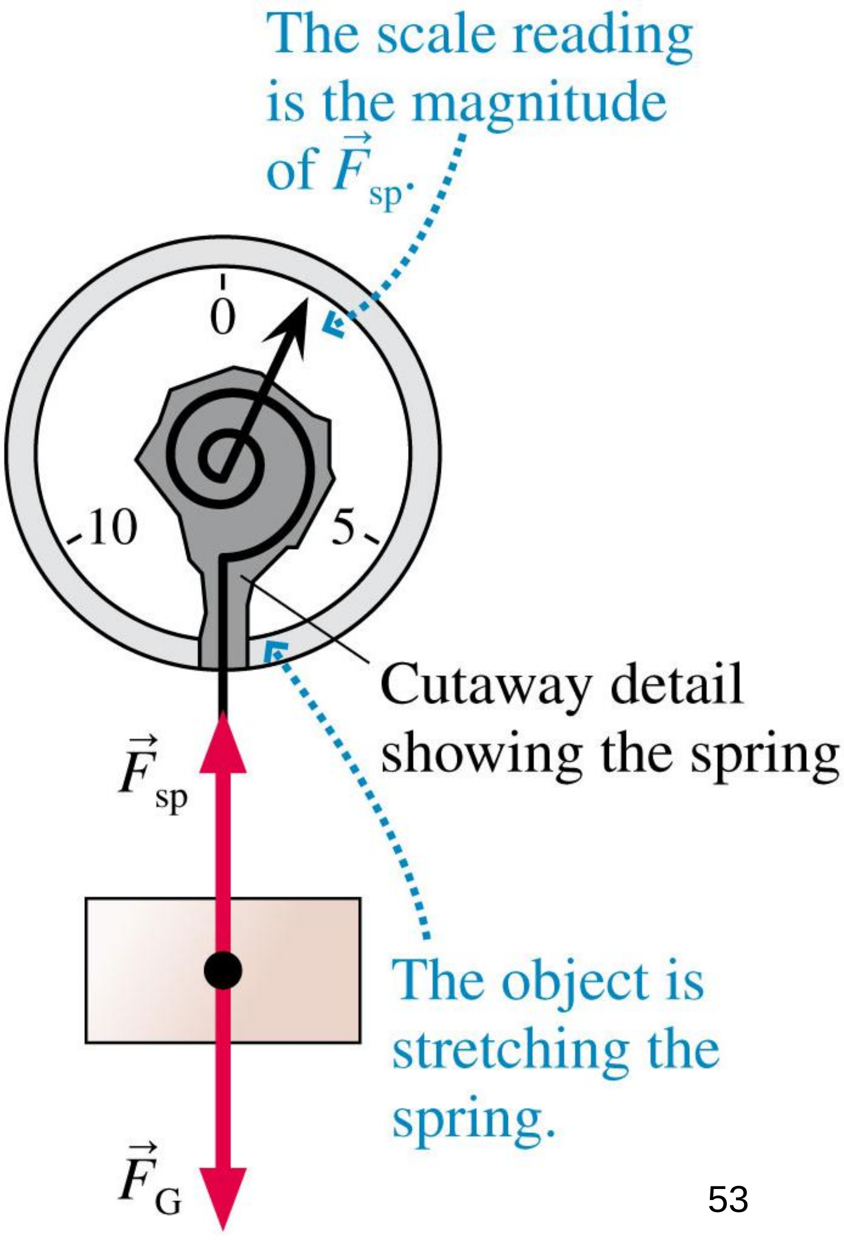
$$\vec{F}_G = (mg, \text{straight down})$$

are the same equation, with different notation!

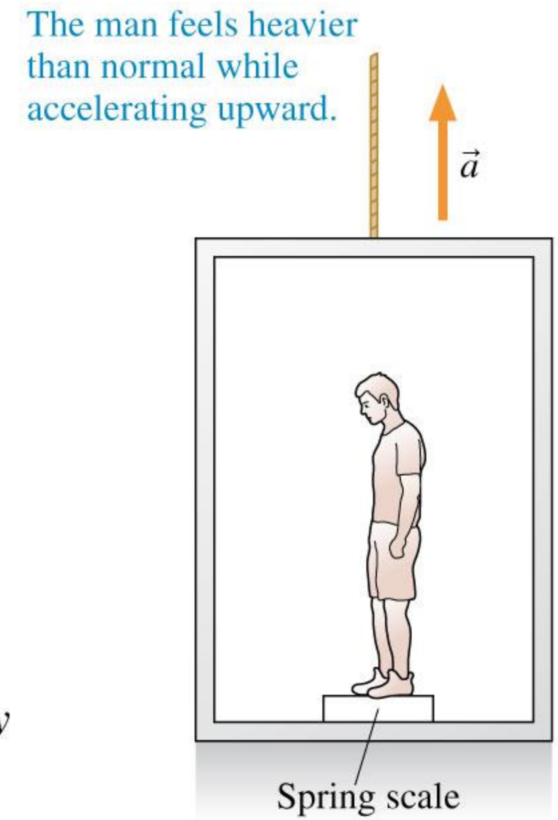
The only difference is that in the second equation we have assumed that $m_2 = M$ (mass of the earth) and $r \approx R$ (radius of the earth).

Weight: A Measurement

- You weigh apples in the grocery store by placing them in a *spring scale* and stretching a spring
- The reading of the spring scale is the magnitude of F_{sp}
- We define the weight of an object as the reading F_{sp} of a calibrated spring scale on which the object is stationary
- Because F_{sp} is a force, weight is measured in newtons



Weight: A Measurement



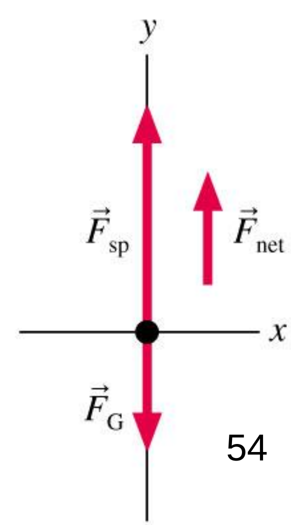
- The figure shows a man weighing himself in an accelerating elevator
- Looking at the free-body diagram, the *y*-component of Newton's second law is:

$$(F_{\text{net}})_y = (F_{\text{sp}})_y + (F_G)_y = F_{\text{sp}} - mg = ma_y$$

- The man's weight as he accelerates vertically is:

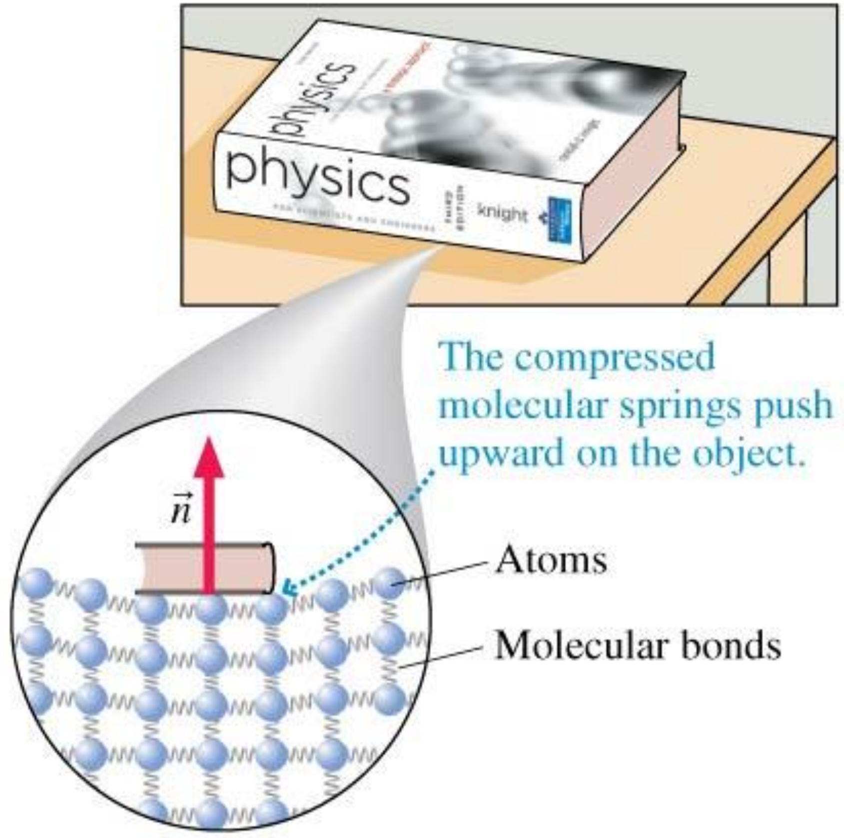
$$w = \text{scale reading } F_{\text{sp}} = mg + ma_y = mg\left(1 + \frac{a_y}{g}\right)$$

- You weigh *more* as an elevator accelerates upward!



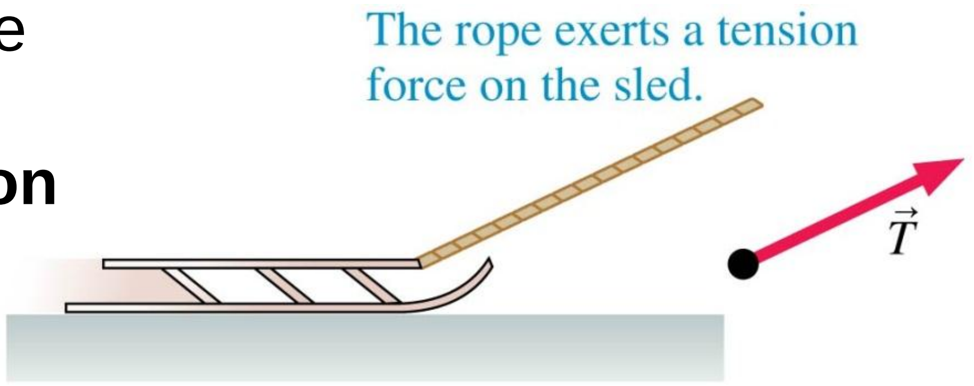
Normal Force

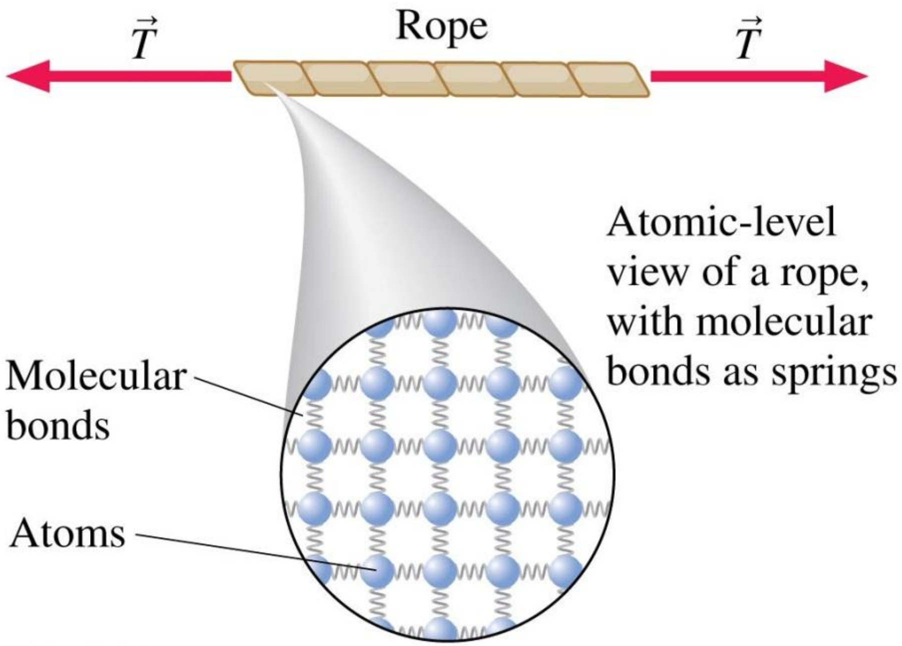
- When an object sits on a table, the table surface exerts an upward contact force on the object
- This pushing force is directed *perpendicular* to the surface, and thus is called the **normal force**
- A table is made of *atoms* joined together by *molecular bonds* which can be modeled as springs
- Normal force is a result of many molecular springs being compressed ever so slightly



Tension Force

- When a string or rope or wire pulls on an object, it exerts a contact force called the **tension force**
- The tension force is in the direction of the string or rope
- A rope is made of *atoms* joined together by *molecular bonds*
- Molecular bonds can be modeled as tiny *springs* holding the atoms together
- Tension is a result of many molecular springs stretching ever so slightly

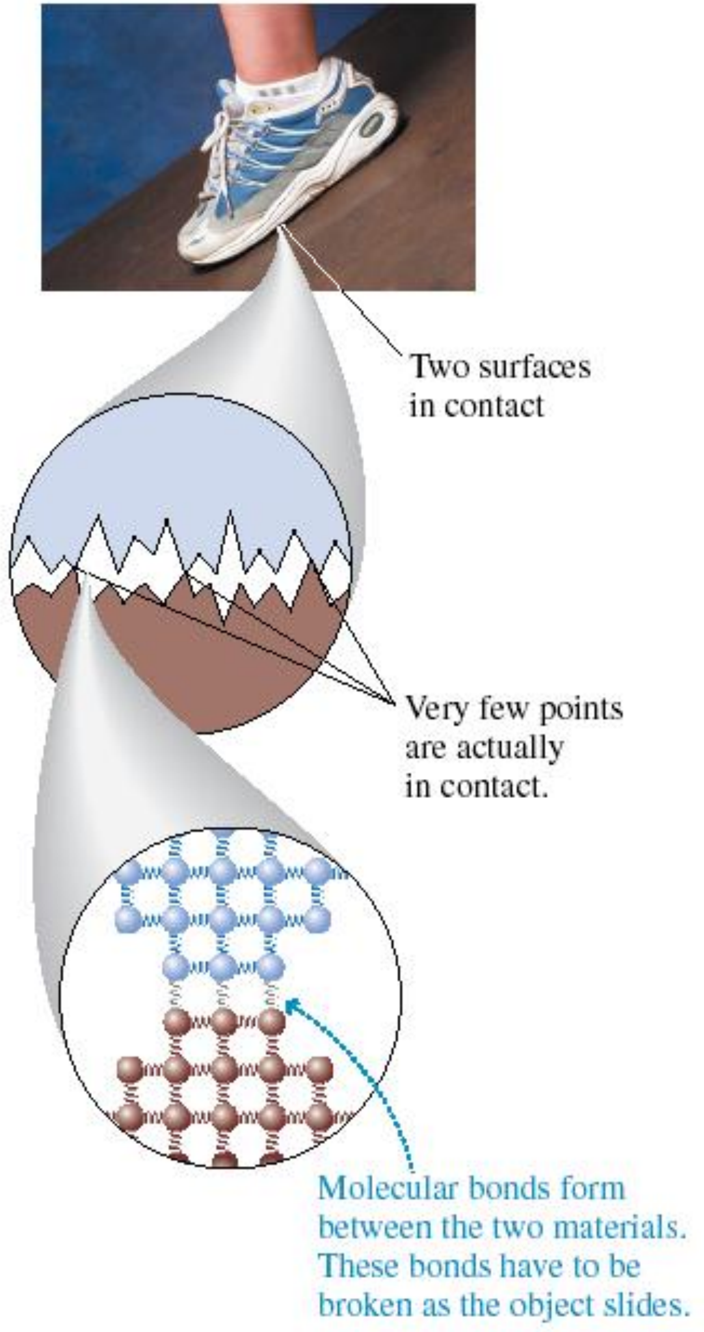




Why does friction exist?

Because at the microscopic level, ***nothing is smooth!***

FIGURE 6.19 An atomic-level view of friction.

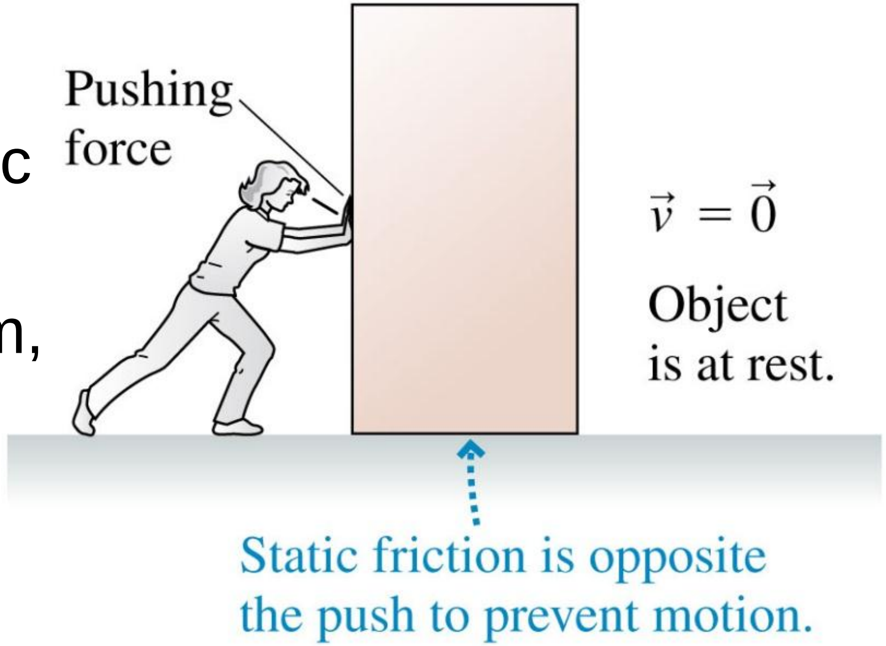


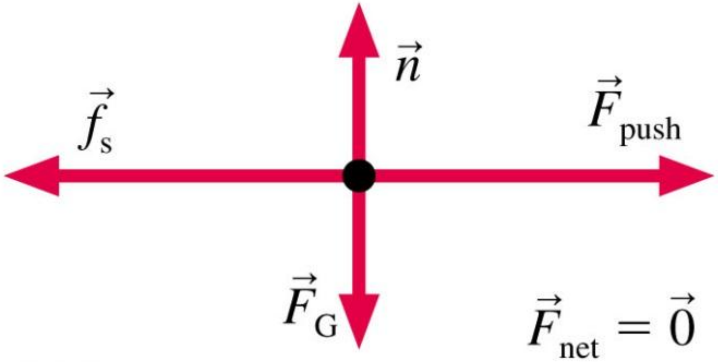
Static Friction

- The figure shows a person pushing on a box that, due to static friction, isn't moving
- Looking at the free-body diagram, the *x*-component of Newton's first law requires that the static friction force must exactly balance the pushing force:

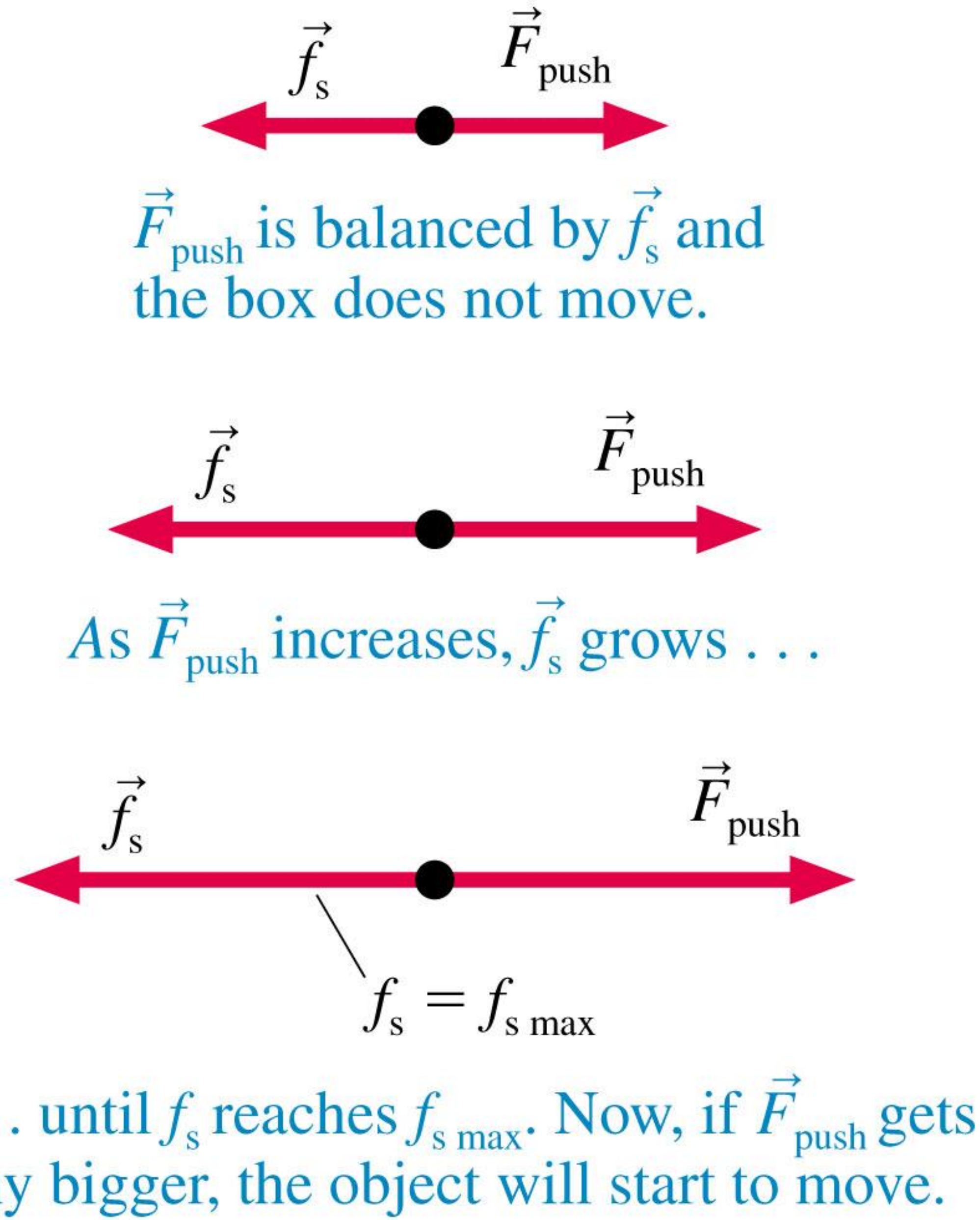
$$f_s = F_{\text{push}}$$

- $\vec{f}_s$ points in the direction *opposite* to the way the object would move if there were no static friction





Static friction acts in *response* to an applied force.



Maximum Static Friction

There's a limit to how big f_s can get. If you push hard enough, the object slips and starts to move. In other words, the static friction force has a *maximum* possible size $f_{s\,max}$.

- The two surfaces don't slip against each other as long as $f_s \leq f_{s\,max}$.
- A static friction force $f_s > f_{s\,max}$ is not physically possible. Many experiments have shown the following approximate relation usually holds:

$$f_{s\,max} = \mu_s n$$

where *n* is the magnitude of the normal force, and the proportionality constant μ_s is called the "coefficient of static friction".

"Kinetic Friction"

$$\vec{f}_k$$

- Also called "sliding friction"
- When two flat surfaces are in contact and sliding relative to one another, heat is created, so it slows down the motion (kinetic energy is being converted to thermal energy).
- Many experiments have shown the following approximate relation usually holds for the magnitude of f_k:

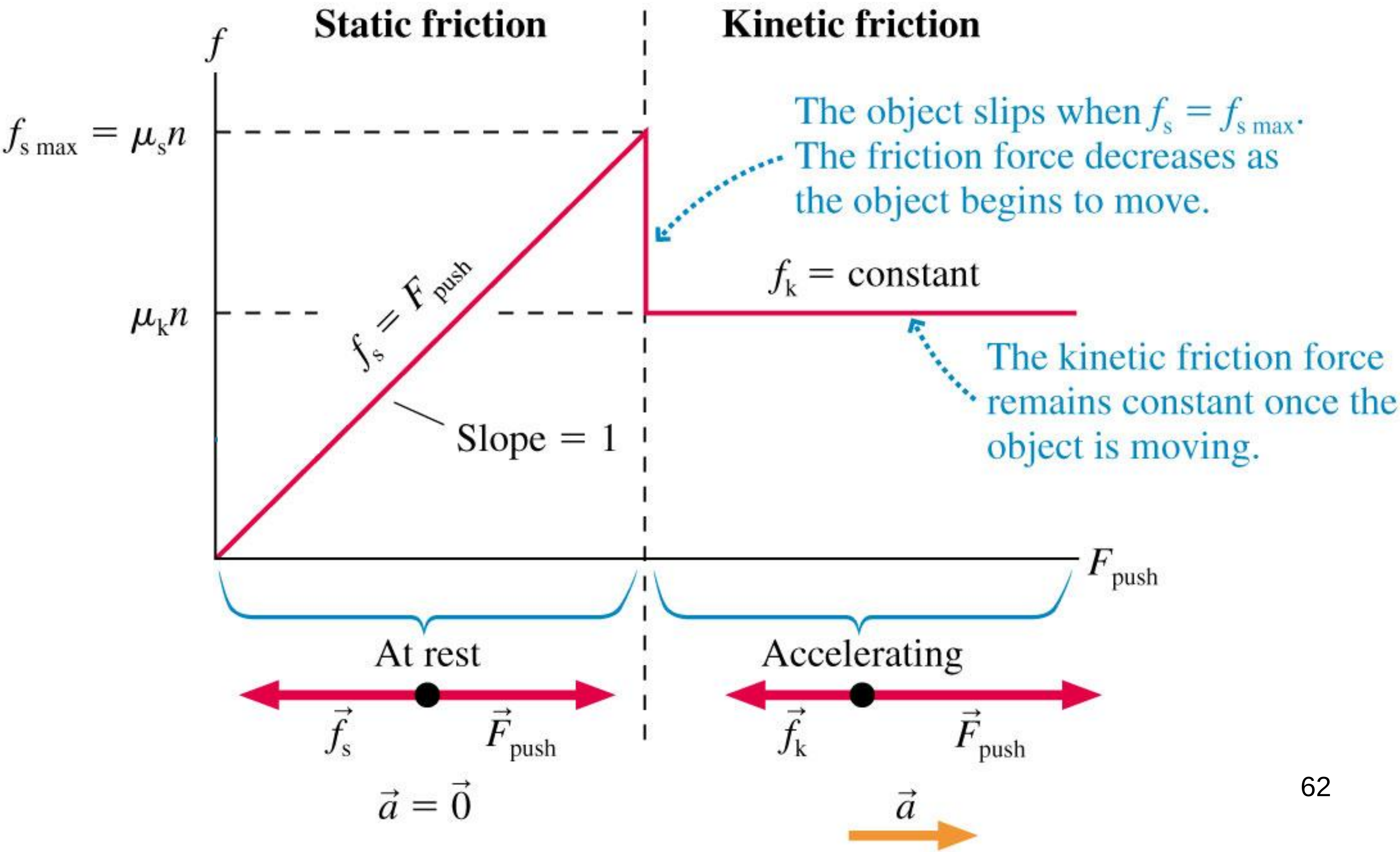
$$f_k = \mu_k n$$

where n is the magnitude of the normal force.

The direction of $\vec{f}_k$ is opposite the direction of motion.

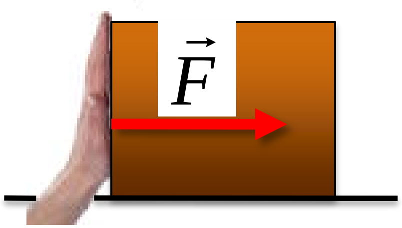
A Model of Friction

The friction force response to an increasing applied force.



A wooden block weighs 100 N, and is sitting stationary on a smooth horizontal concrete surface. The coefficient of static friction between wood and concrete is 0.2.

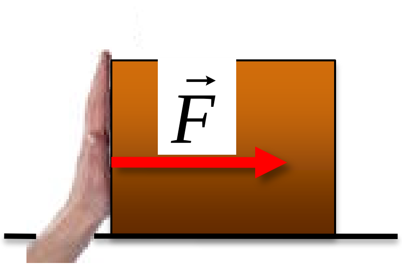
A 5 N horizontal force is applied to the block, pushing toward the right, but the block does not move. What is the force of static friction of the concrete on the block?



A. 100 N, to the left
B. 20 N, to the left
C. 5 N, to the left
D. 20 N, to the right
E. 5 N, to the right

A wooden block weighs 100 N, and is sitting stationary on a smooth horizontal concrete surface. The coefficient of static friction between wood and concrete is 0.2.

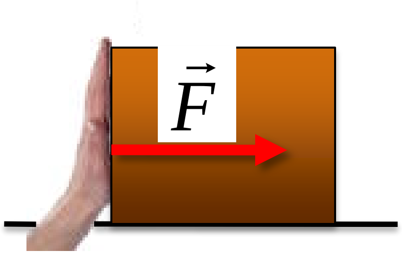
A 5 N horizontal force is applied to the block, pushing toward the right, but the block does not move. What is the force of static friction of the concrete on the block?



A. 100 N, to the left
B. 20 N, to the left
C. 5 N, to the left
D. 20 N, to the right
E. 5 N, to the right

A wooden block weighs 100 N, and is sitting stationary on a smooth horizontal concrete surface. The coefficient of static friction between wood and concrete is 0.2.

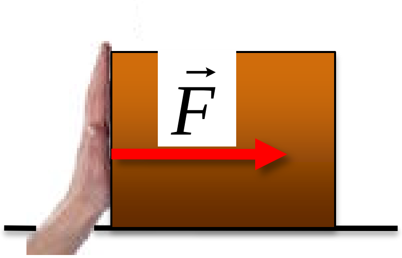
A horizontal force is applied to the block, pushing toward the right. What is the magnitude of the maximum pushing force you can apply and have the block remain stationary?



A. 200 N
B. 100 N
C. 20 N
D. 10 N
E. 5 N

A wooden block weighs 100 N, and is sitting stationary on a smooth horizontal concrete surface. The coefficient of static friction between wood and concrete is 0.2.

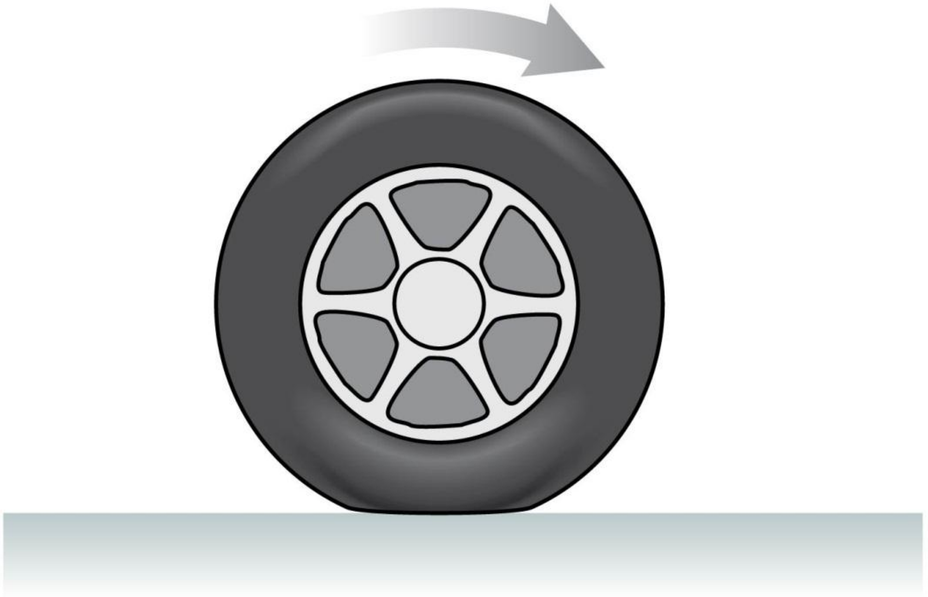
A horizontal force is applied to the block, pushing toward the right. What is the magnitude of the maximum pushing force you can apply and have the block remain stationary?



A. 200 N
B. 100 N
C. 20 N
D. 10 N
E. 5 N

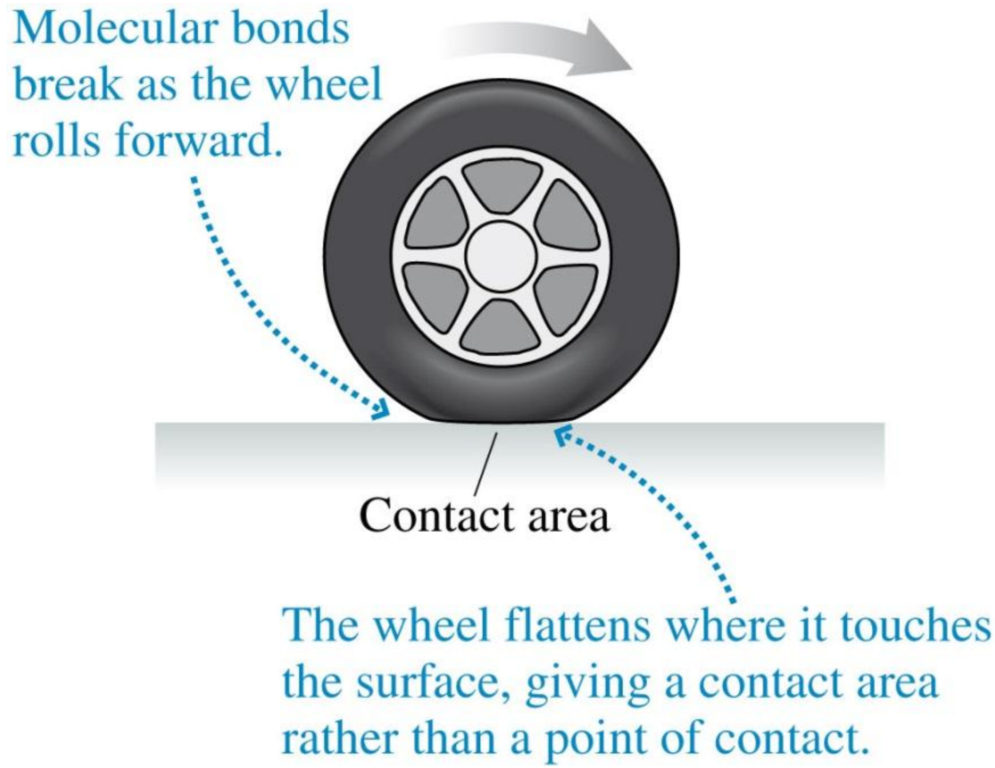
Rolling Motion

- If you slam on the brakes so hard that the car tires slide against the road surface, this is kinetic friction
- Under normal driving conditions, the portion of the rolling wheel that contacts the surface is *stationary*, not sliding



- If your car is accelerating or decelerating or turning, it is *static friction* of the road on the wheels that provides the net force which accelerates the car

Rolling Friction



- A car with no engine or brakes applied does not roll forever; it gradually slows down
- This is due to rolling friction

- The force of rolling friction can be calculated as

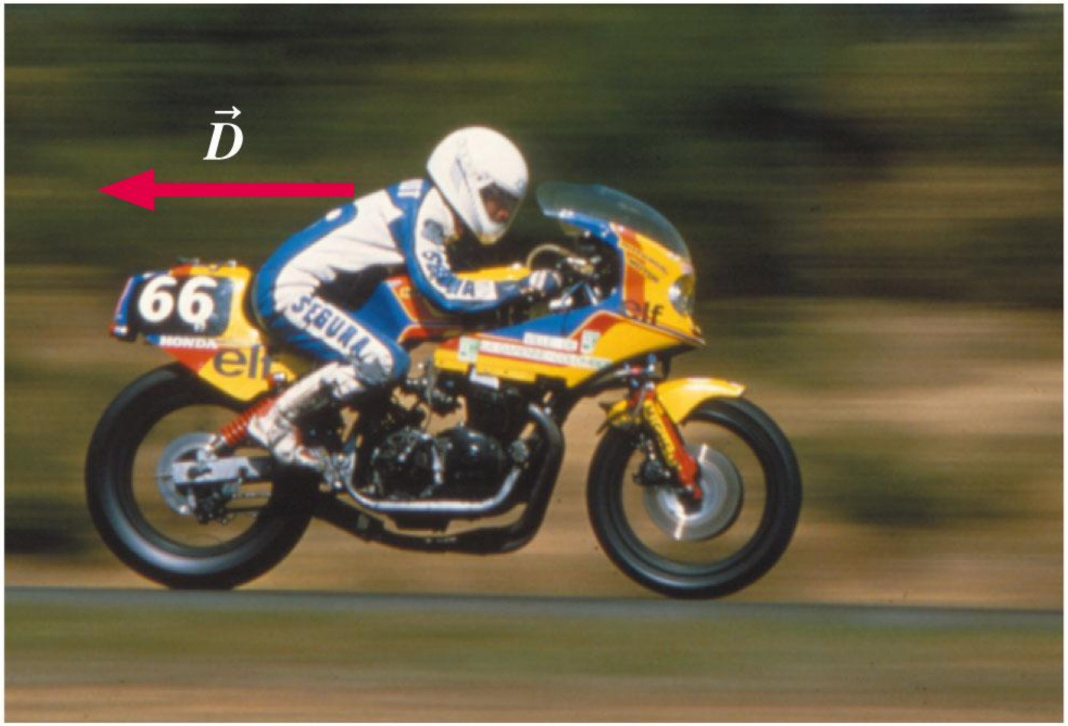
$$f_r = \mu_r n$$

where μ_r is called the coefficient of rolling friction.

- The rolling friction direction is opposite to the velocity of the rolling object relative to the surface

Drag

- The air exerts a drag force on objects as they move through the air
- Faster objects experience a greater drag force than slower objects
- The drag force on a high-speed motorcyclist is significant
- The drag force direction is opposite the object’s velocity



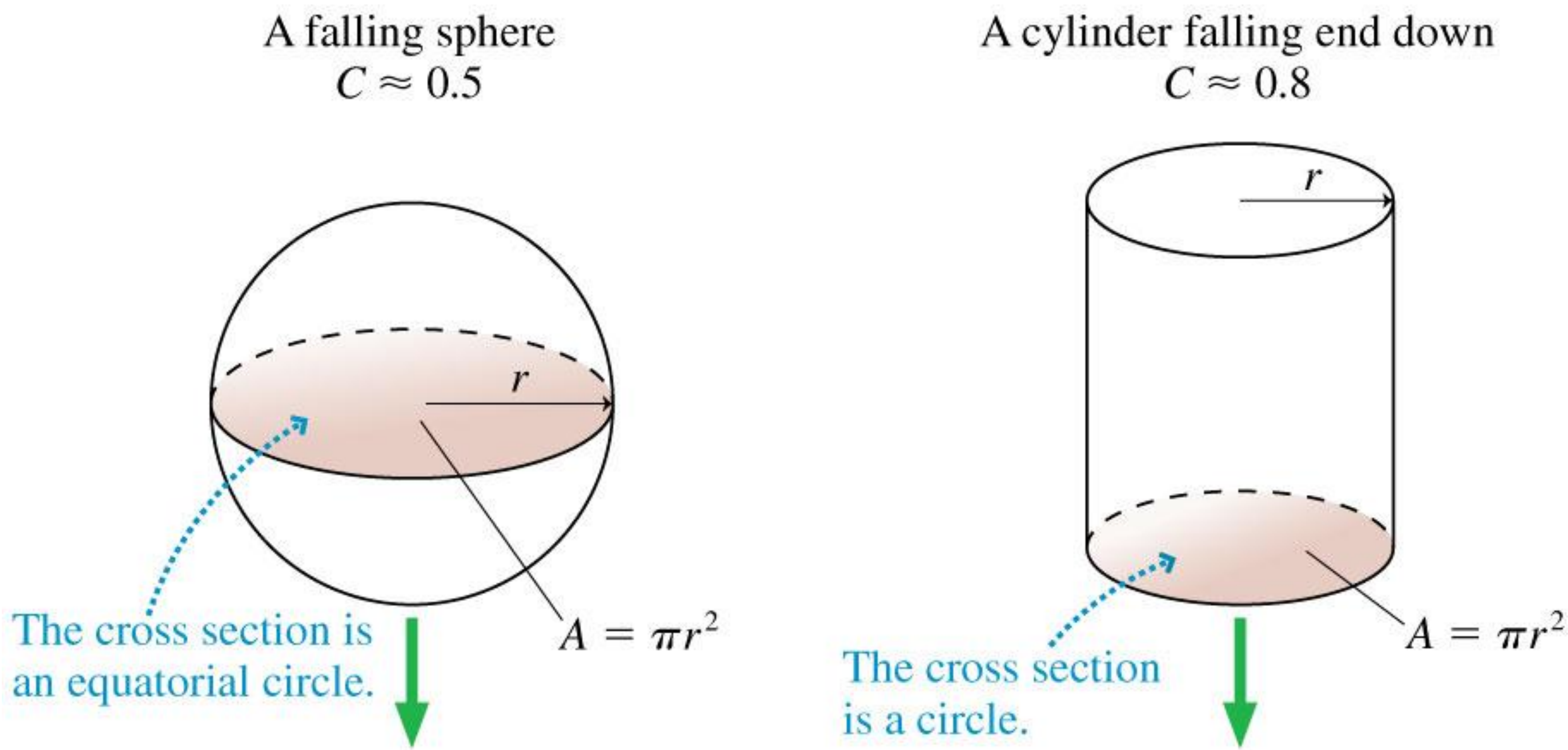
Drag

- For normal sized objects on earth traveling at a speed *v* which is less than a few hundred meters per second, air resistance can be modeled as:

$$\vec{D} = (\tfrac{1}{2}C\rho A v^2, \text{direction opposite the motion})$$

  - *A* is the *cross-section area* of the object
  - ρ is the density of the air, which is about 1.2 kg/m^3
  - *C* is the drag coefficient, which is a dimensionless number that depends on the shape of the object

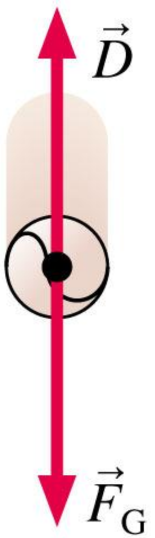
Cross Sectional Area depends on size, shape, and direction of motion.



…Consider the forces on a falling piece of paper, crumpled and not crumpled.

Terminal Speed

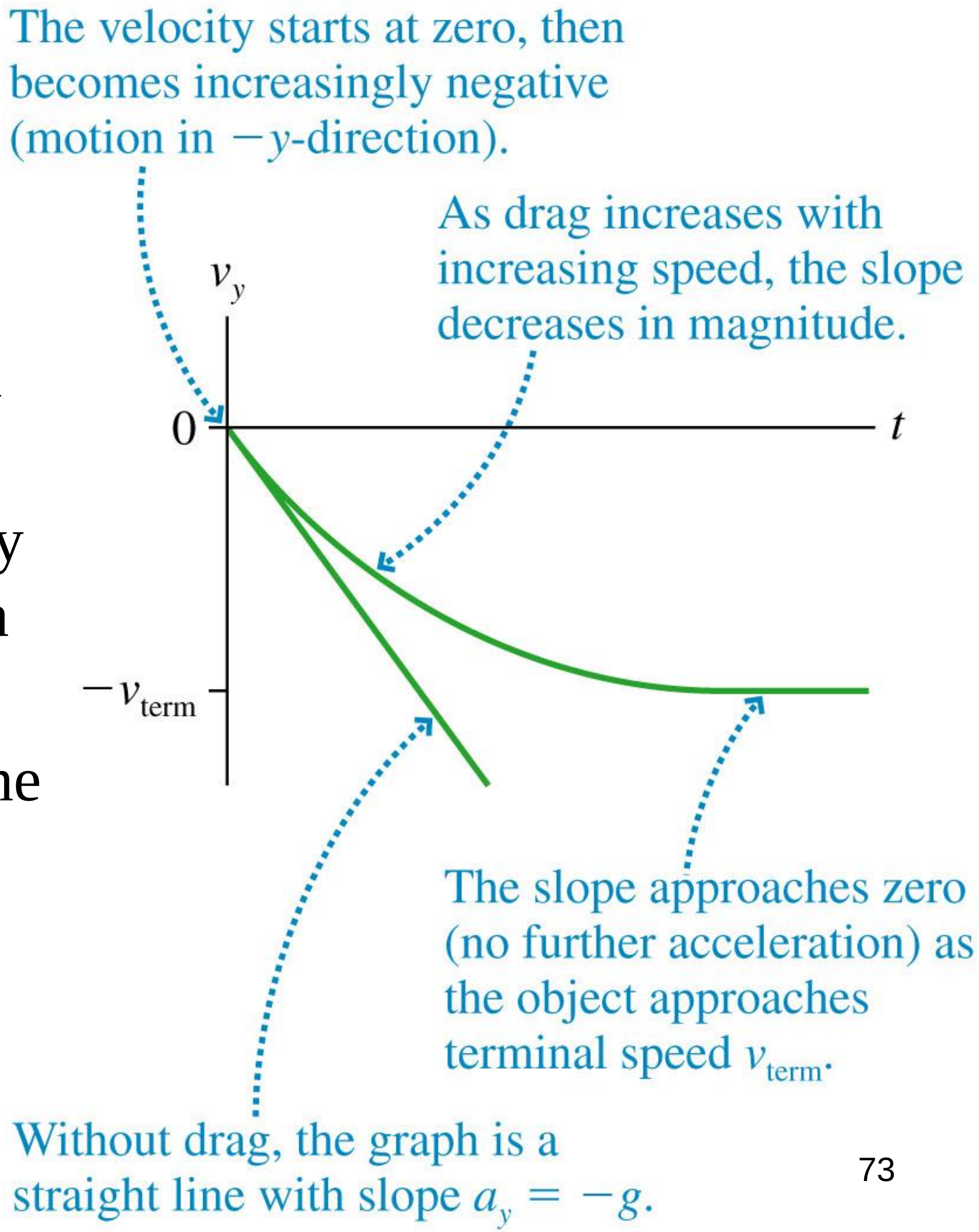
- The drag force from the air increases as an object falls and gains speed
- If the object falls far enough, it will eventually reach a speed at which $D = F_G$
- At this speed, the net force is zero, so the object falls at a *constant* speed, called the terminal speed v_{term}



$$v_{term} = \sqrt{\frac{2mg}{C\rho A}}$$

Terminal Speed

- The figure shows the velocity-versus-time graph of a falling object with and without drag
- Without drag, the velocity graph is a straight line with $a_y = -g$
- When drag is included, the vertical component of the velocity asymptotically approaches $-v_{term}$



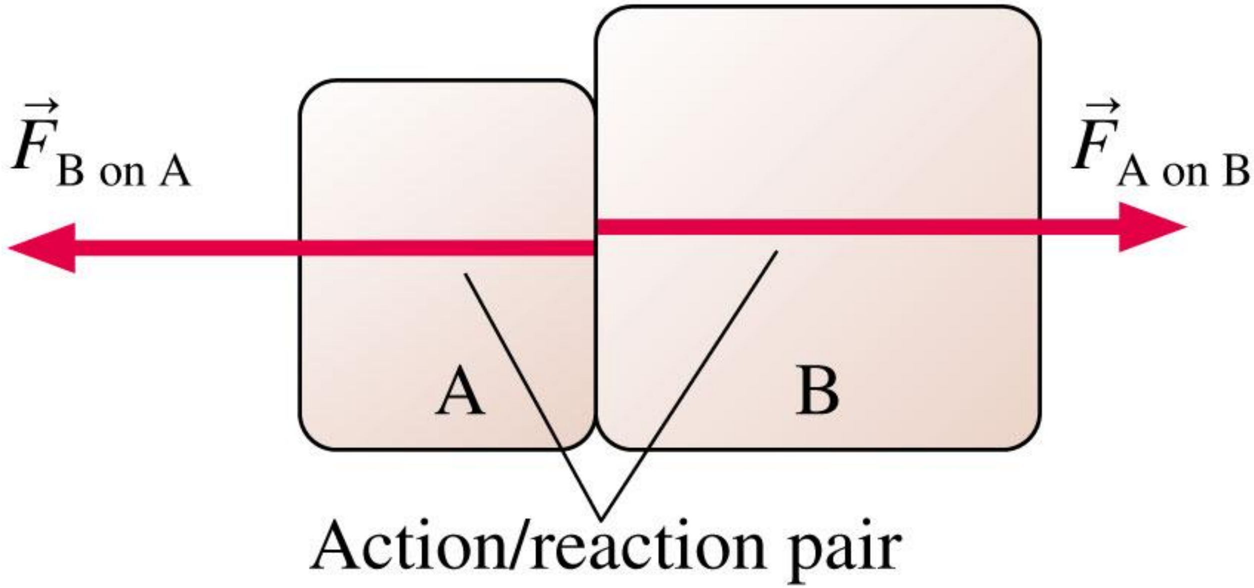
Propulsion

- If you try to walk across a frictionless floor, your foot slips and slides *backward*
- In order to walk, your foot must *stick* to the floor as you straighten your leg, moving your body forward
- The force that prevents slipping is *static friction*
- The static friction force points in the *forward* direction
- It is static friction that propels you forward!

What force causes this sprinter to accelerate?

Interacting Objects

- If object A exerts a force on object B, then object B exerts a force on object A.
- The pair of forces, as shown, is called an **action/reaction pair**.



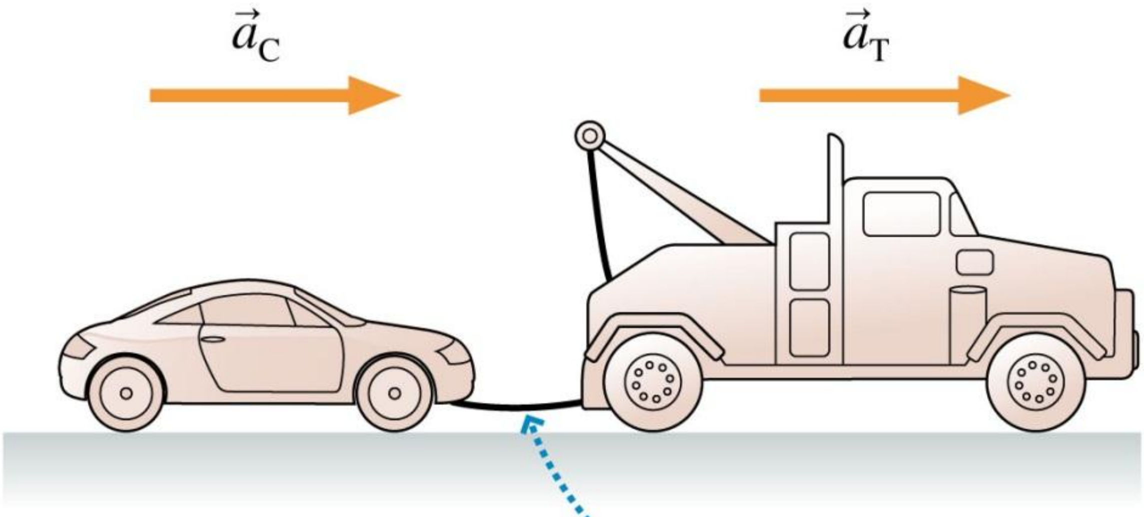
N3 Newton's Third Law

If object 1 acts on object 2 with a force, then object 2 acts on object 1 with an equal force in the opposite direction.

$$\vec{F}_{1\,\mathrm{on}\,2} = -\vec{F}_{2\,\mathrm{on}\,1}$$

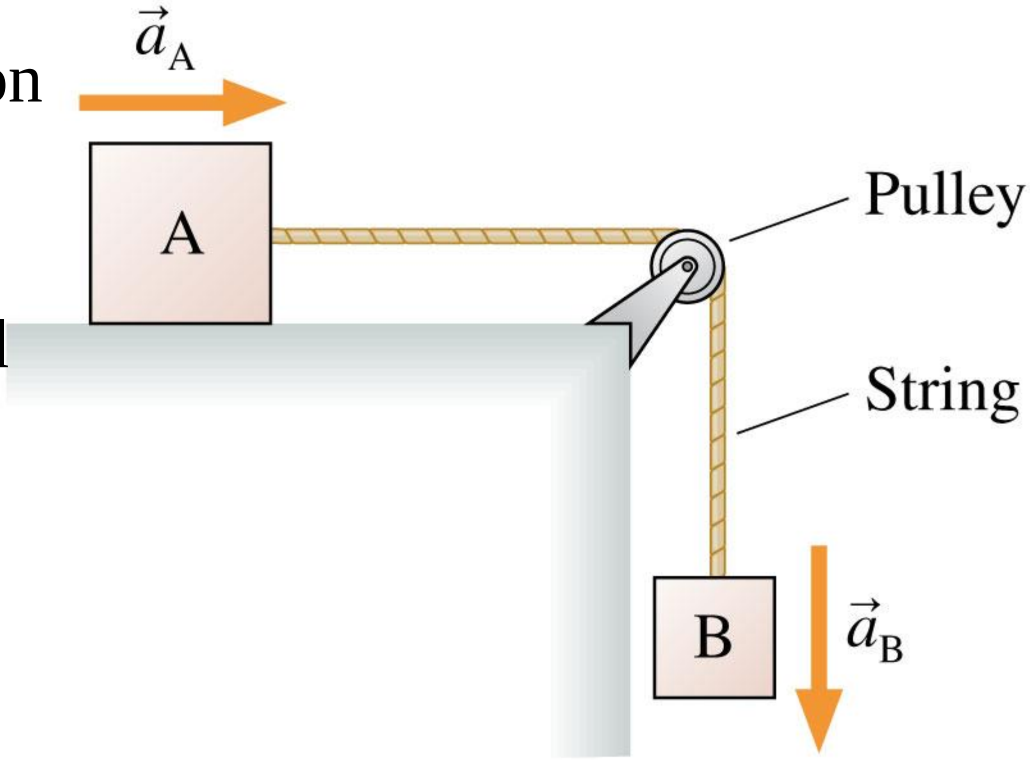
Acceleration Constraints

- If two objects A and B move together, their accelerations are *constrained* to be equal: $a_A = a_B$
- This equation is called an **acceleration constraint**
- Consider a car being towed by a truck
- In this case, the acceleration constraint is $a_{Cx} = a_{Tx} = a_x$
- Because the accelerations of both objects are equal, we can drop the subscripts C and T and call both of them a_x



Acceleration Constraints

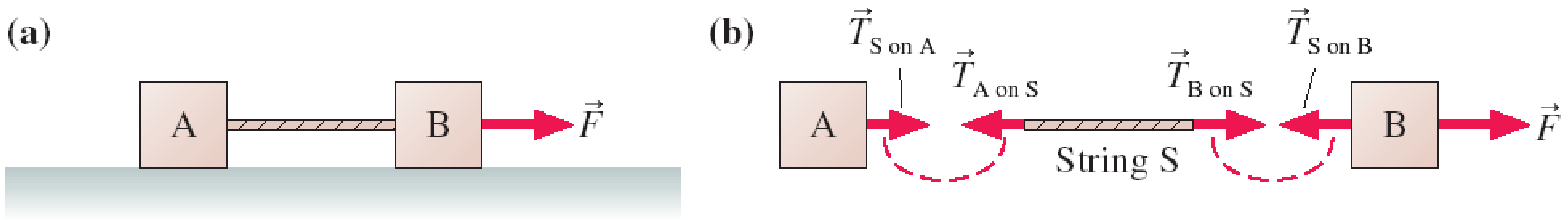
- Sometimes the acceleration of A and B may have different signs
- Consider the blocks A and B in the figure
- The string constrains the two objects to accelerate together



- But, as A moves to the right in the $+x$ direction, B moves down in the $-y$ direction
- In this case, the acceleration constraint is $a_{Ax} = -a_{By}$

The Massless String Approximation

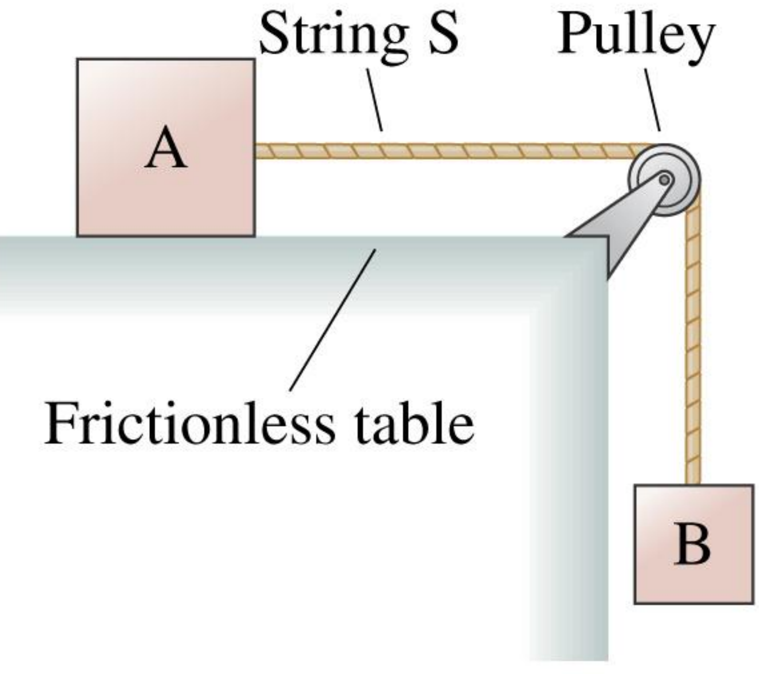
FIGURE 7.22 The string's tension pulls forward on block A, backward on block B.



Often in physics problems the mass of the string or rope is much less than the masses of the objects that it connects. In such cases, we can adopt the following **massless string approximation**:

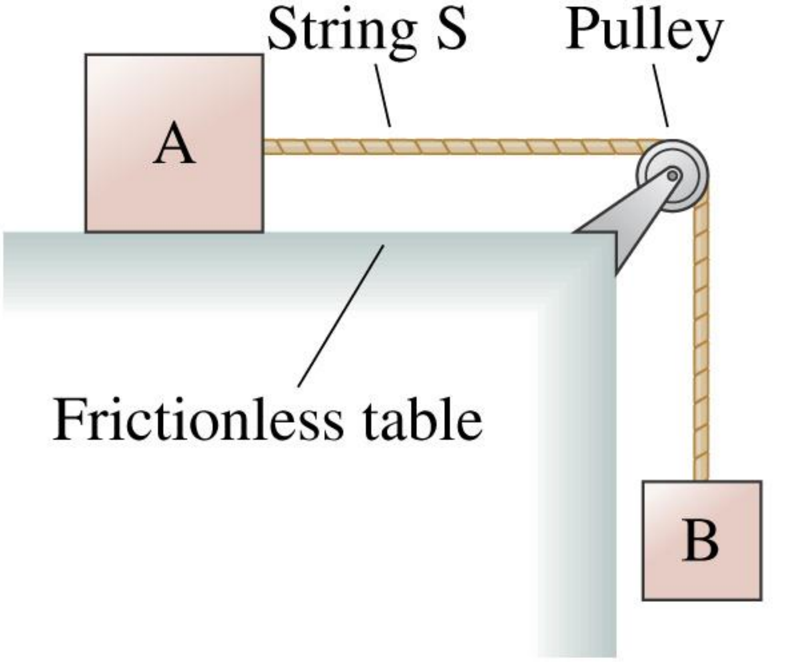
$$T_{\text{B on S}} = T_{\text{A on S}} \qquad \text{(massless string approximation)}$$

In the figure to the right, is the tension in the string greater than, less than, or equal to the force of gravity on block B?



A. Equal to
B. Greater than
C. Less than

In the figure to the right, is the tension in the string greater than, less than, or equal to the force of gravity on block B?

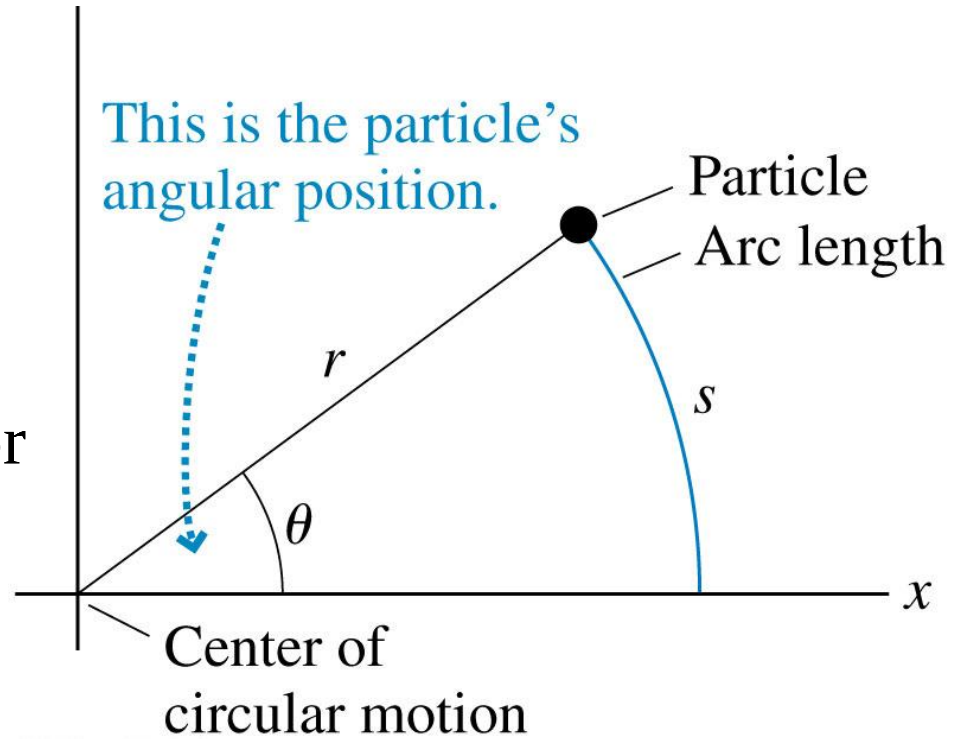


A. Equal to
B. Greater than
C. Less than

Motion on a Circular Path

- Consider a particle at a distance r from the origin, at an angle θ from the positive x axis
- The angle may be measured in degrees, revolutions (rev) or **radians** (rad), that are related by:

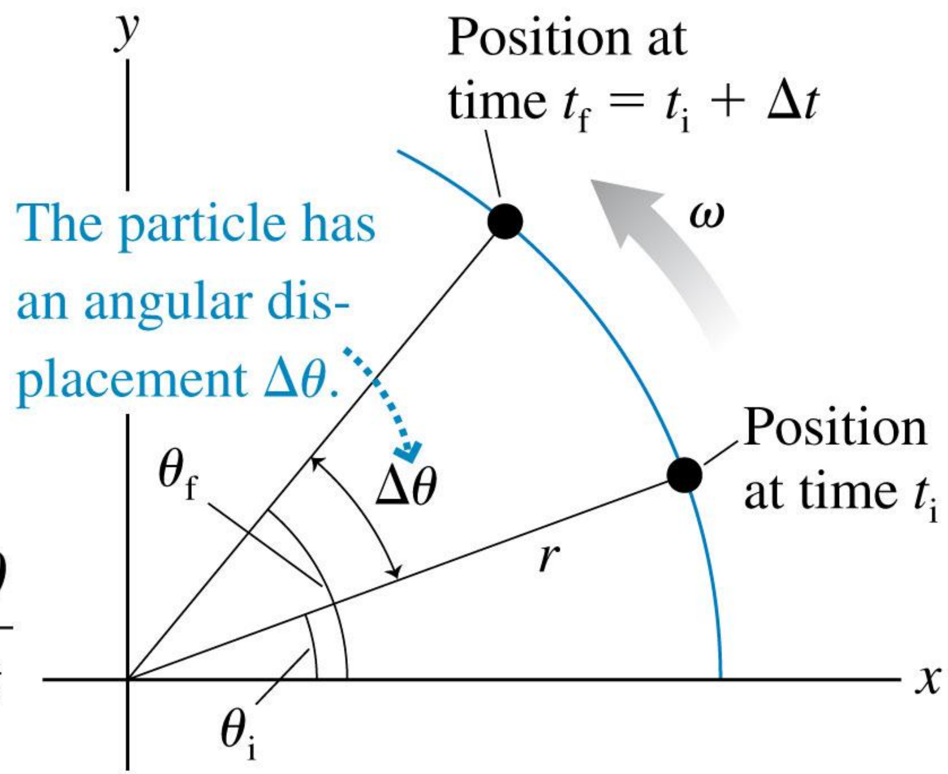
$$1 \text{ rev} = 360° = 2\pi \text{ rad}$$



- If the angle is measured in radians, then there is a simple relation between θ and the **arc length** s that the particle travels along the edge of a circle of radius r:

$$s = r\theta \qquad \text{(with } \theta \text{ in rad)}$$

Angular Velocity



$$\text{average angular velocity} \equiv \frac{\Delta\theta}{\Delta t}$$

- As the time interval Δt becomes very small, we arrive at the definition of instantaneous **angular velocity**

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad \text{(angular velocity)}$$

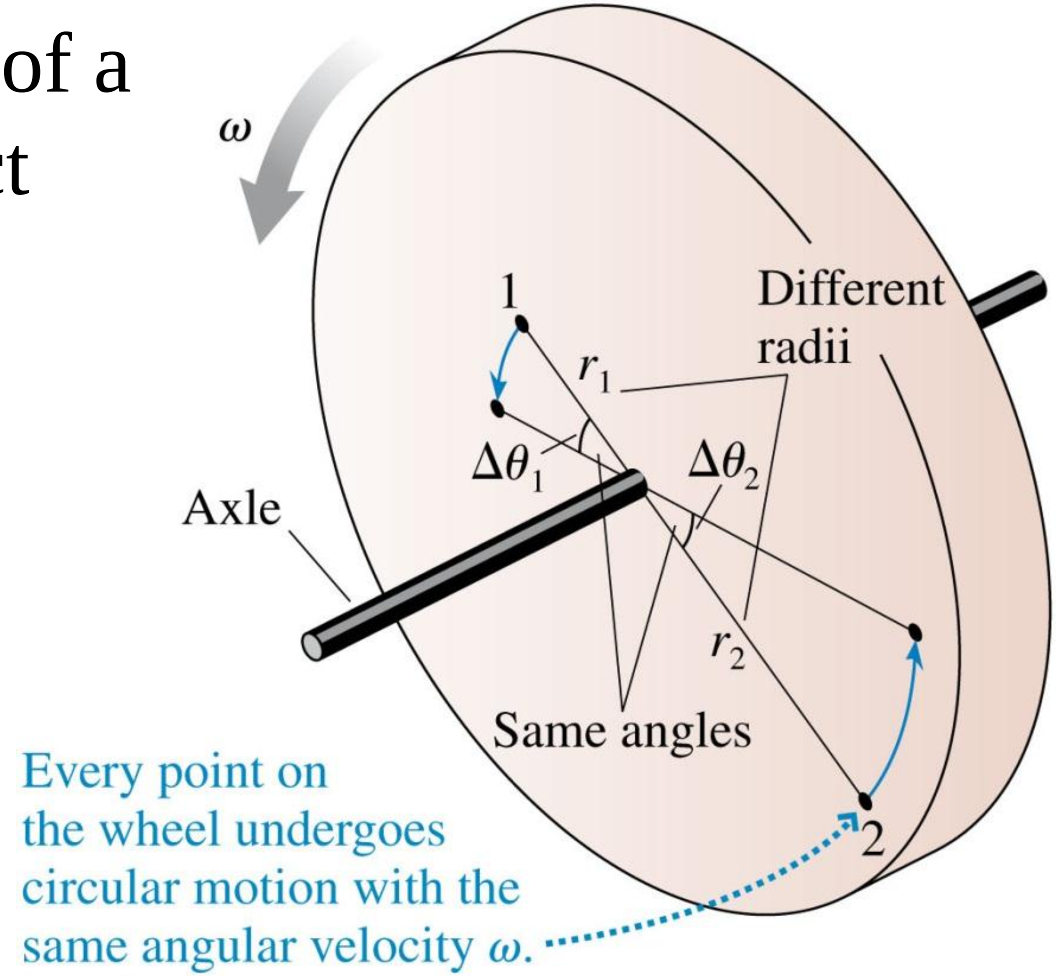
Angular Velocity in Uniform Circular Motion

- When angular velocity ω is constant, this is uniform circular motion
- In this case, as the particle goes around a circle one time, its angular displacement is $\Delta\theta = 2\pi$ during one period $\Delta t = T$
- The absolute value of the constant angular velocity is related to the period of the motion by

$$|\omega| = \frac{2\pi \text{ rad}}{T} \quad \text{or} \quad T = \frac{2\pi \text{ rad}}{|\omega|}$$

Angular Velocity of a Rotating Object

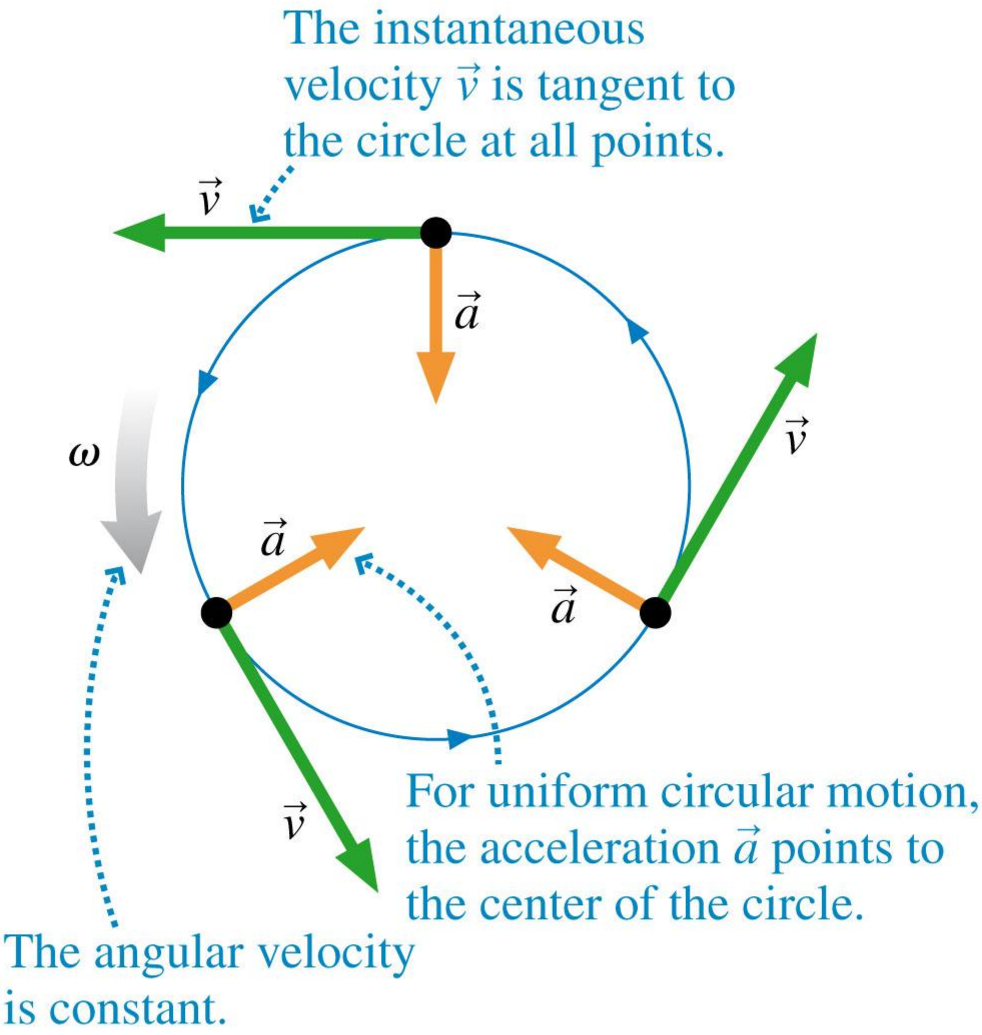
- The figure shows a wheel rotating on an axle
- Points 1 and 2 turn through the *same angle* as the wheel rotates
- That is, $\Delta\theta_1 = \Delta\theta_2$ during some time interval Δt
- Therefore $\omega_1 = \omega_2 = \omega$



- All points on the wheel rotate with the same angular velocity
- We can refer to ω as the angular velocity of the wheel

Tangential Velocity

- The tangential velocity component v_t is the rate ds/dt at which the particle moves around the circle, where s is the arc length
- The tangential velocity and the angular velocity are related by

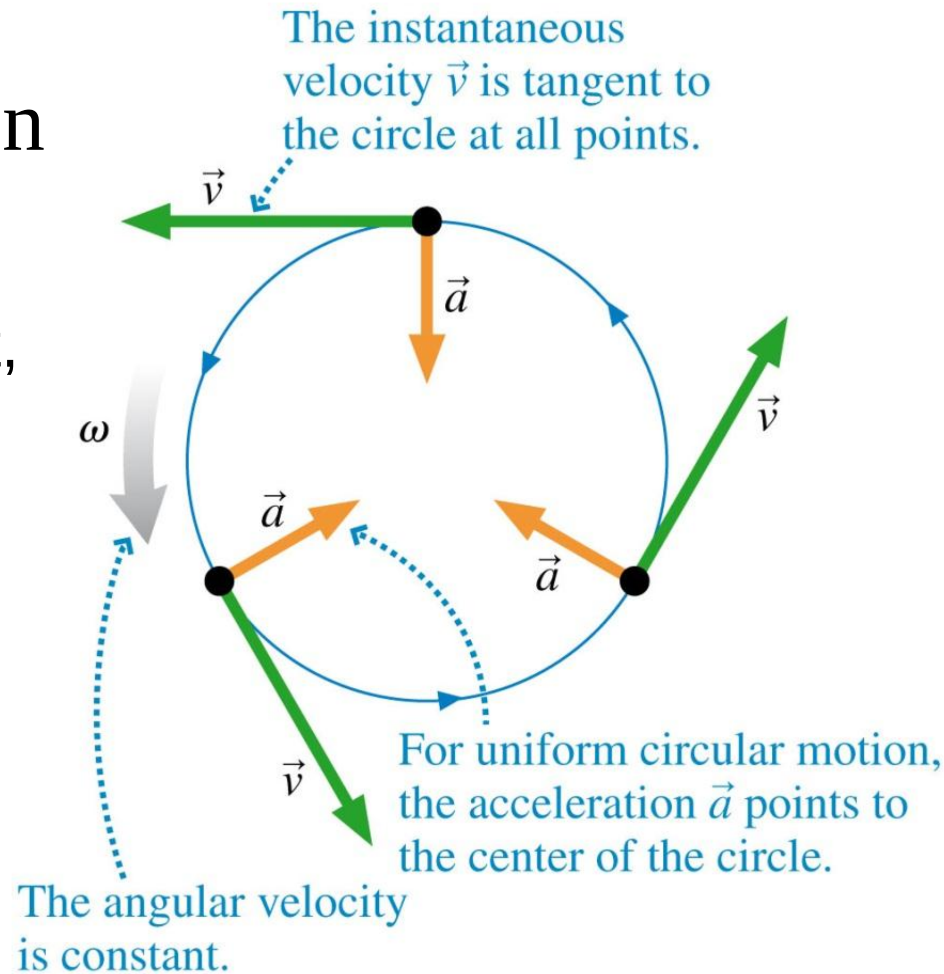


$$v_t = \omega r \qquad \text{(with } \omega \text{ in rad/s)}$$

- In this equation, the units of v_t are m/s, the units of ω are rad/s, and the units of r are m

Centripetal Acceleration

- In uniform circular motion, although the speed is constant, there is an acceleration because the *direction* of the velocity vector is always changing
- The acceleration of uniform circular motion is called **centripetal acceleration**



- The direction of the centripetal acceleration is toward the center of the circle:

$$\vec{a} = \left(\frac{v^2}{r}, \text{toward center of circle}\right) \qquad \text{(centripetal acceleration)}$$

Dynamics of Uniform Circular Motion

- An object in uniform circular motion is *not* traveling at a constant velocity in a straight line
- Consequently, the particle must have a net force acting on it

$$\vec{F}_{\text{net}} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle}\right)$$

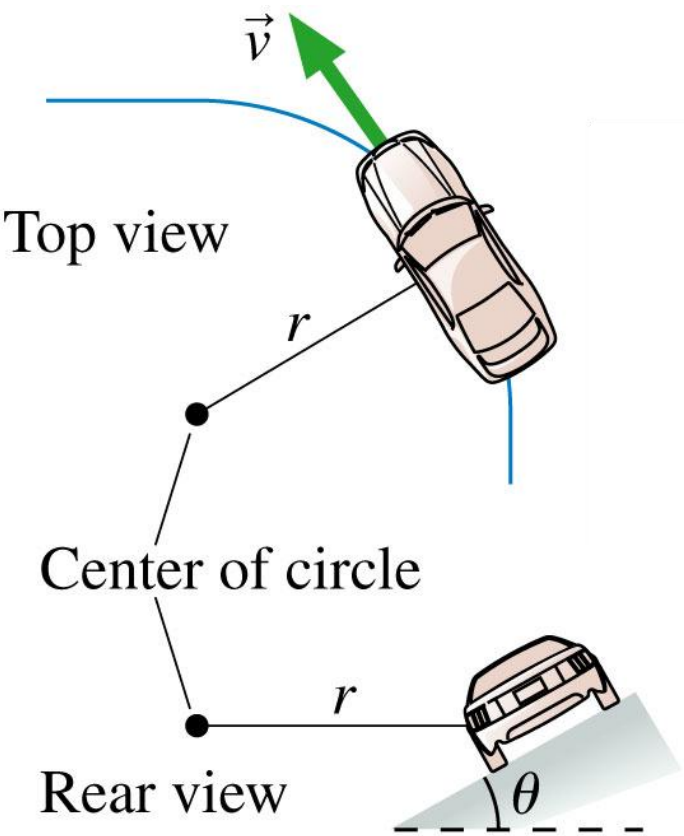
- Without such a force, the object would move off in a straight line tangent to the circle
- The car would end up in the ditch!

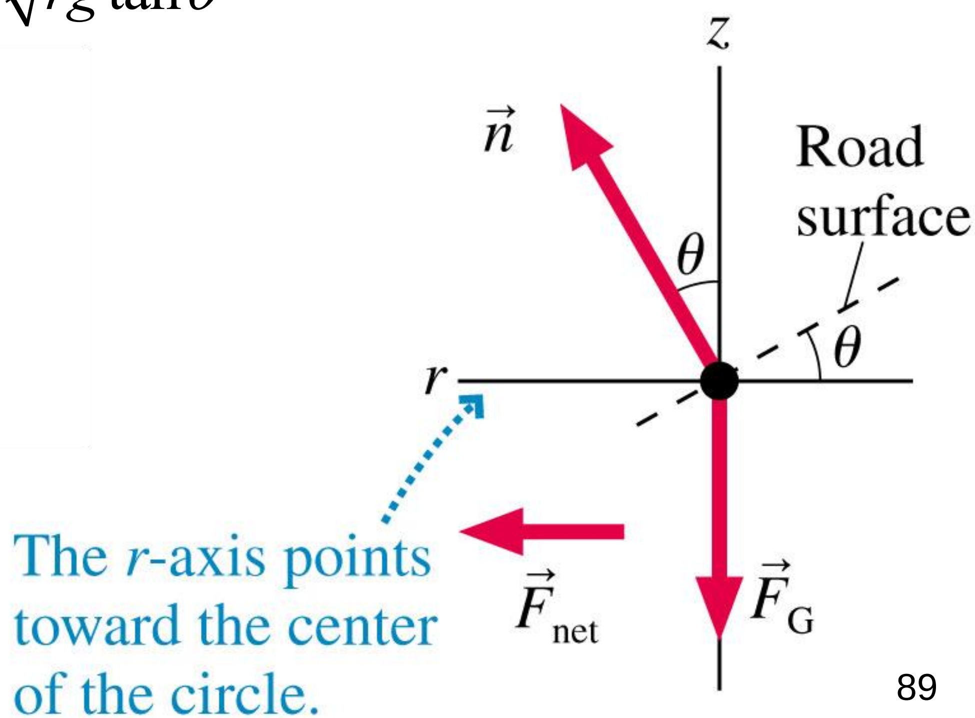
Highway and racetrack curves are banked to allow the normal force of the road to provide the centripetal acceleration of the turn.

Banked Curves

- Real highway curves are *banked* by being tilted up at the outside edge of the curve
- The radial component of the normal force can provide centripetal acceleration needed to turn the car
- For a curve of radius r banked at an angle θ, the exact speed at which a car must take the curve without assistance from friction is

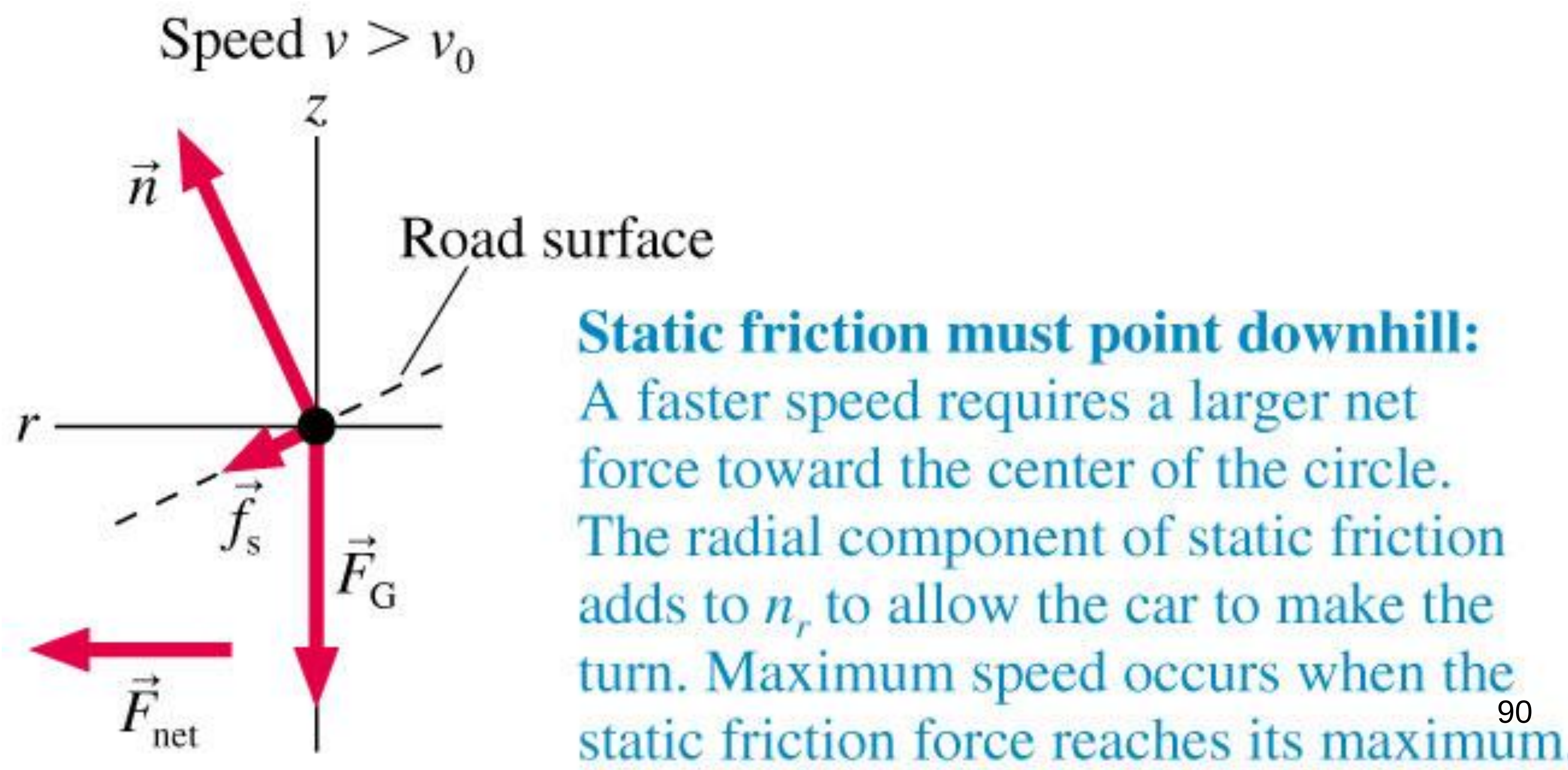
$$v_0 = \sqrt{rg\tan\theta}$$





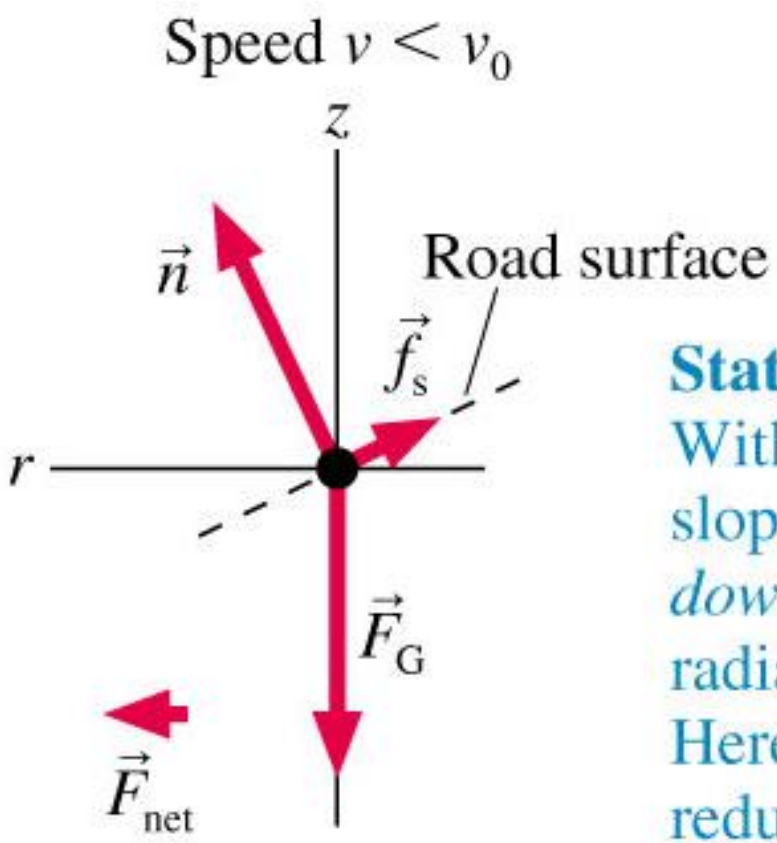
Banked Curves

- Consider a car going around a banked curve at a speed *higher* than $v_0 = \sqrt{rg\tan\theta}$
- In this case, static friction must prevent the car from slipping *up* the hill



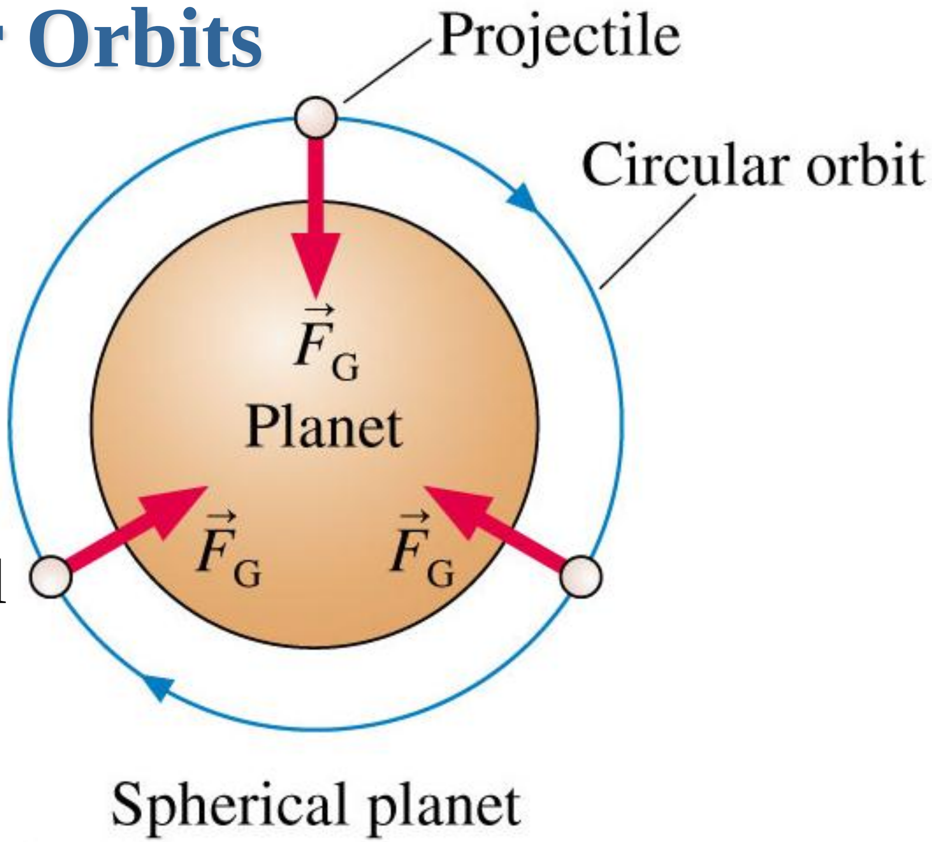
Banked Curves

- Consider a car going around a banked curve at a speed *slower* than $v_0 = \sqrt{rg \tan\theta}$
- In this case, static friction must prevent the car from slipping *down* the hill



Static friction must point uphill: Without a static friction force *up* the slope, a slow-moving car would slide *down* the incline! Further, n_r is too much radial force for circular motion at $v < v_0$. Here the radial component of static friction reduces the net radial force.

Circular Orbits



- An object in a low circular orbit has acceleration:

$$\vec{a} = \frac{\vec{F}_{net}}{m} = (g, \text{toward center})$$

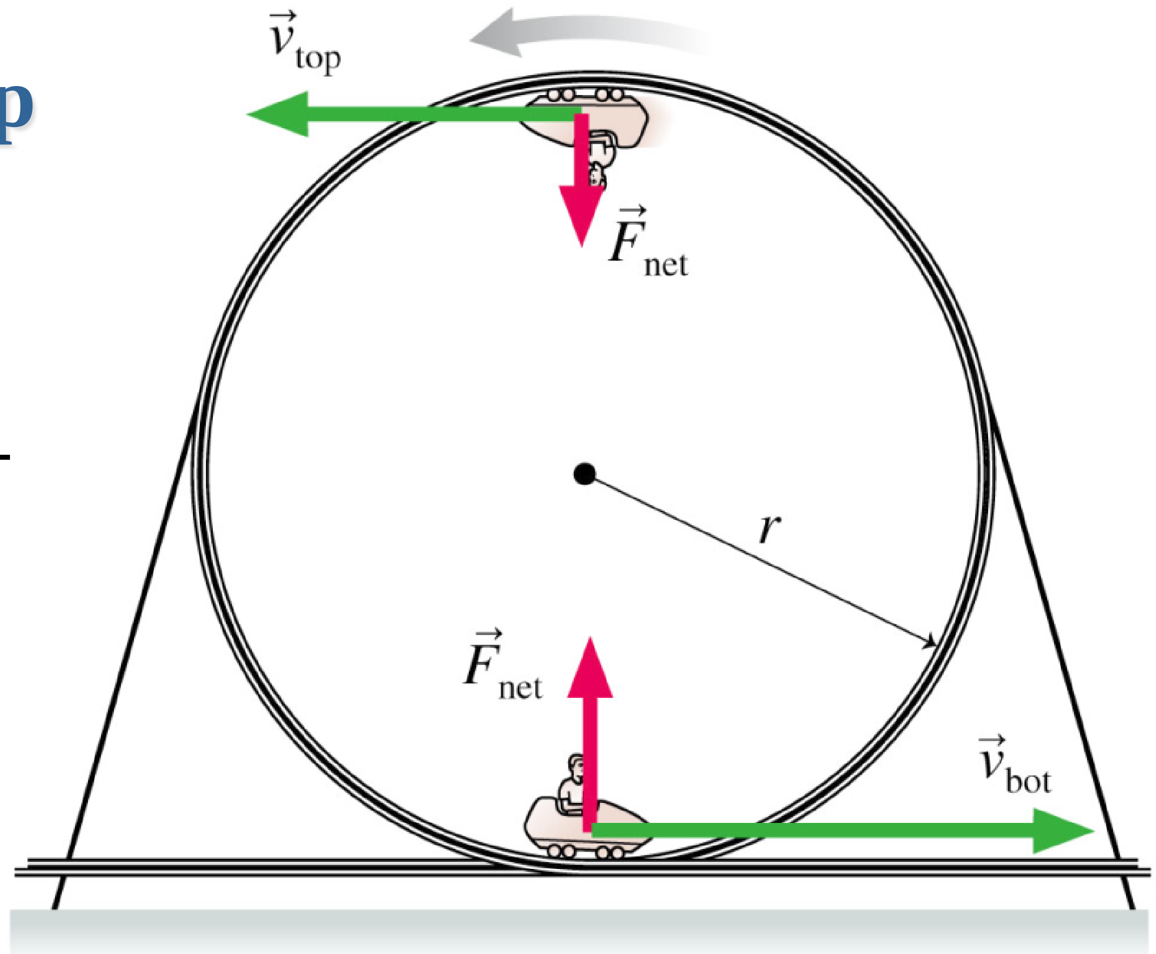
- If the object moves in a circle of radius r at speed v_{orbit} the centripetal acceleration is:

$$a_r = \frac{(v_{orbit})^2}{r} = g$$

- The required speed for a circular orbit near a planet's surface, neglecting air resistance, is:

$$v_{orbit} = \sqrt{rg}$$

Loop-the-loop

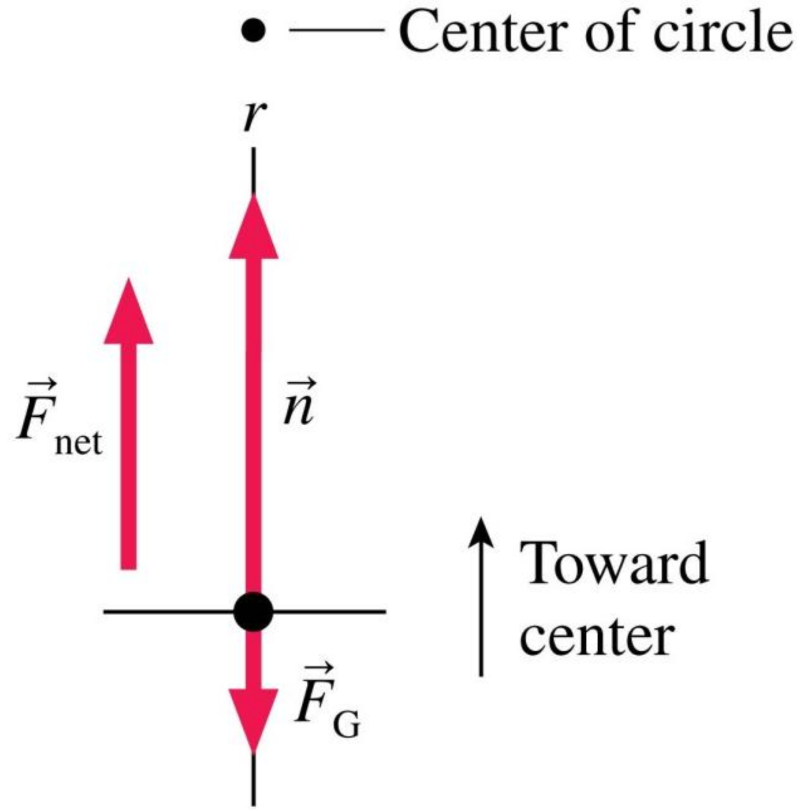


- The figure shows a roller-coaster going around a vertical loop-the-loop of radius *r*

- Because the car is moving in a circle, there must be a net force toward the center of the circle

Loop-the-loop

- The figure shows the roller-coaster free body diagram at the *bottom* of the loop
- Since the net force is toward the center (upward at this point) $n > F_G$
- This is why you "feel heavy" at the bottom of the valley on a roller coaster

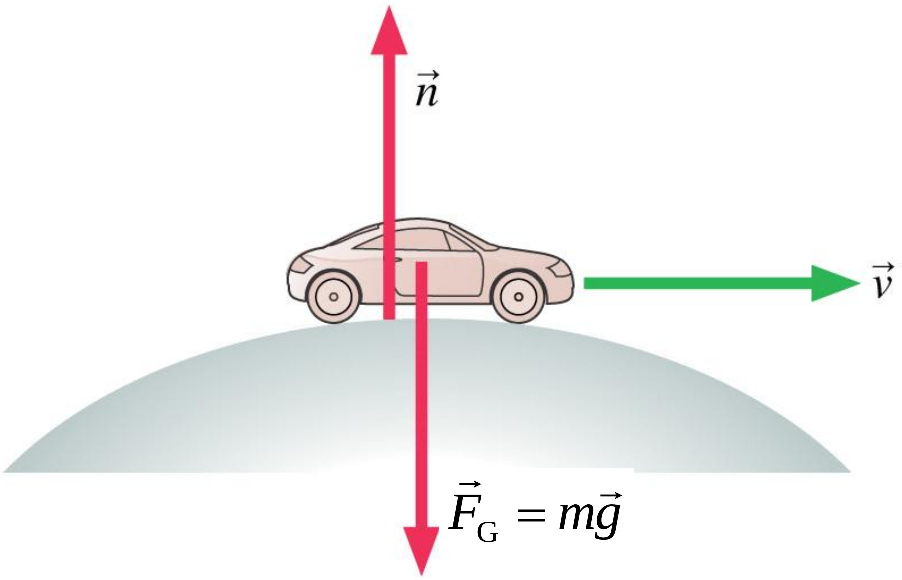


$$\sum F_r = n_r + (F_G)_r = n - mg = ma_r = \frac{m(v_{\text{bot}})^2}{r}$$

$$n = mg + \frac{m(v_{\text{bot}})^2}{r}$$

- The normal force at the bottom is *larger* than *mg*

A car is rolling over the top of a hill at speed *v*. At this instant,



A. $n > F_G$.
B. $n < F_G$.
C. $n = F_G$.
D. We can't tell about n without knowing v.

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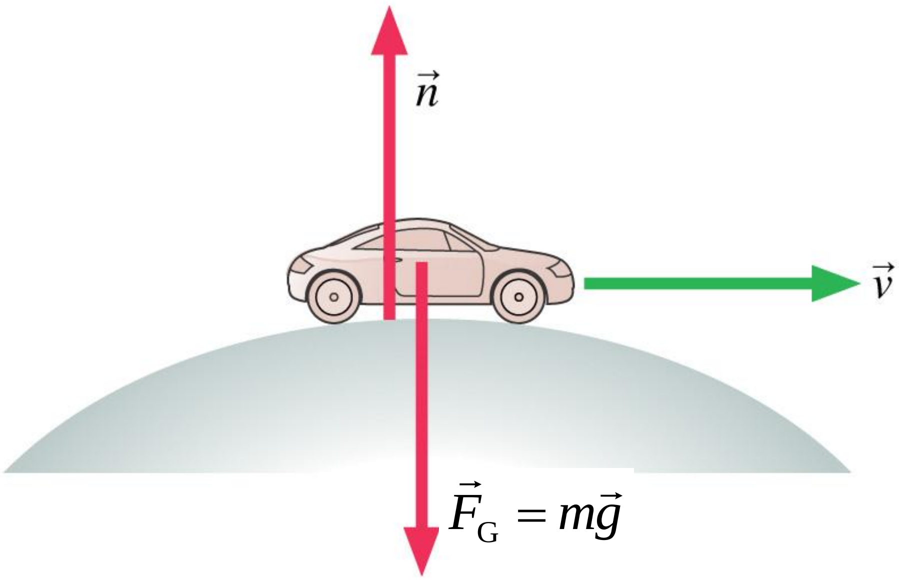


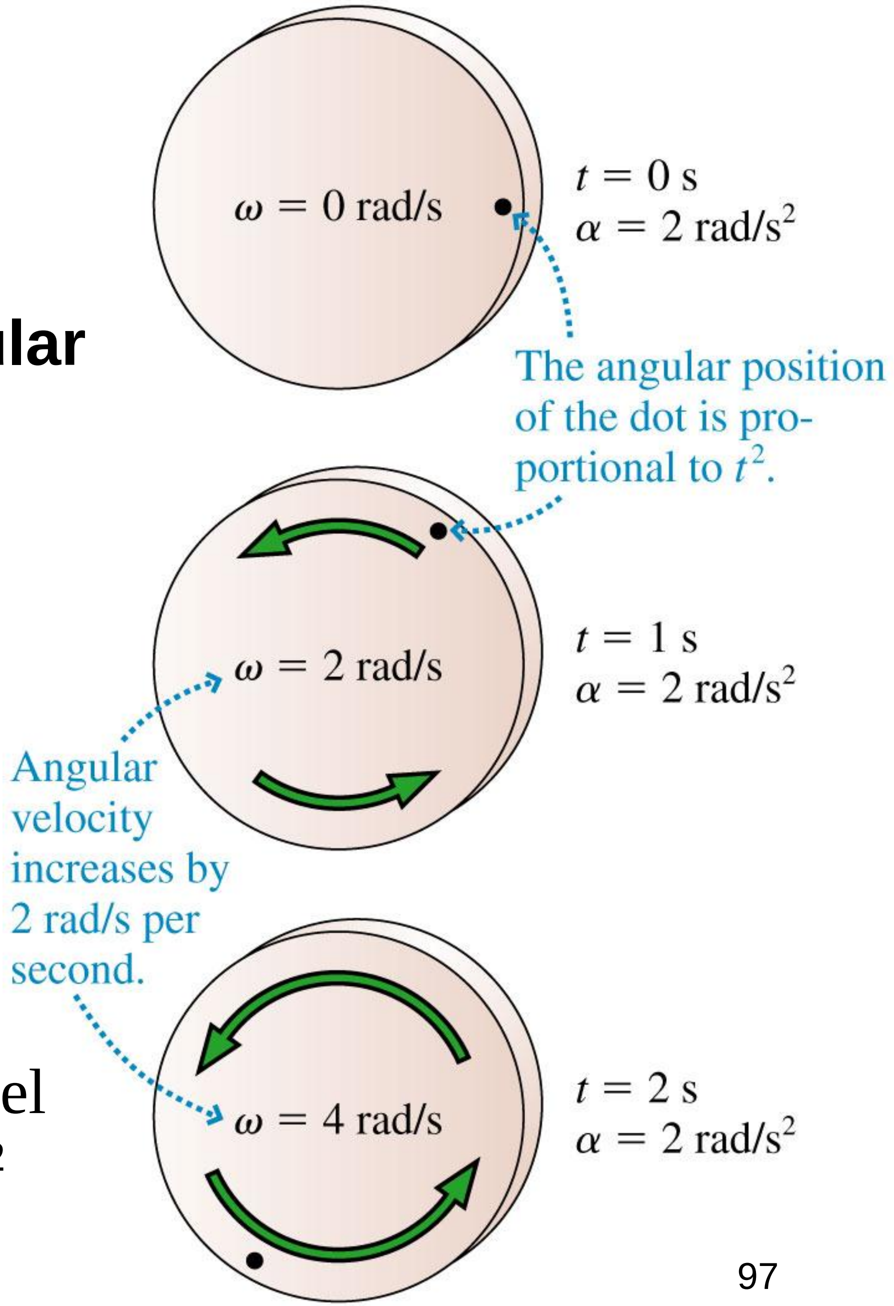
A. $n > F_G$.
B. $n < F_G$.
C. $n = F_G$.
D. We can't tell about n without knowing v.

Angular Acceleration

- Suppose a wheel's rotation is speeding up or slowing down
- This is called **nonuniform circular motion**
- We can define the *angular acceleration* as

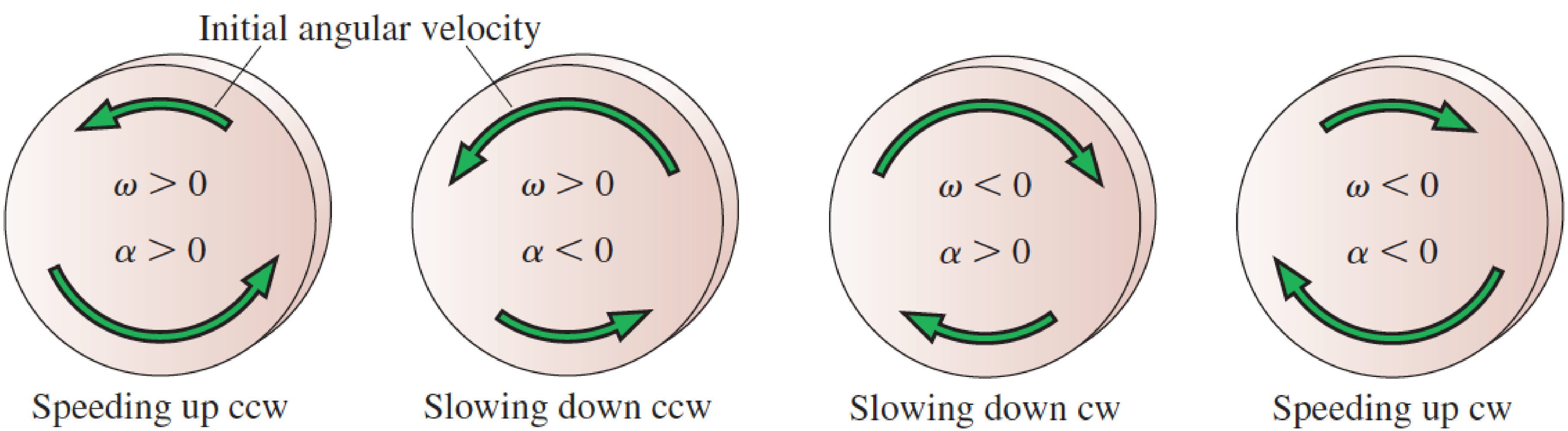
$$\alpha \equiv \frac{d\omega}{dt} \qquad \text{(angular acceleration)}$$

- The units of α are rad/s^2
- The figure to the right shows a wheel with angular acceleration $\alpha = 2$ rad/s^2



The Sign of Angular Acceleration

- If ω is counter-clockwise and $|\omega|$ is increasing, then α is positive
- If ω is counter-clockwise and $|\omega|$ is decreasing, then α is negative
- If ω is clockwise and $|\omega|$ is decreasing, then α is positive
- If ω is clockwise and $|\omega|$ is increasing, then α is negative



Angular Kinematics

- The same relations that hold for linear motion between a_x, v_x and x apply analogously to rotational motion for α, ω and θ
- There is a graphical relationship between α and ω:

α = slope of the ω-versus-t graph at time t

$\omega_f = \omega_i$ + area under the α-versus-t curve between t_i and t_f

- The table shows a comparison of the rotational and linear kinematics equations for constant α or constant a_s:

Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha\,\Delta t$	$v_{fs} = v_{is} + a_s\,\Delta t$
$\theta_f = \theta_i + \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$	$v_{fs}^2 = v_{is}^2 + 2a_s\,\Delta s$

Nonuniform Circular Motion

- The particle in the figure is speeding up as it moves around the circle
- The tangential acceleration is

$$a_t = \frac{dv_t}{dt}$$

$$a_t = r\alpha$$

- The centripetal acceleration is

$$a_r = v^2/r = \omega^2 r$$