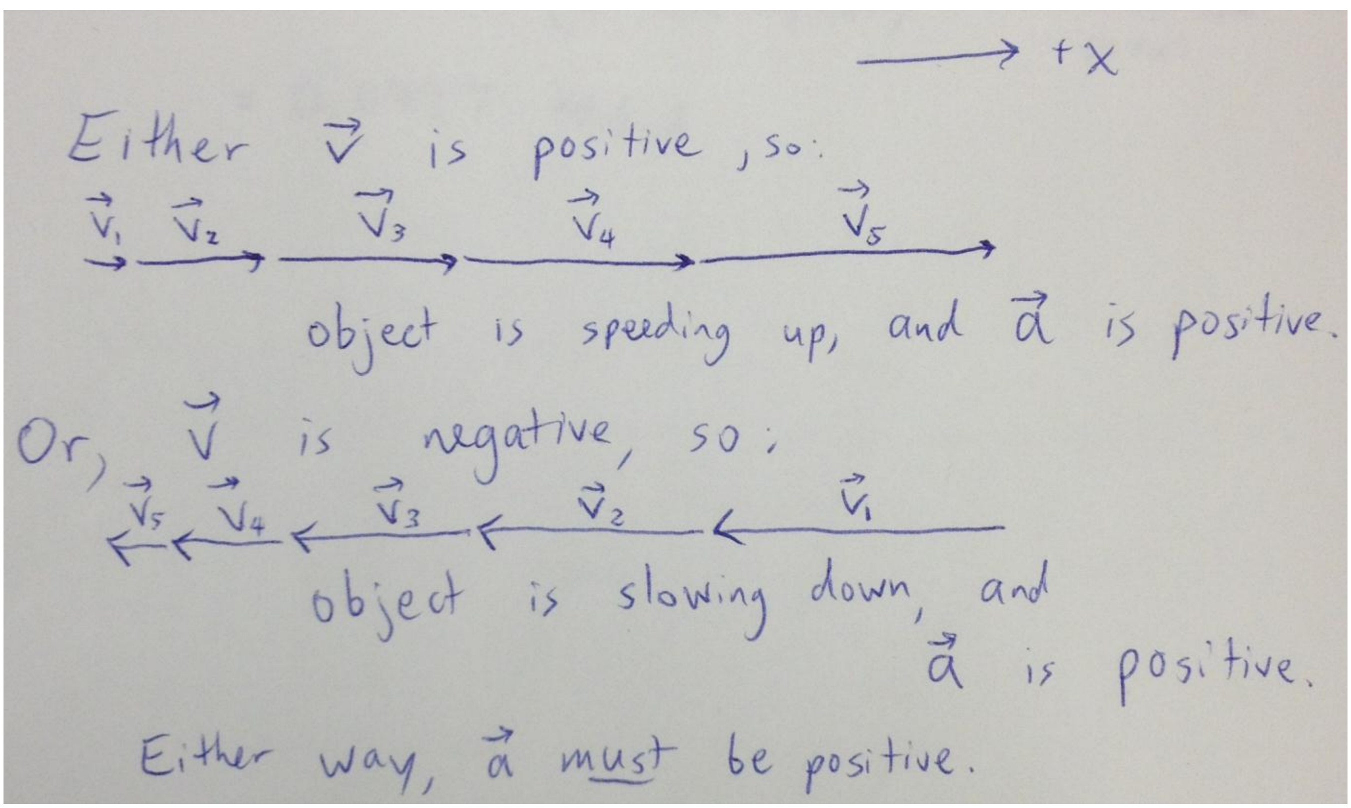
PHY131H1F
Term Test —version 1
Tuesday, October 2, 2012
Duration: 80 minutes
SOLUTIONS – by Jason Harlow

Version 2 Multiple Choice Correspondence Chart		
Version 2 number	**Version 2 Correct Answer**	**Version 1 Solution number below**
1	**A**	**6**
2	**A**	**4**
3	**C**	**7**
4	**B**	**2**
5	**B**	**1**
6	**B**	**8**
7	**E**	**3**
8	**D**	**5**

1. Below is a motion diagram for an object with smooth motion and a constant value of acceleration. We define positive displacements as being toward the right. Note the dots are not numbered, so it is not known whether the object has positive or negative velocity. What can you say about the sign of the acceleration and whether the object is speeding up or slowing down?
   A. The acceleration must be negative, and the object must be slowing down.
   B. The acceleration must be positive, and the object must be speeding up.
   C. The acceleration must be positive, and the object may be speeding up or slowing down.
   D. The acceleration may be positive or negative, and the object must be speeding up.
   E. The acceleration may be positive or negative, and the object may be speeding up or slowing down.



2. The standard kilogram is a polished platinum-iridium cylinder stored in Paris. Imagine a system of units in which the volume of this standard kilogram is defined to be exactly 1 "ford". Note that the density of platinum-iridium is 21,500 kg/m^3, and the density of ice is 919 kg/m^3. What is the density of ice in kg/ford?

A. 4.65×10^{-5}
B. 1.09×10^{-3}
C. 4.27×10^{-2}
D. 23.4
E. 919

Mass of standard kilogram = 1 kg.

Volume of standard Kilogram = 1 ford.

Desity of platinum-iridium = $1 \frac{kg}{ford} = 21{,}500\ kg/m^3$

Conversion factor equal to one: $1 = \left(\frac{1\ kg/ford}{21{,}500\ kg/m^3}\right) = \left(\frac{21{,}500\ kg/m^3}{1\ kg/ford}\right)$

Density of ice = $919\ kg/m^3 \left(\frac{1\ kg/ford}{21{,}500\ kg/m^3}\right)$ ← top & bottom units of kg/m^3 Cancel

$= 0.0427\ kg/ford$

3. You wish to measure the average speed of cars driving down St. George St. Over a distance of 125 ± 2 m you measure an average travel time of 11.3 ± 1.2 seconds. From these measurements you conclude that the average speed of cars is 11.1 m/s. What is the error in this value?

A. 0.11 m/s
B. 1.2 m/s
C. 1.6 m/s
D. 2 m/s
E. 2.3 m/s

$$V_{avg} = \frac{d}{t}$$

Product or division rule:

$$\frac{\Delta V_{avg}}{V_{avg}} = \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

$$= \sqrt{\underbrace{\cancel{\left(\frac{2}{125}\right)^2}}_{\text{negligible}} + \left(\frac{1.2}{11.3}\right)^2}$$

$$\frac{\Delta V_{avg}}{V_{avg}} = \frac{1.2}{11.3}$$

$$\Delta V_{avg} = \cancel{11.1}\left(\frac{1.2}{\cancel{11.3}}\right) \simeq 1.2 \text{ m/s}$$

[Note: When computing errors, you know your final answer will have only 1 or 2 significant figures, so approximate calculations are sufficient, and can save time.]

4. An extremely accurate digital thermometer gives a reading of 37.2°C when placed in a tissue sample. Repeated measurements give the same value of 37.2°C. What is the error in this value?

A. The error is 0.0 °C
B. The error is 0.05 °C ← Reading error
C. The error is 0.1 °C
D. There is no scatter in the measurements, so no error can be reported.
E. In this situation you should take more measurements until you have a large enough sample to compute a standard deviation.

The standard deviation is much less than the reading error, so use the reading error.

5. A ball is dropped from rest and another ball is dropped from rest a short time later. At the instant the second ball is dropped, the distance between the two balls is d. What is the distance between the balls at a time t after the second ball was dropped?

A. d
B. $d-\frac{1}{2}gt^2$
C. $d+\frac{1}{2}gt^2$
D. $d-t\sqrt{2dg}$
E. $d+t\sqrt{2dg}$

Instant second ball is dropped:

define $+y$ = down $\Rightarrow a_y = g$.

← 2nd ball $v_{2i} = 0$, $y_{2i} = 0$

d

← 1st ball $y_{1i} = d$, $v_{1i} = ?$

What is the speed of the first ball after having fallen a distance, d?

Known: $a_y = g$ $v_i = 0$ $\Delta y = d$

Need: v_f

Don't care about Δt

Use: $v_f^2 = v_i^2 + 2a_y\Delta y = 0 + 2gd$

$\longrightarrow v_f = \sqrt{2gd}$

This is v_{1i}, the initial speed of first ball when second ball is dropped

At time t later

$$y_2 = \cancel{y_{2i}}^{\,0} + \cancel{v_{2i}}^{\,0}t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

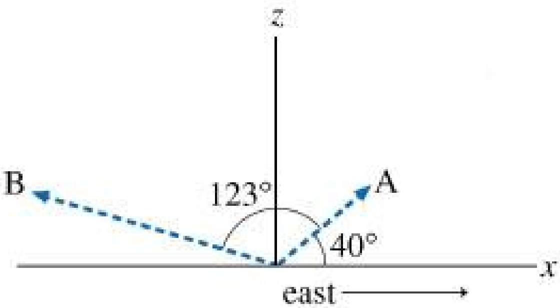
$$y_1 = y_{1i} + v_{1i}t + \frac{1}{2}gt^2$$

$$y_1 = d + \sqrt{2gd}\,t + \frac{1}{2}gt^2$$

$$y_1 - y_2 = d + \sqrt{2gd}\,t + \cancel{\frac{1}{2}gt^2} - \cancel{\frac{1}{2}gt^2}$$

6. A radar station, located at the origin of *xz* plane, as shown in the figure below, detects an airplane coming straight at the station from the east. When the airplane is first observed, its position relative to the origin is $\vec{R}_A$. The position vector $\vec{R}_A$ has a magnitude of 445 m and is located at exactly 40.0º above the horizon. The airplane is tracked for another 123º in the vertical east-west plane, until it has passed directly over the station and reached point B. The position of point B relative to the origin is $\vec{R}_B$ (the magnitude of $\vec{R}_B$ is 908 m). The position vectors are shown in the diagram, where the *x* axis represents the ground and the positive *z* direction is upward. What is the magnitude of the airplane's displacement $|\vec{R}_B - \vec{R}_A|$?

A. 20.6 m
B. 763 m
C. 1.10×10^3 m
D. 1210 m
E. 1330 m



	magnitude	θ	x-comp.	z-comp
$\vec{R}_A$	445 m	40°	$445 \cos 40° = 340.8899$	$445 \sin 40° = 286.0403$
$\vec{R}_B$	908 m	$123+40 = 163°$	$908 \cos 163° = -868.324$	$908 \sin 163° = 265.4756$
$\vec{R}_B - \vec{R}_A$			-1209.21 m	-20.5647 m

$$|\vec{R}_B - \vec{R}_A| = \sqrt{(-1209.21)^2 + (-20.5647)^2}$$

$= 1209.4$ m

Round to 3 sig figs

$= 1210$ m

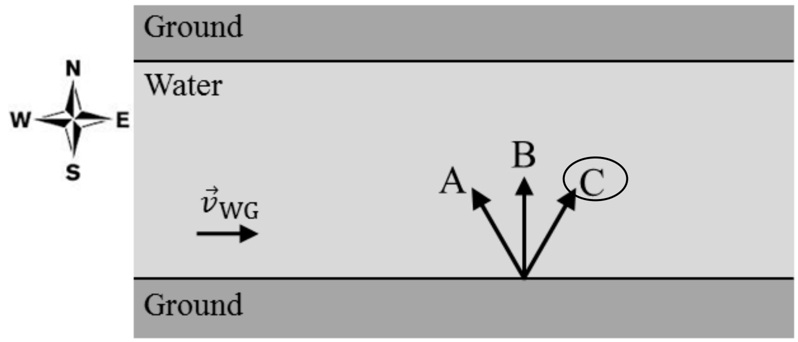
7. An object is dropped and freely falls to the ground with an acceleration of magnitude 1 *g*. (Assume air resistance is negligible and the object is near the surface of the Earth.) If, instead, it is thrown upward at an angle from the same initial location, after the object leaves your hand its acceleration will be

A. 0 *g*.
B. 1 *g* downward.
C. 1 *g* upward.
D. Larger than *g*.
E. Smaller than *g*, but not zero.

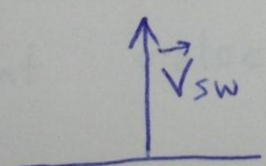
Projectile motion and freefall have the same acceleration: $\vec{a} = g$, downward

8. A river runs from west to east. The velocity of the Water relative to the Ground has a constant value of $\vec{v}_{WG}$. A Swimmer can swim at some maximum constant speed relative to the water v_{SW}. The Swimmer chooses the direction of his velocity so he crosses the river from the south bank to the north bank in the minimum amount of time. Which of the three labeled arrows on the diagram best indicates the direction of the velocity of the Swimmer relative to the Ground, $\vec{v}_{SG}$?

A. A.
B. B.
C. C.
D. While crossing the river, the direction of $\vec{v}_{SG}$ starts as C, but then curves toward B.
E. While crossing the river, the direction of $\vec{v}_{SG}$ starts as A, but then curves toward B.

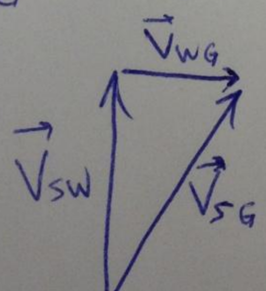


In the reference frame of the Water
(Water is stationary):



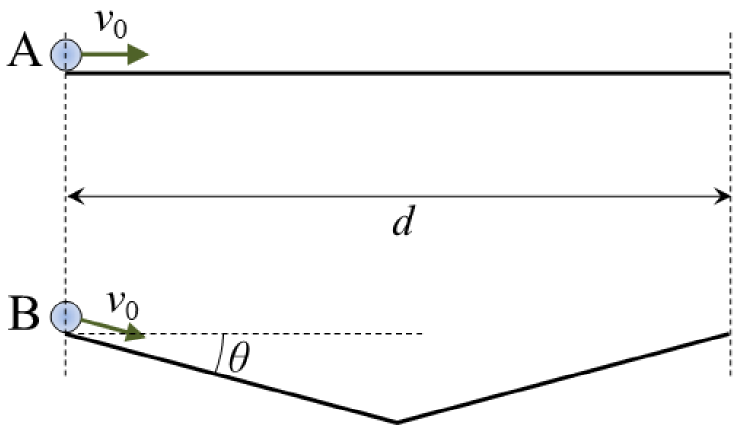
← should be directly North, so time for crossing is minimum.

$$\vec{V}_{SG} = \vec{V}_{SW} + \vec{V}_{WG}$$



FREE-FORM IN TWO UNRELATED PARTS (16 points total)
Clearly show your reasoning and work as some part marks may be awarded. Write your final answers in the boxes provided.

PART A
Two balls roll on frictionless tracks, both starting with the same initial speed v_0 = 1.40 m/s. Ball A rolls on a straight, horizontal track a distance d = 2.20 m. Ball B rolls down a straight track which is angled at θ = 10.0° below the horizontal. Half-way along the track, the track for ball B turns upward at an angle of θ = 10.0° above the horizontal, so that the two tracks reach the same height at the end. At the instant Ball B encounters the sharp upward turn half-way along its track, assume it changes its direction without changing its speed. The *horizontal* distance that ball B travels is d = 2.20 m.



A1. [**3 points**] How much time does it take Ball A to roll the distance d = 2.20 m? [Please express your answer to the correct number of significant figures, with units in the box provided.]

Constant velocity $v = v_0 = \frac{d}{t_A}$

$$t_A = \frac{d}{V_0} = \frac{2.20\ \text{m}}{1.40\ \text{m/s}}$$

$= 1.5714$ s

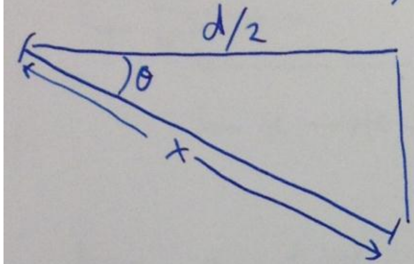
3 sig figs.

$\boxed{1.57\ \text{s}}$

PART A – Continued from previous page

A2. [**5 points**] How much time does it take Ball B to roll the full length of its track, a total *horizontal* distance of d = 2.20 m? [Please express your answer to the correct number of significant figures, with units in the box provided.]

By symmetry, the time to roll down the track is the same as the time to roll up the track, which is $t_B/2$.



$$x = \frac{d/2}{\cos\theta} = \frac{1.10}{\cos 10^\circ} = 1.1170 \text{ m}$$

$$a_x = g \sin\theta = 9.8 \sin 10^\circ = 1.7018 \text{ m/s}^2$$

$$V_0 = 1.40 \text{ m/s}$$

$$V_f^2 = V_0^2 + 2a_x X$$

$$V_f = \sqrt{1.40^2 + 2(1.7018)(1.1170)}$$

$$V_f = 2.4004 \text{ m/s}$$

Average speed is $V_{avg} = \frac{V_i + V_f}{2} = \frac{1.4 + 2.4004}{2} = 1.9002 \text{ m/s}$

$$V_{avg} = \frac{x}{(t_B/2)} = \frac{2x}{t_B}$$

$$t_B = \frac{2x}{V_{avg}} = \frac{2(1.1170)}{1.9002}$$

$= 1.1757$ s

3 sig figs

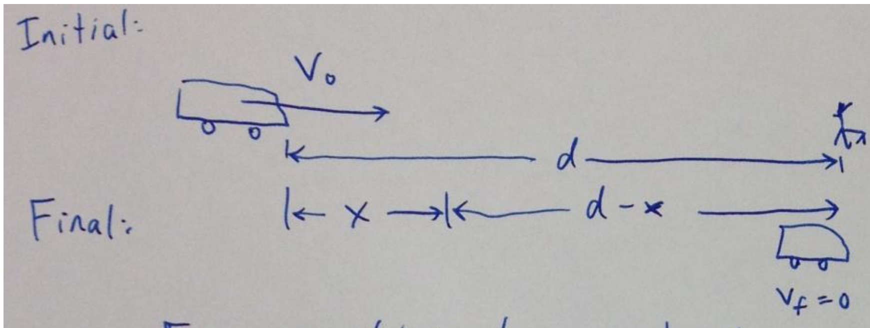
$$\boxed{1.18 \text{ s}}$$

A3. [**1 point**] Which ball reaches the end of its track first? [Circle the letter of the best answer below]

A. Ball A
B. Ball B Since 1.18 s < 1.57 s
C. both balls reach the end at the same time

PART B [7 points]

You're driving down the highway late one night at speed v_0 when a deer jumps onto the road a distance d in front of you. Your reaction time before stepping on the brakes is t_1. While stepping on the brakes, your car slows down, and the maximum magnitude of the car's acceleration during this time is a. What is the minimum value of the distance d so that you come to a stop before hitting the deer? [Please express your answer in the box provided using any or all of the following: v_0, t_1, a and any numerical constants necessary.]



For a time t_1, velocity is constant and the car travels a distance x.

$$V_0 = \frac{x}{t_1} \Rightarrow x = V_0 t_1$$

After that, the car travels a distance $d - x = d - v_0 t_1$. We don't care how long that takes. Let's assume constant acceleration in the negative direction: $a_x = -a$.

Use: $$V_f^2 = V_0^2 + 2a_x(\Delta x)$$

$$0 = V_0^2 - 2a(d - v_0 t_1)$$

Solve for d

$$2ad - 2v_0 a t_1 = V_0^2$$

$$2ad = V_0^2 + 2v_0 a t_1$$

$$\boxed{d = \frac{V_0^2}{2a} + V_0 t_1}$$