Physics for Scientists and Engineers

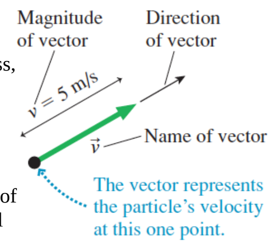
A Strategic Approach with Modern Physics
Third Edition by Randall D. Knight

Chapter 3. Vectors and Coordinate Systems

- 3.1 Vectors
- 3.2 Properties of Vectors
- Please read pages 70 through 74

Vectors

- A quantity that is fully described by a single number is called a **scalar quantity** (ie mass, temperature, volume)
- A quantity having both a magnitude and a direction is called a **vector quantity**
- The *geometric representation* of a vector is an arrow with the tail of the arrow placed at the point where the measurement is made

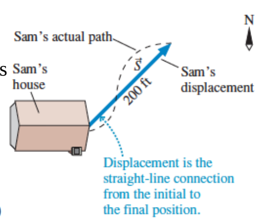


- We label vectors by drawing a small arrow over the letter that represents the vector, ie: $\vec{r}$ for position, $\vec{v}$ for velocity, $\vec{a}$ for acceleration

Properties of Vectors

- Suppose Sam starts from his front door, takes a walk, and ends up 200 ft to the northeast of where he started
- We can write Sam's displacement as

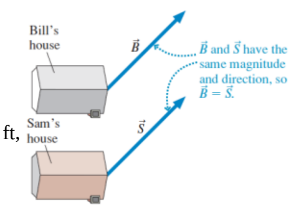
$$\vec{S} = (200 \text{ ft, northeast})$$



- The magnitude of Sam's displacement is $S = |\vec{S}| = 200$ ft, the distance between his initial and final points

Properties of Vectors

- Sam and Bill are neighbors
- They both walk 200 ft to the northeast of their own front doors
- Bill's displacement $\vec{B}$ = (200 ft, northeast) has the same magnitude and direction as Sam's displacement $\vec{S}$

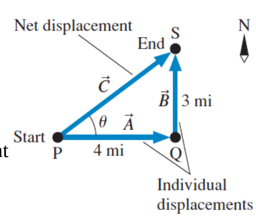


- Two vectors are equal if they have the same magnitude and direction
- This is true regardless of the starting points of the vectors
- $\vec{B} = \vec{S}$

Vector Addition

- A hiker's displacement is 4 miles to the east, then 3 miles to the north, as shown
- Vector $\vec{C}$ is the net displacement

$$\vec{C} = \vec{A} + \vec{B}$$



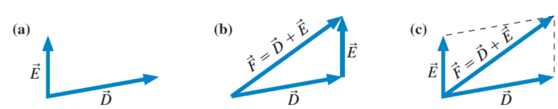
- Because $\vec{A}$ and $\vec{B}$ are at right angles, the magnitude of C is given by the Pythagorean theorem:

$$C = \sqrt{A^2 + B^2} = \sqrt{(4 \text{ mi})^2 + (3 \text{ mi})^2} = 5 \text{ mi}$$

- To describe the direction of $\vec{C}$, we find the angle: $\theta = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{3 \text{ mi}}{4 \text{ mi}}\right) = 37°$
- Altogether, the hiker's net displacement is: $\vec{C} = \vec{A} + \vec{B} = (5 \text{ mi, } 37° \text{ north of east})$

Parallelogram Rule for Vector Addition

- It is often convenient to draw two vectors with their tails together, as shown in (a) below
- To evaluate $\vec{F} = \vec{D} + \vec{E}$, you could move $\vec{E}$ over and use the tip-to-tail rule, as shown in (b) below
- Alternatively, $\vec{F} = \vec{D} + \vec{E}$ can be found as the diagonal of the parallelogram defined by $\vec{D}$ and $\vec{E}$, as shown in (c) below



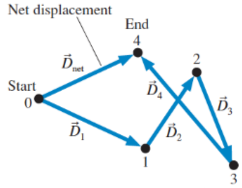
What is $\vec{D} + \vec{E}$?

Tip-to-tail rule: Slide the tail of $\vec{E}$ to the tip of $\vec{D}$.

Parallelogram rule: Find the diagonal of the parallelogram formed by $\vec{D}$ and $\vec{E}$.

Addition of More than Two Vectors

- Vector addition is easily extended to more than two vectors
- The figure shows the path of a hiker moving from initial position 0 to position 1, then 2, 3, and finally arriving at position 4

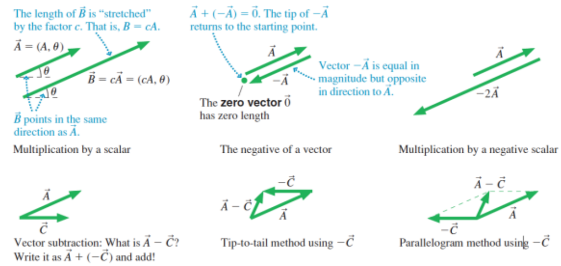


- The four segments are described by displacement vectors $\vec{D}_1$, $\vec{D}_2$, $\vec{D}_3$ and $\vec{D}_4$
- The hiker's net displacement, an arrow from position 0 to 4, is

$$\vec{D}_{net} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4$$

- The vector sum is found by using the tip-to-tail method three times in succession

More Vector Mathematics



Try:
Stop To Think 3.1 and 3.2
(Answers are at the very end of the chapter.)
Work Through:
Examples 3.1 and 3.2

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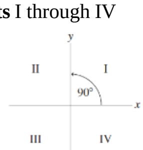
Chapter 3. Vectors and Coordinate Systems

- 3.3 Coordinate Systems and Vector Components
- Please read pages 74 through 77

Coordinate Systems and Vector Components

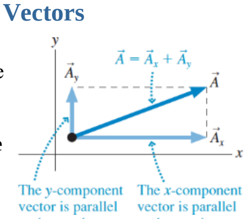
- A *coordinate system* is an artificially imposed grid that you place on a problem
- You are free to choose:
  - Where to place the origin, and
  - How to orient the axes
- Below is a conventional *xy*-coordinate system and the four **quadrants** I through IV



Component Vectors

- The figure shows a vector $\vec{A}$ and an *xy*-coordinate system that we've chosen
- We can define two new vectors *parallel to the axes* that we call the **component vectors** of $\vec{A}$, such that:

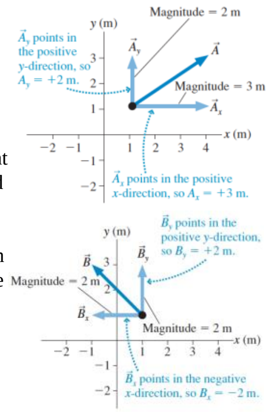
$$\vec{A} = \vec{A}_x + \vec{A}_y$$



- We have broken $\vec{A}$ into two perpendicular vectors that are parallel to the coordinate axes
- This is called the **decomposition** of $\vec{A}$ into its component vectors

Components

- Suppose a vector $\vec{A}$ has been decomposed into component vectors $\vec{A}_x$ and $\vec{A}_y$ parallel to the coordinate axes
- We can describe each component vector with a single number called the component
- The component tells us how big the component vector is, and, with its sign, which ends of the axis the component vector points toward
- Shown to the right are two examples of determining the components of a vector

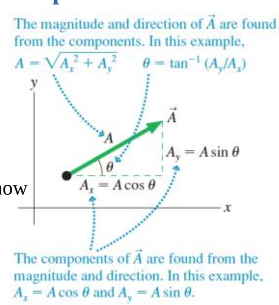


Tactics: Determining the components of a vector

1. The absolute value $|A_x|$ of the x-component A_x is the magnitude of the component vector $\vec{A}_x$.
2. The *sign* of A_x is positive if $\vec{A}_x$ points in the positive x-direction, negative if $\vec{A}_x$ points in the negative x-direction.
3. The y-component A_y is determined similarly.

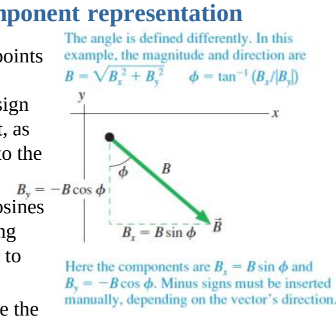
Moving between the geometric representation and the component representation

- We will frequently need to decompose a vector into its components
- We will also need to “reassemble” a vector from its components
- The figure to the right shows how to move back and forth between the geometric and component representations of a vector



Moving between the geometric representation and the component representation

- If a component vector points left (or down), you must *manually* insert a minus sign in front of the component, as done for B_y in the figure to the right
- The role of sines and cosines can be reversed, depending upon which angle is used to define the direction
- The angle used to define the direction is almost always between 0° and 90°



Try:
Stop To Think 3.3
(Answer is at the very end of the chapter.)
Work Through:
Examples 3.3 and 3.4

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Chapter 3. Vectors and Coordinate Systems

- 3.4 Vector Algebra
- Please read pages 77 through 80

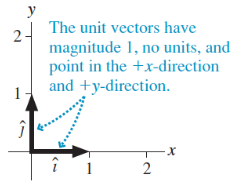
Unit Vectors

- Each vector in the figure to the right has a magnitude of 1, no units, and is parallel to a coordinate axis
- A vector with these properties is called a **unit vector**
- These unit vectors have the special symbols

$$\hat{\imath} \equiv (1, \text{positive } x\text{-direction})$$

$$\hat{\jmath} \equiv (1, \text{positive } y\text{-direction})$$

- Unit vectors establish the directions of the positive axes of the coordinate system



Vector Algebra

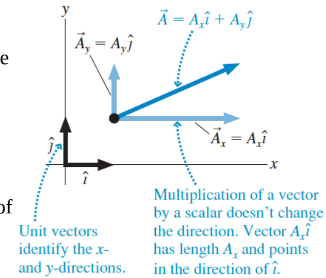
- When decomposing a vector, unit vectors provide a useful way to write component vectors:

$$\vec{A}_x = A_x\hat{\imath}$$

$$\vec{A}_y = A_y\hat{\jmath}$$

- The full decomposition of the vector $\vec{A}$ can then be written

$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath}$$



Working with Vectors

- We can perform vector addition by adding the *x*- and *y*-components separately
- This method is called **algebraic addition**
- For example, if $\vec{D} = \vec{A} + \vec{B} + \vec{C}$, then

$$D_x = A_x + B_x + C_x$$

$$D_y = A_y + B_y + C_y$$

- Similarly, to find $\vec{R} = \vec{P} - \vec{Q}$ we would compute

$$R_x = P_x - Q_x$$

$$R_y = P_y - Q_y$$

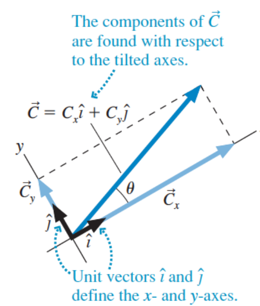
- To find $\vec{T} = c\vec{S}$, where *c* is a scalar, we would compute

$$T_x = cS_x$$

$$T_y = cS_y$$

Tilted Axes and Arbitrary Directions

- For some problems it is convenient to tilt the axes of the coordinate system
- The axes are still perpendicular to each other, but there is no requirement that the *x*-axis has to be horizontal



- Tilted axes are useful if you need to determine component vectors "parallel to" and "perpendicular to" an arbitrary line or surface

Try:
Stop To Think 3.4
(Answer is at the very end of the chapter.)
Work Through:
Examples 3.5 through 3.8