PHY131H1F
University of Toronto

Class 7 Preclass Video
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Based on Knight 3rd edition
Ch. 4, sections 4.1 to 4.4,
pgs. 85-97

Section 4.1

Acceleration

The *average acceleration* of a moving object is defined as the vector:

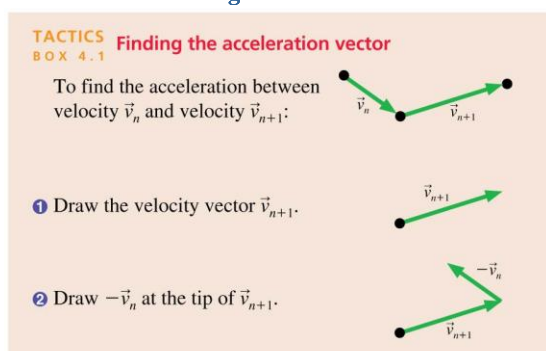
$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t}$$

The acceleration $\vec{a}$ **points in the same direction as** $\Delta\vec{v}$, the change in velocity

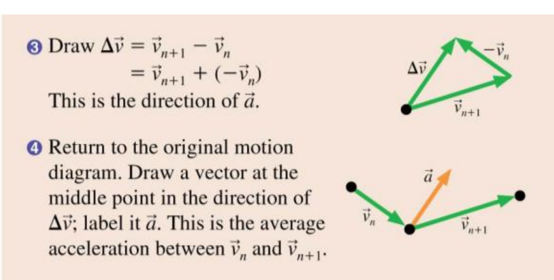
As an object moves, its velocity vector can change in two possible ways:

1. The magnitude of the velocity can change, indicating a change in speed, or
2. The direction of the velocity can change, indicating that the object has changed direction.

Tactics: Finding the acceleration vector

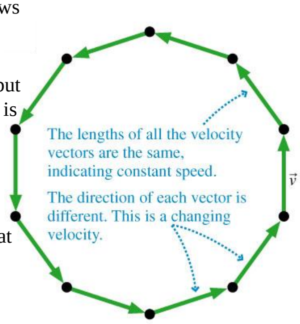


Tactics: Finding the acceleration vector



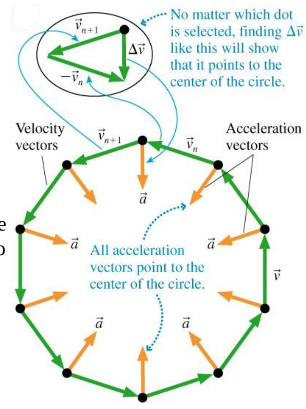
Acceleration

- The figure to the right shows a motion diagram of Maria riding a Ferris wheel
- Maria has constant speed but *not* constant velocity, so she is accelerating.
- For every pair of adjacent velocity vectors, we can subtract them to find the average acceleration near that point



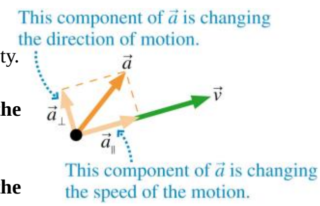
Acceleration

- At every point Maria's acceleration points toward the center of the circle.
- This is an acceleration due to changing direction, not to changing speed.



Analyzing the acceleration vector

- An object's acceleration can be decomposed into components parallel and perpendicular to the velocity.
- **$\vec{a}_{\parallel}$ is the piece of the acceleration that causes the object to change speed**
- **$\vec{a}_{\perp}$ is the piece of the acceleration that causes the object to change direction**
- An object changing direction *always* has a component of acceleration perpendicular to the direction of motion.

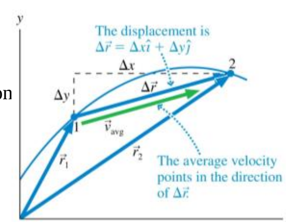


Section 4.2

Two-Dimensional Kinematics

- The figure to the right shows the *trajectory* of a particle moving in the *x-y* plane
- The particle moves from position $\vec{r}_1$ at time t_1 to position $\vec{r}_2$ at a later time t_2
- The average velocity points in the direction of the displacement $\Delta\vec{r}$ and is

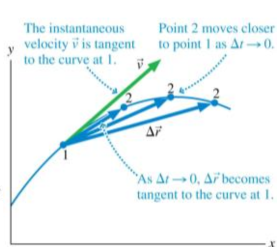
$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath}$$



Two-Dimensional Kinematics

- The instantaneous velocity is the limit of $\vec{v}_{\text{avg}}$ as $\Delta t \rightarrow 0$
- As shown the instantaneous velocity vector is tangent to the trajectory
- Mathematically:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$



which can be written:

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

where: $v_x = \frac{dx}{dt}$ and $v_y = \frac{dy}{dt}$

Two-Dimensional Kinematics

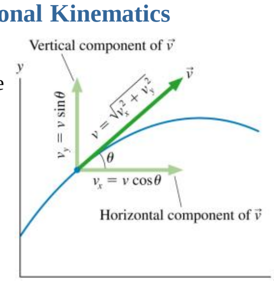
- If the velocity vector's angle θ is measured from the positive *x*-direction, the velocity components are

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

where the particle's *speed* is

$$v = \sqrt{v_x^2 + v_y^2}$$

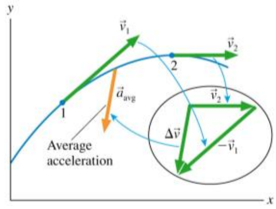


- Conversely, if we know the velocity components, we can determine the direction of motion:

$$\tan\theta = \frac{v_y}{v_x}$$

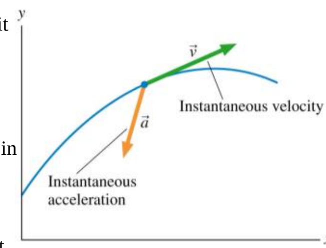
Two-Dimensional Acceleration

- The figure to the right shows the trajectory of a particle moving in the *x*-*y* plane
- The instantaneous velocity is $\vec{v}_1$ at time t_1 and $\vec{v}_2$ at a later time t_2
- We can use vector subtraction to find a_{avg} during the time interval $\Delta t = t_2 - t_1$



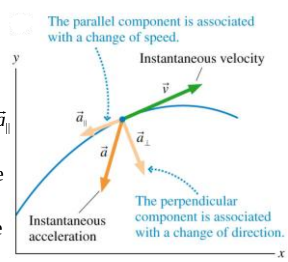
Two-Dimensional Acceleration

- The *instantaneous acceleration* is the limit of $\vec{a}_{avg}$ as $\Delta t \to 0$.
- The instantaneous acceleration vector is shown along with the instantaneous velocity in the figure.
- By definition, $\vec{a}$ is the rate at which $\vec{v}$ is changing at that instant.



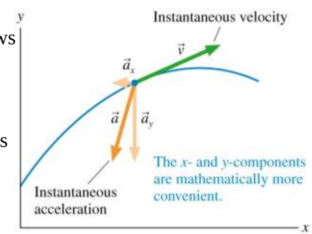
Decomposing Two-Dimensional Acceleration

- The figure to the right shows the trajectory of a particle moving in the *x*-*y* plane
- The acceleration $\vec{a}$ is decomposed into components $\vec{a}_{\parallel}$ and $\vec{a}_{\perp}$
- $\vec{a}_{\parallel}$ is associated with a change in speed
- $\vec{a}_{\perp}$ is associated with a change of direction
- $\vec{a}_{\perp}$ always points toward the "inside" of the curve because that is the direction in which $\vec{v}$ is changing



Decomposing Two-Dimensional Acceleration

- The figure to the right shows the trajectory of a particle moving in the *x*-*y* plane
- The acceleration $\vec{a}$ is decomposed into components a_x and a_y



- If v_x and v_y are the *x*- and *y*- components of velocity, then

$$a_x = \frac{dv_x}{dt} \quad \text{and} \quad a_y = \frac{dv_y}{dt}$$

Constant Acceleration

- If the acceleration $\vec{a} = a_x\hat{i} + a_y\hat{j}$ is constant, then the two components a_x and a_y are both constant
- In this case, everything from Chapter 2 about constant-acceleration kinematics applies to the *components*
- The *x*-components and *y*-components of the motion can be treated independently
- They remain connected through the fact that Δt must be the same for both

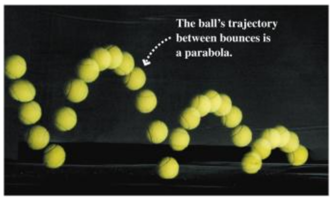
$$x_f = x_i + v_{ix}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad y_f = y_i + v_{iy}\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x\,\Delta t \qquad v_{fy} = v_{iy} + a_y\,\Delta t$$

Section 4.3

Projectile Motion

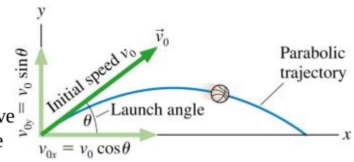
- Baseballs, tennis balls, Olympic divers, etc, all exhibit *projectile motion*
- A **projectile** is an object that moves in two dimensions under the influence of *only* gravity
- Projectile motion extends the idea of free-fall motion to include a horizontal component of velocity
- Air resistance is neglected



- Projectiles in two dimensions follow a *parabolic trajectory* as shown in the photo

Projectile Motion

- The start of a projectile's motion is called the *launch*
- The angle θ of the initial velocity v_0 above the x-axis is called the **launch angle**



- The initial velocity vector can be broken into components

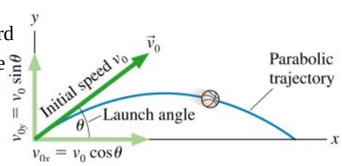
$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$

where v_0 is the initial speed

Projectile Motion

- Gravity acts downward
- Therefore, a projectile has no horizontal acceleration
- Thus



$a_x = 0$

(projectile motion)

$a_y = -g$

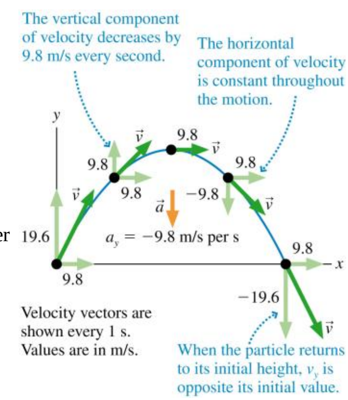
- The vertical component of acceleration a_y is $-g$ of free fall
- The horizontal component of a_x is zero
- Projectiles are in free fall

Projectile Motion

- The figure shows a projectile launched from the origin with initial velocity

$\vec{v}_0 = (9.8\hat{i} + 19.6\hat{j})$ m/s

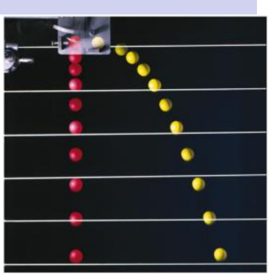
- The value of v_x never changes because there's no horizontal acceleration
- v_y decreases by 9.8 m/s every second



Reasoning About Projectile Motion

A heavy ball is launched exactly horizontally at height h above a horizontal field. At the exact instant that the ball is launched, a second ball is simply dropped from height h. Which ball hits the ground first?

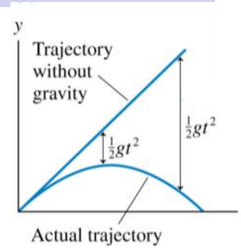
- If air resistance is neglected, the balls hit the ground *simultaneously*
- The initial horizontal velocity of the first ball has *no* influence over its vertical motion
- Neither ball has any initial vertical motion, so both fall distance h in the same amount of time



Reasoning About Projectile Motion

A hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He points his arrow directly at the coconut, but the coconut falls from the branch at the *exact* instant the hunter shoots the arrow. Does the arrow hit the coconut?

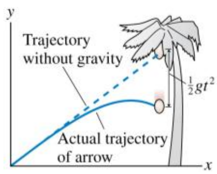
- Without gravity, the arrow would follow a straight line
- Because of gravity, the arrow at time t has "fallen" a distance $\frac{1}{2}gt^2$ below this line
- The separation grows as $\frac{1}{2}gt^2$, giving the trajectory its parabolic shape



Reasoning About Projectile Motion

A hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He points his arrow directly at the coconut, but the coconut falls from the branch at the *exact* instant the hunter shoots the arrow. Does the arrow hit the coconut?

- Had the coconut stayed on the tree, the arrow would have curved under its target as gravity cases it to fall a distance $\frac{1}{2}gt^2$ below the straight line
- But $\frac{1}{2}gt^2$ is also the distance the coconut falls while the arrow is in flight
- So *yes*, the arrow hits the coconut!



Range of a Projectile

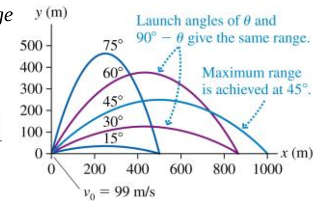
A projectile with initial speed v_0 has a launch angle of θ above the horizontal. How far does it travel over level ground before it returns to the same elevation from which it was launched?

- This distance is sometimes called the *range* of a projectile
- Example 4.5 from your textbook shows:

$$\text{distance} = \frac{v_0^2 \sin(2\theta)}{g}$$

- The maximum distance occurs for $\theta = 45°$

Trajectories of a projectile launched at different angles with a speed of 99 m/s.



PROBLEM-SOLVING STRATEGY 4.1 **Projectile motion problems**

MODEL Make simplifying assumptions, such as treating the object as a particle. Is it reasonable to ignore air resistance?

VISUALIZE Use a pictorial representation. Establish a coordinate system with the *x*-axis horizontal and the *y*-axis vertical. Show important points in the motion on a sketch. Define symbols and identify what the problem is trying to find.

SOLVE The acceleration is known: $a_x = 0$ and $a_y = -g$. Thus the problem is one of two-dimensional kinematics. The kinematic equations are

$$x_f = x_i + v_{ix}\,\Delta t \qquad y_f = y_i + v_{iy}\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$v_{fx} = v_{ix} = \text{constant} \qquad v_{fy} = v_{iy} - g\,\Delta t$$

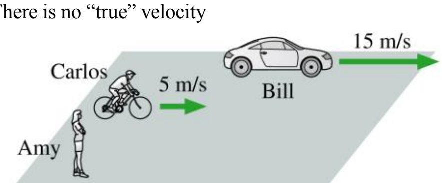
Δt is the same for the horizontal and vertical components of the motion. Find Δt from one component, then use that value for the other component.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Section 4.4

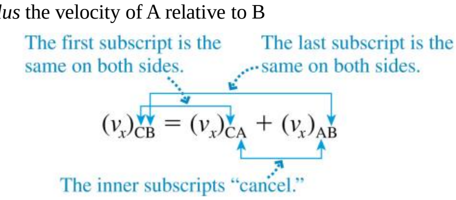
Relative Motion

- The figure below shows Amy and Bill watching Carlos on his bicycle
- According to Amy, Carlos's velocity is $(v_x)_{CA}$ = +5 m/s
- The CA subscript means "C relative to A"
- According to Bill, Carlos's velocity is $(v_x)_{CB}$ = − 10 m/s
- Every velocity is measured *relative* to a certain observer
- There is no "true" velocity



Relative Motion

- The velocity of C relative to B is the velocity of C relative to A *plus* the velocity of A relative to B

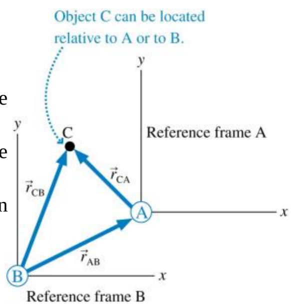


- If B is moving to the right relative to A, then A is moving to the left relative to B
- Therefore, $(v_x)_{AB} = -(v_x)_{BA}$

Reference Frames

- A coordinate system in which an experimenter makes position measurements is called a **reference frame**
- In the figure, Object C is measured in two different reference frames, A and B
- $\vec{r}_{CA}$ is the position of C relative to the origin of A
- $\vec{r}_{CB}$ is the position of C relative to the origin of B
- $\vec{r}_{AB}$ is the position of the origin of A relative to the origin of B

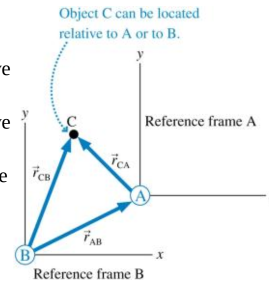
$$\vec{r}_{CB} = \vec{r}_{CA} + \vec{r}_{AB}$$



Reference Frames

- Relative velocities are found as the time derivative of the relative positions
- $\vec{v}_{CA}$ is the velocity of C relative to A
- $\vec{v}_{CB}$ is the velocity of C relative to B
- $\vec{v}_{AB}$ is the velocity of reference frame A relative to reference frame B

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$$



- This is known as the **Galilean transformation of velocity**