PHY131H1F
University of Toronto

Class 8 Preclass Video
by Jason Harlow

Based on Knight 3rd edition
Ch. 4, sections 4.5 to 4.7,
pgs. 98-108

Section 4.5

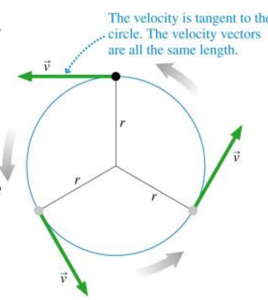
Circular Motion

- Consider a ball on a roulette wheel
- It moves along a circular path of radius *r*
- Other examples of circular motion are a satellite in an orbit, or a ball on the end of a string
- Circular motion is an example of two-dimensional motion in a plane

Uniform Circular Motion

- To begin the study of circular motion, consider a particle that moves at *constant speed* around a circle of radius *r*
- This is called **uniform circular motion**
- The time interval to complete one revolution is called the period, *T*
- The period *T* is related to the speed *v*:

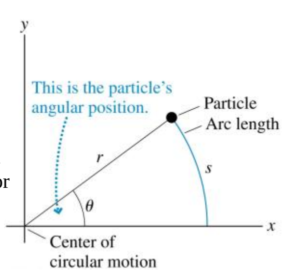
$$v = \frac{1 \text{ circumference}}{1 \text{ period}} = \frac{2\pi r}{T}$$



Angular Position

- Consider a particle at a distance *r* from the origin, at an angle θ from the positive *x* axis
- The angle may be measured in degrees, revolutions (rev) or **radians** (rad), that are related by:

1 rev = 360° = 2π rad



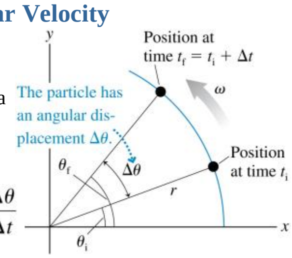
- If the angle is measured in radians, then there is a simple relation between θ and the **arc length** *s* that the particle travels along the edge of a circle of radius *r*:

$$s = r\theta \qquad \text{(with } \theta \text{ in rad)}$$

Angular Velocity

- A particle on a circular path moves through an **angular displacement** $\Delta\theta = \theta_f - \theta_i$ in a time interval $\Delta t = t_f - t_i$
- In analogy with linear motion, we define:

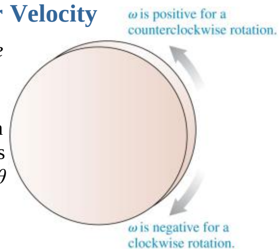
$$\text{average angular velocity} \equiv \frac{\Delta\theta}{\Delta t}$$



- As the time interval Δt becomes very small, we arrive at the definition of instantaneous **angular velocity**

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad \text{(angular velocity)}$$

Angular Velocity



- Angular velocity ω is the *rate* at which a particle's angular position is changing
- As shown in the figure, ω can be positive or negative, and this follows from our definition of θ
- A particle moves with uniform circular motion if ω is constant
- ω and θ are related graphically:

ω = slope of the θ-versus-t graph at time t

$\theta_f = \theta_i$ + area under the ω-versus-t curve between t_i and t_f

$= \theta_i + \omega \Delta t$

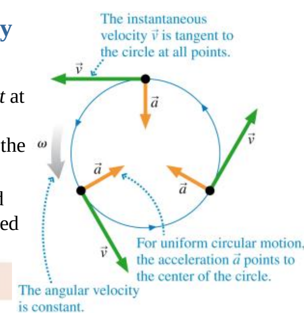
Angular Velocity in Uniform Circular Motion

- When angular velocity ω is constant, this is uniform circular motion
- In this case, as the particle goes around a circle one time, its angular displacement is $\Delta\theta = 2\pi$ during one period $\Delta t = T$
- The absolute value of the constant angular velocity is related to the period of the motion by

$$|\omega| = \frac{2\pi \text{ rad}}{T} \qquad \text{or} \qquad T = \frac{2\pi \text{ rad}}{|\omega|}$$

Section 4.6

Tangential Velocity

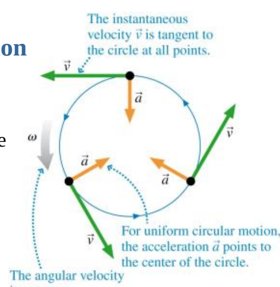


- The tangential velocity component v_t is the rate ds/dt at which the particle moves around the circle, where s is the arc length
- The tangential velocity and the angular velocity are related by

$v_t = \omega r \qquad$ (with ω in rad/s)

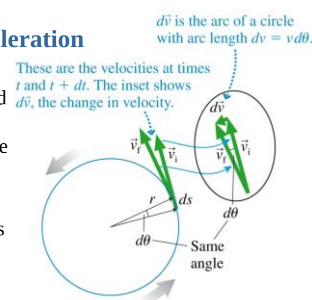
- In this equation, the units of v_t are m/s, the units of ω are rad/s, and the units of r are m

Centripetal Acceleration



- In uniform circular motion, although the speed is constant, there is an acceleration because the *direction* of the velocity vector is always changing
- The acceleration of uniform circular motion is called **centripetal acceleration**
- The direction of the centripetal acceleration is toward the center of the circle
- The magnitude of the centripetal acceleration is constant for uniform circular motion

Centripetal Acceleration



- The figure shows the velocity $\vec{v}_i$ at one instant and the velocity $\vec{v}_f$ an infinitesimal amount of time dt later
- By definition, $\vec{a} = d\vec{v}/dt$
- By analyzing the isosceles triangle of velocity vectors, we can show that:

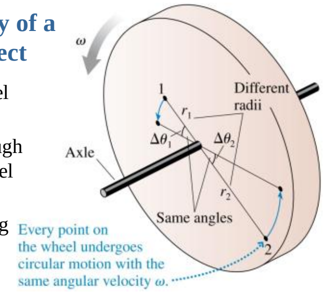
$$\vec{a} = \left(\frac{v^2}{r}, \text{ toward center of circle}\right) \qquad \text{(centripetal acceleration)}$$

which can be written in terms of angular velocity as: $a = \omega^2 r$

Section 4.7

Angular Velocity of a Rotating Object

- The figure shows a wheel rotating on an axle
- Points 1 and 2 turn through the *same angle* as the wheel rotates
- That is, $\Delta\theta_1 = \Delta\theta_2$ during some time interval Δt
- Therefore $\omega_1 = \omega_2 = \omega$



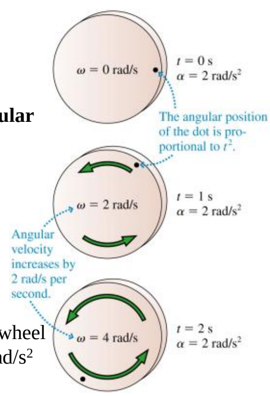
- All points on the wheel rotate with the same angular velocity
- We can refer to ω as the angular velocity of the wheel

Angular Acceleration

- Suppose a wheel's rotation is speeding up or slowing down
- This is called **nonuniform circular motion**
- We can define the *angular acceleration* as

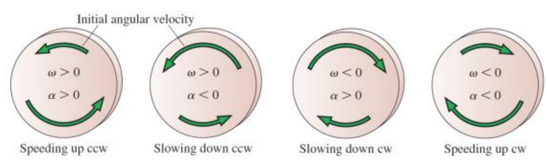
$$\alpha \equiv \frac{d\omega}{dt} \qquad \text{(angular acceleration)}$$

- The units of α are rad/s²
- The figure to the right shows a wheel with angular acceleration $\alpha = 2$ rad/s²



The Sign of Angular Acceleration

- If ω is counter-clockwise and $|\omega|$ is increasing, then α is positive
- If ω is counter-clockwise and $|\omega|$ is decreasing, then α is negative
- If ω is clockwise and $|\omega|$ is decreasing, then α is positive
- If ω is clockwise and $|\omega|$ is increasing, then α is negative



Angular Kinematics

- The same relations that hold for linear motion between a_x, v_x and x apply analogously to rotational motion for α, ω and θ
- There is a graphical relationship between α and ω:

α = slope of the ω-versus-t graph at time t

$\omega_f = \omega_i$ + area under the α-versus-t curve between t_i and t_f

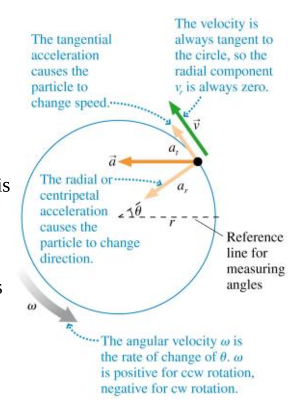
- The table shows a comparison of the rotational and linear kinematics equations for constant α or constant a_s:

Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha\,\Delta t$	$v_{fs} = v_{is} + a_s\,\Delta t$
$\theta_f = \theta_i + \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$	$v_{fs}^2 = v_{is}^2 + 2a_s\,\Delta s$

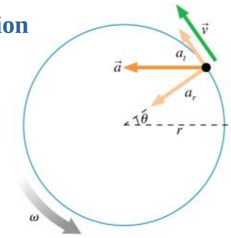
Acceleration in Nonuniform Circular Motion

- The particle in the figure is moving along a circle and is speeding up
- The centripetal acceleration is $a_r = v_t^2/r$, where v_t is the tangential speed
- There is also a tangential acceleration a_t which is always tangent to the circle
- The magnitude of the total acceleration is

$$a = \sqrt{a_r^2 + a_t^2}$$



Nonuniform Circular Motion



- A particle moves along a circle and may be changing speed
- The *distance traveled* along the circle is related to θ:

$$s = r\theta \qquad \text{(with } \theta \text{ in rad)}$$

- The *tangential velocity* is related to the angular velocity:

$$v_t = \omega r \qquad \text{(with } \omega \text{ in rad/s)}$$

- The *tangential acceleration* is related to the angular acceleration:

$$a_t = \frac{dv_t}{dt} = \frac{d(\omega r)}{dt} = \frac{d\omega}{dt} r = \alpha r$$