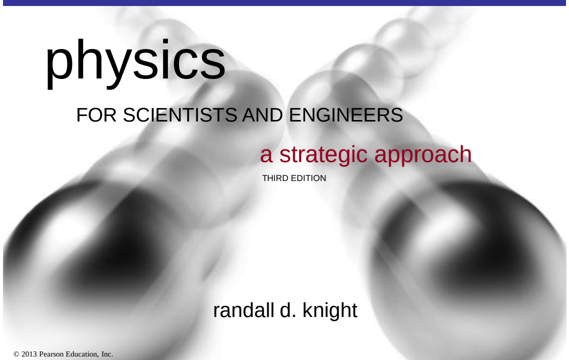
Class 13 Sections 8.1-8.3



ALWAYS LEARNING PEARSON

Chapter 8. Dynamics II: Motion in a Plane

Chapter Goal: To learn how to solve problems about motion in a plane.

Dynamics in Two Dimensions

- Suppose the x- and y-components of acceleration are *independent* of each other.
- That is, a_x does not depend on y or v_y, and a_y does not depend on x or v_x.
- Your problem-solving strategy is to:
  1. Draw a pictorial representation: a motion diagram (if needed) and a free-body diagram.
  2. Use Newton's second law in component form:

$$(F_{\text{net}})_x = \sum F_x = ma_x \quad \text{and} \quad (F_{\text{net}})_y = \sum F_y = ma_y$$

The force components (including proper signs) are found from the free-body diagram

Dynamics in Two Dimensions

3. Solve for the acceleration. If the acceleration is constant, use the two-dimensional kinematic equations of Chapter 4 to find velocities and positions:

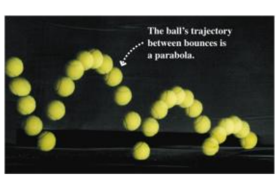
$$x_f = x_i + v_{ix}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad y_f = y_i + v_{iy}\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x\,\Delta t \qquad v_{fy} = v_{iy} + a_y\,\Delta t$$

Projectile Motion: Review

- In the absence of air resistance, a projectile moves under the influence of only gravity.
- If we choose a coordinate system with a vertical y-axis, then

$$\vec{F}_G = -mg\,\hat{j}$$



- Consequently, from Newton's second law, the acceleration is

$$a_x = \frac{(F_G)_x}{m} = 0$$

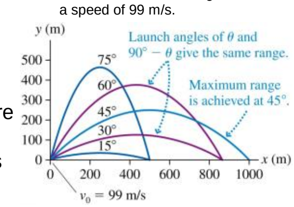
$$a_y = \frac{(F_G)_y}{m} = -g$$

Projectile Motion: Review

- Consider a projectile with initial speed v_0, and a launch angle of θ above the horizontal.
- In Chapter 4 we found that the distance it travels before it returns to the same elevation from which it was launched (the *range*) is:

$$\text{distance} = \frac{v_0^2 \sin(2\theta)}{g}$$

Trajectories of a projectile launched at different angles with a speed of 99 m/s.



- The maximum range occurs for $\theta = 45°$.
- All of these results *neglect* the effect of air resistance.

Projectile Motion with Air Resistance (Drag)

- For low-mass projectiles on earth, the effects of air resistance, or drag, are too large to ignore.
- When drag is included, the angle for maximum range of a projectile depends both on its size and mass.
- The optimum angle is roughly 35° for baseballs.
- The flight of a golf ball is even more complex, because of the dimples and effects of spin.
- Professional golfers achieve their maximum range at launch angles of barely 15°!

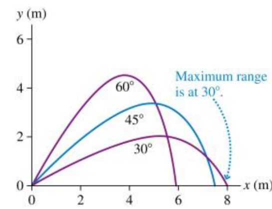
Projectile Motion with Air Resistance (Drag)

- The acceleration of a typical projectile subject to drag force from the air is:

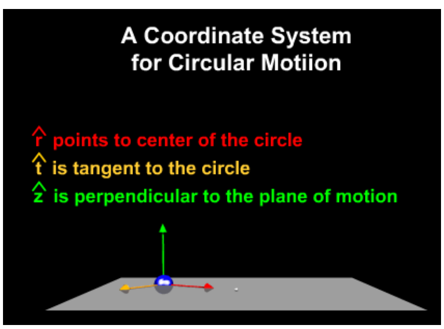
$$a_x = -\frac{\rho CA}{2m} v_x \sqrt{v_x^2 + v_y^2}$$

$$a_y = -g - \frac{\rho CA}{2m} v_y \sqrt{v_x^2 + v_y^2}$$

- The components of acceleration are *not* independent of each other.
- These equations can only be solved numerically.
- The figure shows the numerical solution for a 5-g plastic ball.

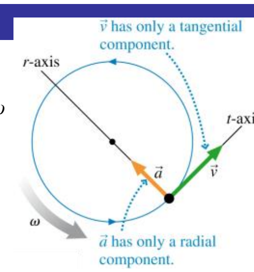


Uniform Circular Motion



Uniform Circular Motion

- A particle in uniform circular motion with angular velocity ω has velocity $v = \omega r$, in the tangential direction.
- The acceleration of uniform circular motion points to the center of the circle.
- The *rtz*-components of $\vec{v}$ and $\vec{a}$ are:



$$v_r = 0 \qquad a_r = \frac{v^2}{r} = \omega^2 r$$

$$v_t = \omega r \qquad a_t = 0$$

$$v_z = 0 \qquad a_z = 0$$

Dynamics of Uniform Circular Motion

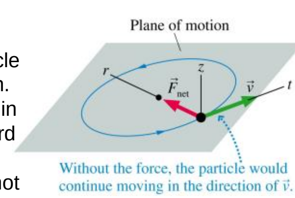
- An object in uniform circular motion is *not* traveling at a constant velocity in a straight line.
- Consequently, the particle must have a net force acting on it

$$\vec{F}_{net} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle}\right)$$

- Without such a force, the object would move off in a straight line tangent to the circle.
- The car would end up in the ditch!

Dynamics of Uniform Circular Motion

- The figure shows a particle in uniform circular motion.
- The net force must point in the radial direction, toward the center of the circle.
- This centripetal force is not a new force; it must be *provided* by familiar forces.



$$v_r = 0 \qquad a_r = \frac{v^2}{r} = \omega^2 r$$

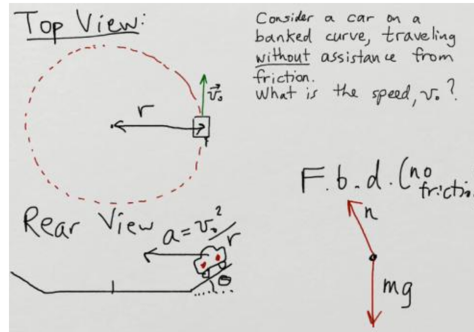
$$v_t = \omega r \qquad a_t = 0$$

$$v_z = 0 \qquad a_z = 0$$

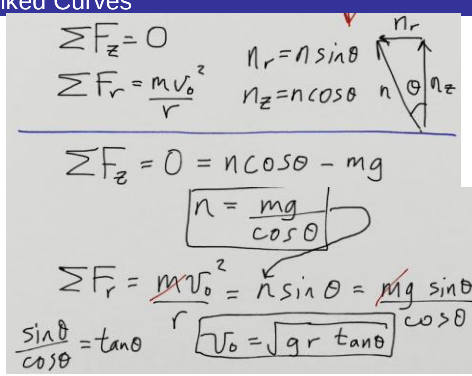
Banked Curves

- Real highway curves are *banked* by being tilted up at the outside edge of the curve.
- The radial component of the normal force can provide centripetal acceleration needed to turn the car.

Banked Curves



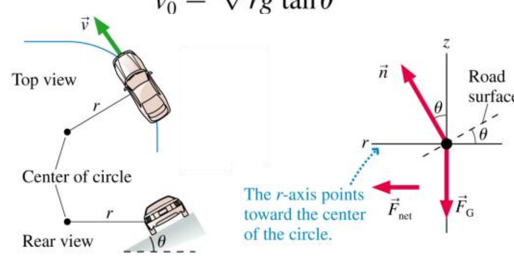
Banked Curves



Banked Curves

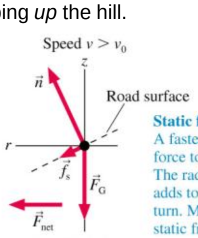
- For a curve of radius r banked at an angle θ, the exact speed at which a car must take the curve without assistance from friction is .

$$v_0 = \sqrt{rg\tan\theta}$$



Banked Curves

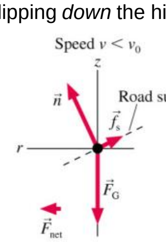
- Consider a car going around a banked curve at a speed *higher* than $v_0 = \sqrt{rg\tan\theta}$.
- In this case, static friction must prevent the car from slipping *up* the hill.



Static friction must point downhill: A faster speed requires a larger net force toward the center of the circle. The radial component of static friction adds to n_r to allow the car to make the turn. Maximum speed occurs when the static friction force reaches its maximum.

Banked Curves

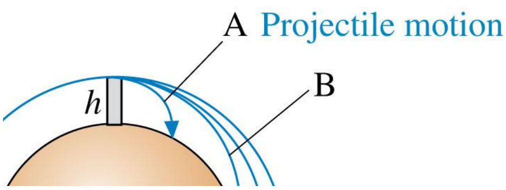
- Consider a car going around a banked curve at a speed *slower* than $v_0 = \sqrt{rg\tan\theta}$.
- In this case, static friction must prevent the car from slipping *down* the hill.



Static friction must point uphill: Without a static friction force *up* the slope, a slow-moving car would slide *down* the incline! Further, n_r is too much radial force for circular motion at $v < v_0$. Here the radial component of static friction reduces the net radial force.

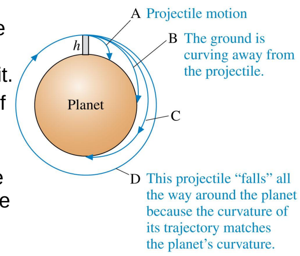
Circular Orbits

- The figure shows a perfectly smooth, spherical, airless planet with one tower of height h
- A projectile is launched parallel to the ground with speed v_0
- If v_0 is very small, as in trajectory A, it simply falls to the ground along a parabolic trajectory
- This is the "flat-earth approximation"



Circular Orbits

- As the initial speed v_0 is increased, the range of the projectile increases as the ground curves away from it.
- Trajectories B and C are of this type.
- If v_0 is sufficiently large, there comes a point where the trajectory and the curve of the earth are parallel.
- In this case, the projectile "falls" but it never gets any closer to the ground!
- This is trajectory D, called an **orbit**.



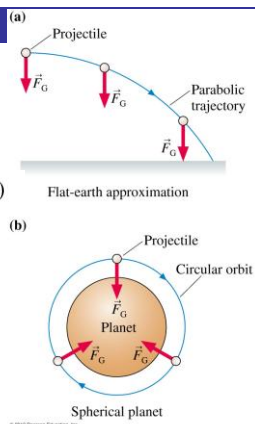
Circular Orbits

- In the flat-earth approximation, shown in figure (a), the gravitational force on an object of mass m is:

$$\vec{F}_G = (mg, \text{vertically downward})$$

- Since actual planets are spherical, the real force of gravity is toward the center of the planet, as shown in figure (b).

$$\vec{F}_G = (mg, \text{toward center})$$



Circular Orbits

- An object in a low circular orbit has acceleration:

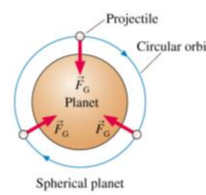
$$\vec{a} = \frac{\vec{F}_{net}}{m} = (g, \text{toward center})$$

- If the object moves in a circle of radius r at speed v_{orbit} the centripetal acceleration is:

$$a_r = \frac{(v_{orbit})^2}{r} = g$$

- The required speed for a circular orbit near a planet's surface, neglecting air resistance, is:

$$v_{orbit} = \sqrt{rg}$$



Circular Orbits

- The period of a low-earth-orbit satellite is:

$$T = \frac{2\pi r}{v_{orbit}} = 2\pi\sqrt{\frac{r}{g}}$$

- If r is approximately the radius of the earth $R_e = 6400$ km, then T is about 90 minutes.

- An orbiting spacecraft is constantly in free fall, falling under the influence only of the gravitational force.
- This is why astronauts feel *weightless* in space.