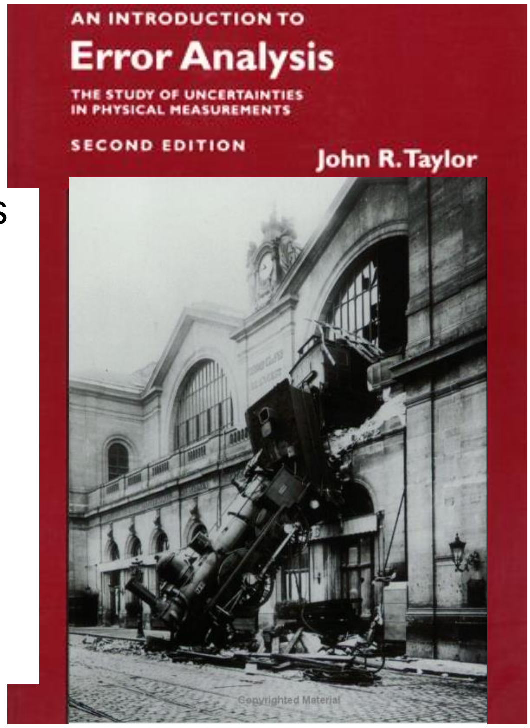
PHY131H1F

Class 3

Today: Error Analysis

- Significant Figures
- Unit Conversion
- Normal Distribution
- Standard Deviation
- Reading Error
- Propagation of Errors
- Error in the Mean



Clicker Question

3 sig figs.

- A cart begins at rest, and accelerates down a ramp with acceleration: $a = 0.518\ \mathrm{m/s^2}$. After 3.2 s, how far has it traveled? $d = \frac{1}{2} a t^2$ gives the result 2.65216 on your calculator. How should you best report this answer in your "Final Answer" box?

2 sig figs

A. 2.65216 m

"weakest link": 3.2 ← 2 sig figs.

B. 2.65 m

C. 2.60 m

D. 2.6 m

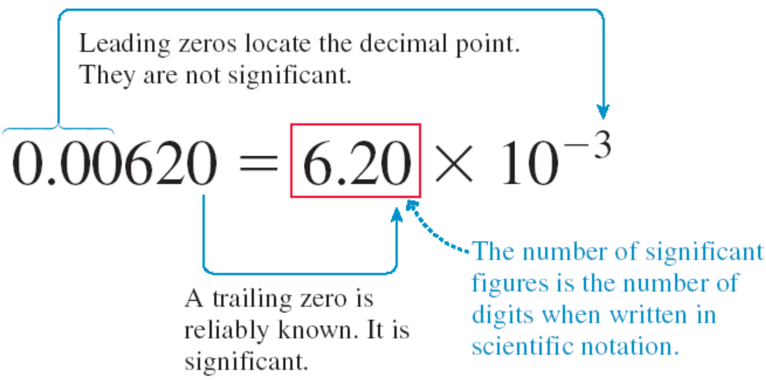
E. 2.7 m

Student comment on preclass quiz

- “The question about ranking Significant Figures was the most challenging for me because I had to let go of the notion that "more is more".”
- **Harlow comment:** Right! It is ***not*** always better to say “2.65216” if “2.7” is actually more honest.

From Knight Chapter 1:

FIGURE 1.25 Determining significant figures.



- The number of significant figures ≠ the number of decimal places.
- Changing units shifts the decimal point but does not change the number of significant figures.

Significant Figures

- Which of the following has the most number of significant figures?

A. 8200

B. 0.0052

C. 0.430

D. 4×10^{-23}

E. 8000.01

Student comment on last week's preclass quiz

- "Does 8200 have two or four significant figures?"
- **Harlow Answer:** I don't know!
- It would depend on the context. If someone was selling oil barrels and charged you for 8200 barrels, probably there are 4 significant figures since there's a lot of money involved. If someone says the distance from here to the nearest hospital is 8200 m, probably there are 2 significant figures, because it was rounded.
- Best practice in science: use scientific notation!
- 8.200×10^3 has 4 sig figs
- 8.2×10^3 has 2 sig figs

When do I round?

- The final answer of a problem should be displayed to the correct number of significant figures
- Numbers in intermediate calculations should ***not*** be rounded off
- It's best to keep lots of digits in the calculations to avoid round-off error, which can compound if there are several steps

Student comment on last week's preclass quiz

- "If the last digit is 5, how do we round the number if the second last digit is odd/even?"
- I always round 0,1,2,3,4 down and 5,6,7,8,9 up (seems fair)
- So 3.45 to two sig figs is 3.5
- And 3.35 to two sig figs 3.4
- The importance of significant figures when it comes to tests, exams, practicals, and homework questions (i.e. will we lose a mark if we have the incorrect number of significant figures?) would be useful to know.
- For Test 1 only, you will lose a small amount of marks for incorrect sig figs

Unit Conversion: What's wrong with this picture?

Maraj Harlow
Age: 10 months

Neither do I, Son, neither do I….

Convert 9 m to feet

Facts you are given:

$$1 = \left(\frac{1\text{ in}}{2.54\text{ cm}}\right) = \left(\frac{2.54\text{ cm}}{1\text{ in}}\right)$$

$$1 = \left(\frac{100\text{ cm}}{1\text{ m}}\right) = \left(\frac{1\text{ m}}{100\text{ cm}}\right) \qquad 1 = \left(\frac{12\text{ in}}{1\text{ foot}}\right) = \left(\frac{1\text{ foot}}{12\text{ in}}\right)$$

$$d = 9\ \text{m}\left(\frac{100\text{ cm}}{1\text{ m}}\right)$$

$$= 900\ \text{cm}\left(\frac{1\text{ in}}{2.54\text{ cm}}\right)$$

$$d = 354.3\ \text{in}\left(\frac{1\text{ foot}}{12\text{ in}}\right)$$

$$d = 29.53\text{ feet}$$

round to 1 sig fig

$$\boxed{d = 30\text{ feet}}$$

Real 2-hour Practicals begin this week!

- All you need is a calculator, your textbook and something to write with.
- Dr. Meyertholen has posted a good 6 minute video with answers to lots of FAQs about Practicals at:
- http://youtu.be/DVsLJgxJVgo

Class 3 Preclass Quiz on MasteringPhysics

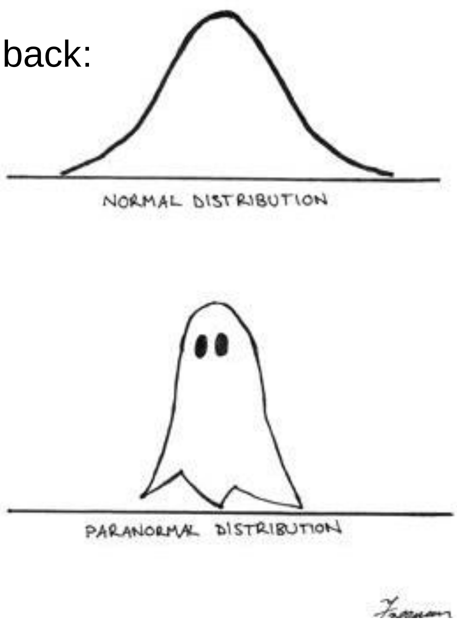
- This was due this morning at 8:00am
- 962 students submitted the quiz on time
- 73% correct: If a digital thermometer reads 12.8°C and the error specified in the manual is "half the last digit", in this case it would be ±0.05°C.
- 79% correct: In the physical sciences, an "error" is the number to the right of the ± symbol, also known as an uncertainty.
- 82% correct: In an equation for the normal distribution (Gaussian) sigma is related to the **width** of the distribution.
- 98% correct: "Mean" is the same as "average".

Class 3 Preclass Quiz on MasteringPhysics

- Some common student comments/feedback:
- I was most confused by the equation used to calculate standard deviation, as well as the meaning of "number of degrees of freedom".
- i was confused by how to use the three rules to propagate errors and find a final value
- I really wanted to chose the song by taylor swift for the definition of the mean, but I didn't.
- The concepts revolving around the histogram are a little confusing to me. I was wondering how some of the statistics were created. e.g., "68%" area under the curve of a Gaussian lies between the mean minus the standard deviation.

Class 3 Preclass Quiz on MasteringPhysics

- Some common student comments/feedback:



Last Wednesday I asked at the end of class:

- If your height is 150 cm, is there an error in that number?
- ANSWER: YES! Almost every measured number has an error, even if it is not stated. If you told me your height was 150 cm, I would guess the error is probably between 1 and 5 cm. [But there is no way to know this, unless you investigate how the 150 was measured.]

Demo and Example: What is the Period of a Swinging Ball? (Pendulum)

- Procedure: Measure the time for 5 oscillations, t_5.
- The period is calculated as $T = t_5 / 5$.

t_5 data: 7.53 s

7.38 s

Clicker Question

Here were Harlow's measurements of t_5:

7.53 s

7.38 s

7.47 s

7.43 s

Which of the following might be a good estimate for the error in Harlow's ***first*** measurement of 7.53 seconds?

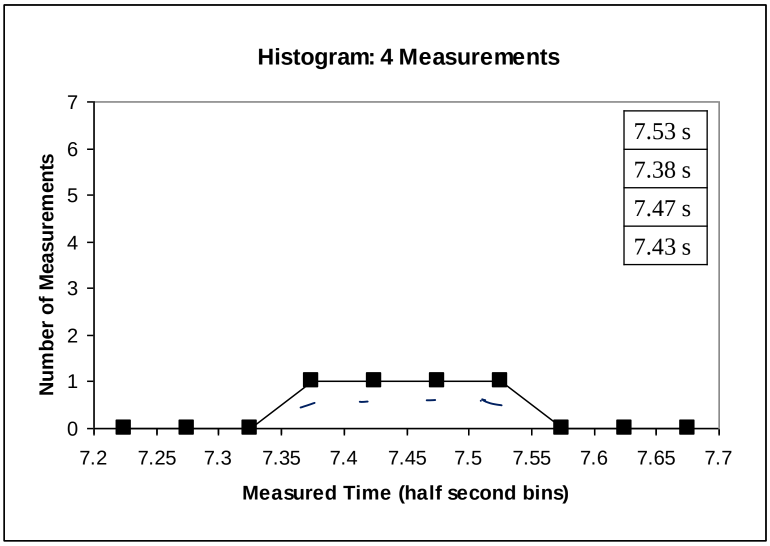
A. 0.005 s

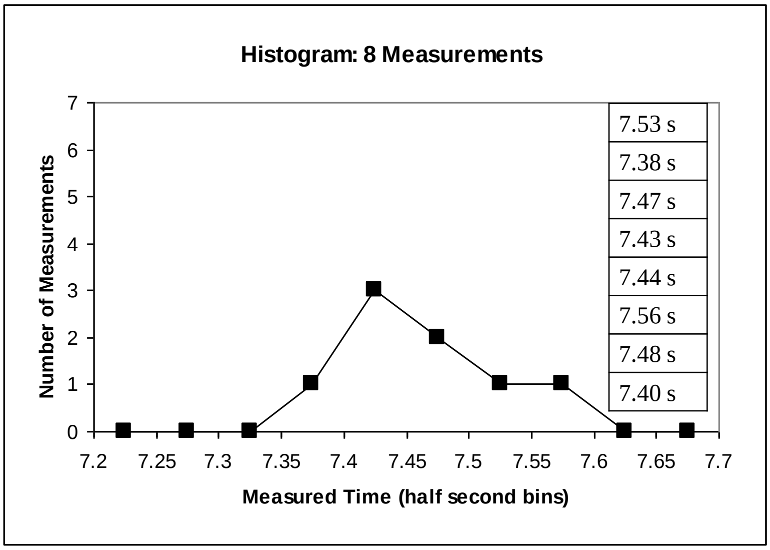
B. 0.05 s

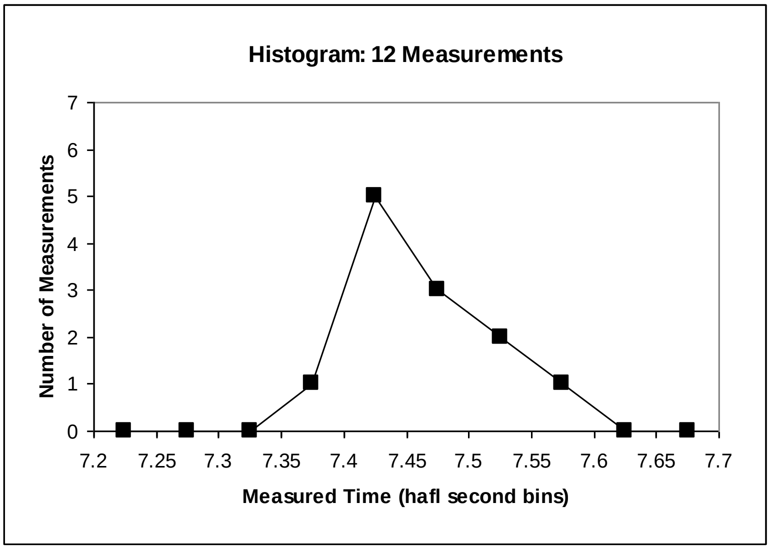
C. 0.5 s

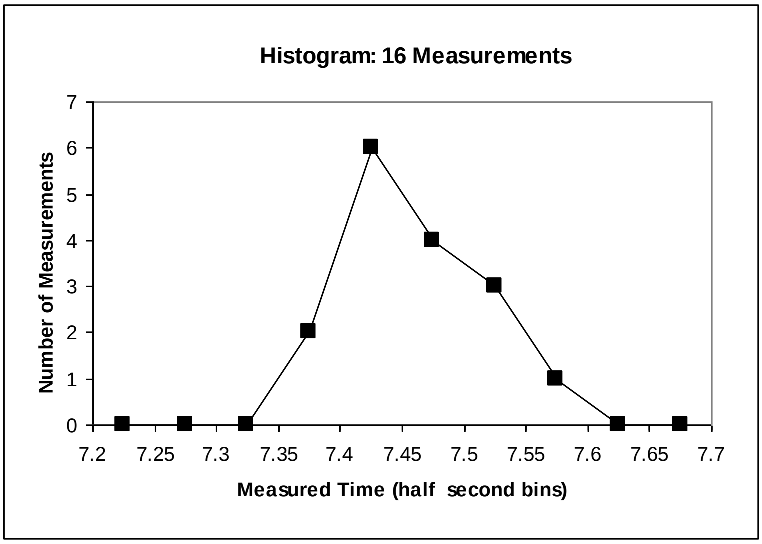
D. 5 s

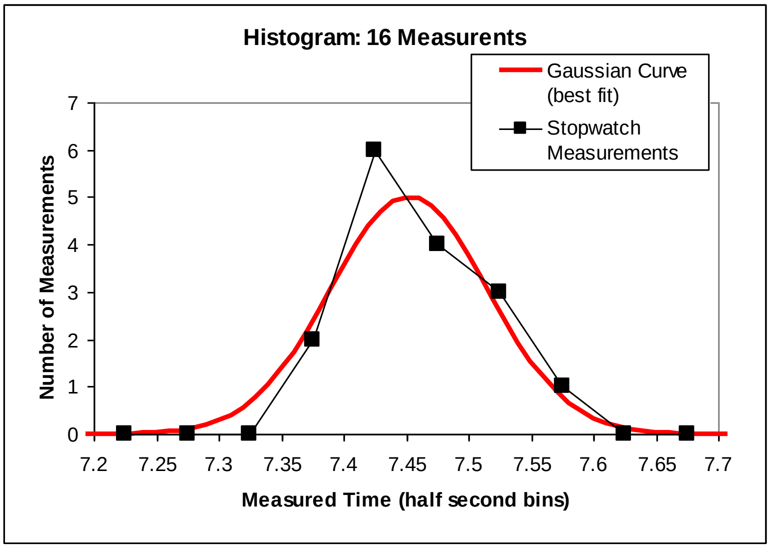
E. Impossible to determine




Histogram: 8 Measurements
Number of Measurements
Measured Time (half second bins)
0
1
2
3
4
5
6
7
7.2
7.25
7.3
7.35
7.4
7.45
7.5
7.55
7.6
7.65
7.7
7.53 s
7.38 s
7.47 s
7.43 s
7.44 s
7.56 s
7.48 s
7.40 s


Histogram: 12 Measurements
Number of Measurements
Measured Time (hafl second bins)
0
1
2
3
4
5
6
7
7.2
7.25
7.3
7.35
7.4
7.45
7.5
7.55
7.6
7.65
7.7

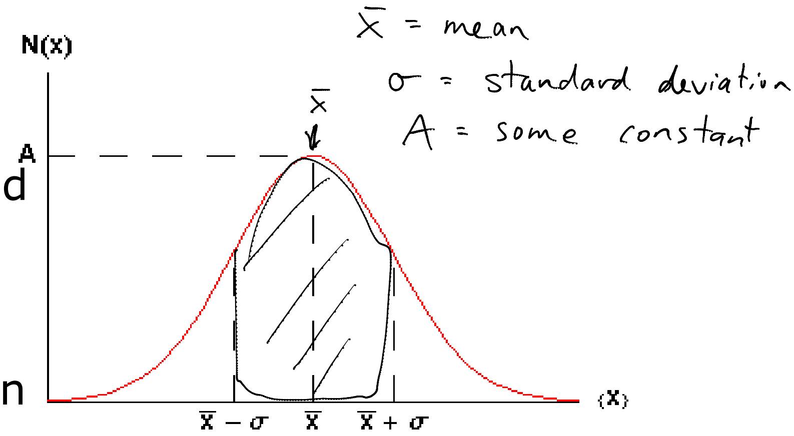

Histogram: 16 Measurements
Number of Measurements
0
1
2
3
4
5
6
7
7.2
7.25
7.3
7.35
7.4
7.45
7.5
7.55
7.6
7.65
7.7
Measured Time (half second bins)


Histogram: 16 Measurents
Gaussian Curve (best fit)
Stopwatch Measurements
Number of Measurements
0
1
2
3
4
5
6
7
7.2
7.25
7.3
7.35
7.4
7.45
7.5
7.55
7.6
7.65
7.7
Measured Time (half second bins)

The Gaussian:

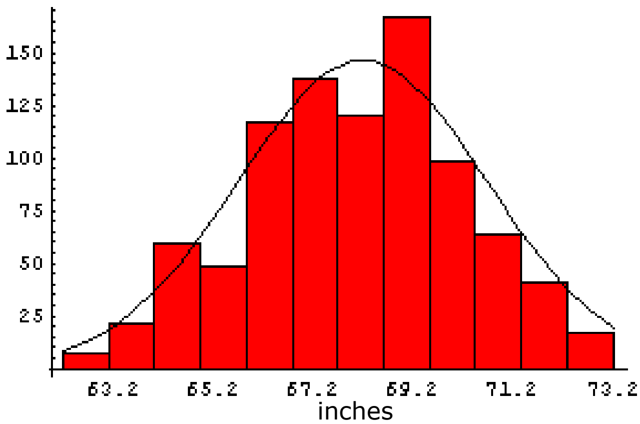
$$N(x) = Ae^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$$

When you integrate this curve, you find that 68% of area lies between the dotted lines on the graph.

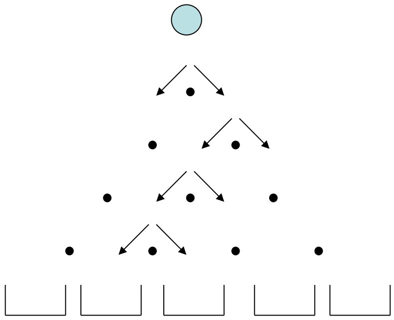


Gaussian Distributions turn up everywhere!

Heights of some People (London, 1886)



Random Walk



Where does an object end up, if it takes N steps randomly left or right?

The final distribution is described by a Gaussian function!

The t_5 data

7.53 s $\pm$ 0.06 s
7.38 s $\pm$ 0.06 s
7.47 s $\pm$ 0.06 s
7.43 s $\pm$ 0.06 s

Numerically:

$\bar{t}_{5,est}$ = 7.45250 s

σ_{est} = 0.0634429 s

σ_{est} = 0.06 s

error in any particular measurement.

Propagation of Errors

- Rule #1 (sum or difference rule):
- If $z = x + y$
- or $z = x - y$
- then $\Delta z = \sqrt{\Delta x^2 + \Delta y^2}$ ← "adding in quadrature"

Δx = error in measurement of x
Δy = " " y
Δz = error in computed value of z.

- Rule #2 (product or division rule):
- If $z = xy$
- or $z = x/y$
- then $$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

Propagation of Errors

- Rule #2.1 (multiply by exact constant rule):
- If $z = xy$
- and x is an exact number, so that Δx=0
- then $\Delta z = |x|(\Delta y)$ ← important in unit conversion

- Rule #3 (exponent rule):
- If $z = x^n$
- then $$\frac{\Delta z}{z} = n\frac{\Delta x}{x}$$

- What is the error in the mean of N measurements with the **same** error Δx ?

exact constant

Mean: $\bar{x} = \dfrac{(x_1 + x_2 + \dots + x_N)}{N} = \dfrac{x_{sum}}{N} = \left(\dfrac{1}{N}\right) x_{sum}$

Use Rule #1 to find error in x_{sum}, Δx_{sum}:

$$\Delta x_{sum} = \sqrt{\Delta x_1^2 + \Delta x_2^2 + \dots + \Delta x_N^2} = \sqrt{\Delta x^2 + \Delta x^2 + \dots + \Delta x^2}$$

$$= \sqrt{N \Delta x^2} = \sqrt{N}\, \Delta x$$

Rule #2.1: Multiply by exact constant:

$$\Delta \bar{x} = \frac{\Delta x_{sum}}{N} = \frac{\sqrt{N}\, \Delta x}{N}\left(\frac{\sqrt{N}}{\sqrt{N}}\right) = \frac{N \Delta x}{N\sqrt{N}} = \frac{\Delta x}{\sqrt{N}}$$

$$\boxed{\Delta \bar{x} = \frac{\Delta x}{\sqrt{N}}}$$ ← Error in the mean

The Error in the Mean

- Many individual, independent measurements are repeated N times
- Each individual measurement has the same error Δx
- Using error propagation you can show that the error in the estimated mean is:

$$\Delta \bar{x}_{\text{est}} = \frac{\Delta x}{\sqrt{N}}$$

Clicker Question

- You wish to know the time it takes to travel from Finch station to Yonge/Bloor by subway. You ask 10 people to take a stopwatch and time the trip. After analyzing all the data you find that it takes an average of 26 minutes and 40 seconds, with an error in this average of ± 100 seconds.
- If you expand your survey and ask 1000 people to time the trip, when you analyze the data, what would you expect to be the error in the average time?

A. 100 seconds
B. 50 seconds
C. 10 seconds
D. 1 second
E. 0.05 seconds

Significant Figures

- The rules for significant figures when errors are involved are:

1. **Errors should be specified to one or two significant figures.**
2. **The most precise column in the number for the error should also be the most precise column in the number for the value.**

Clicker Question

- Example: If a calculated result is d = 7.056 +/- 0.705 m, how should you report this?

A. 7.1 ± 0.7 m
B. 7.06 ± 0.71 m
C. 7.056 ± 0.705 m
D. Any of the above
E. Either A or B, but not C

Before Class 4 on Wednesday

- Please read Chapter 2, Sections 2.1-2.4 of Knight (or at least watch the pre-class video for Class 4)
- Please do the pre-class quiz before Wednesday at 8:00am.
- Don’t forget to start the Problem Set due Sunday – there’s no time limit, just a deadline
- Something to think about: Does constant velocity imply constant acceleration? Does constant acceleration imply constant velocity?