PHY131H1F - Class 8

Today, finishing off Chapter 4:

- Circular Motion
- Rotation

Clicker Question

Angular Notation: it's all Greek to me! $\boxed{\frac{d\theta}{dt} = \omega}$

θ is an angle, and the S.I. unit of angle is radians. (**NOT** degrees!)

The time derivative of θ is ω.

What are the S.I. units of ω ?

A. m/s^2

B. rad / s

C. N/m

D. rad

E. rad /s^2

Clicker Question

Angular Notation: it's all Greek to me!

$$\boxed{\frac{d\omega}{dt} = \alpha}$$

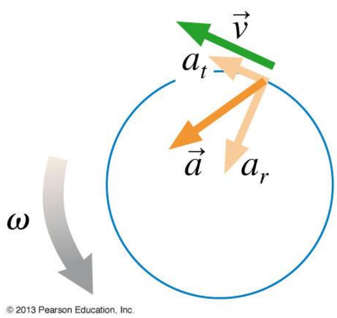
		analogous to:
θ =	"theta"	x
ω =	"omega"	v
α =	"alpha"	a

The time derivative of ω is α.
What are the S.I. units of α?

A. m/s^2
B. rad / s
C. N/m
D. rad
E. rad /s^2

Last day at the end of class I asked:

- Consider a wheel that is rotating, and speeding up.
- Is a point on the edge of the wheel accelerating toward the centre? [**Yes**, it must have a centre-pointing component in order to stay on the circular path!]
- Is this point accelerating in the forward direction? [**Yes**, it must have a forward component in order to speed up!]
- Or is it doing both? [**Yes** – the actual acceleration vector is on a diagonal!]



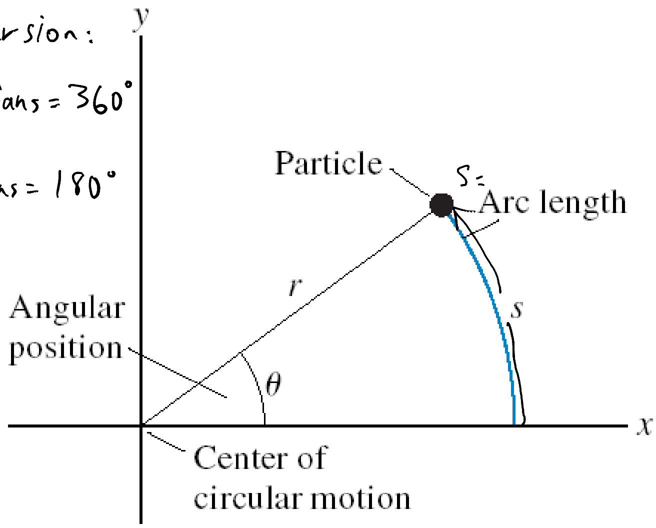
Circular Motion

Conversion:

2π radians $= 360°$

or

π radians $= 180°$



r = constant

units: m

s and θ both change as the particle moves

s = "arc length" m

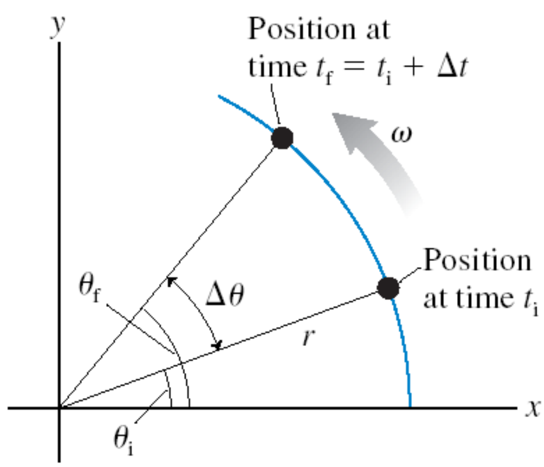
θ = "angular position" degrees or radians

$\boxed{s = r\theta}$ when θ is measured in radians

This eq. is not true if θ is measured in degrees.

Angular Velocity

$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$



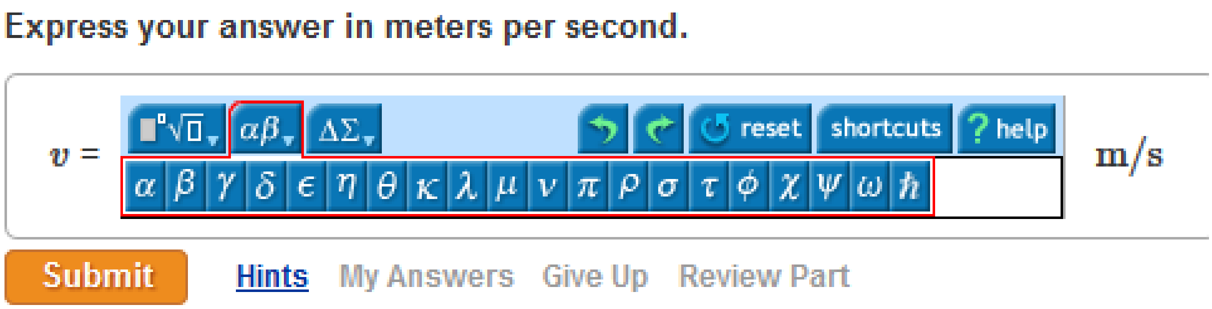
$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$\boxed{v_t = r\omega}$ when ω is measured in rad/s

Note on MasteringPhysics

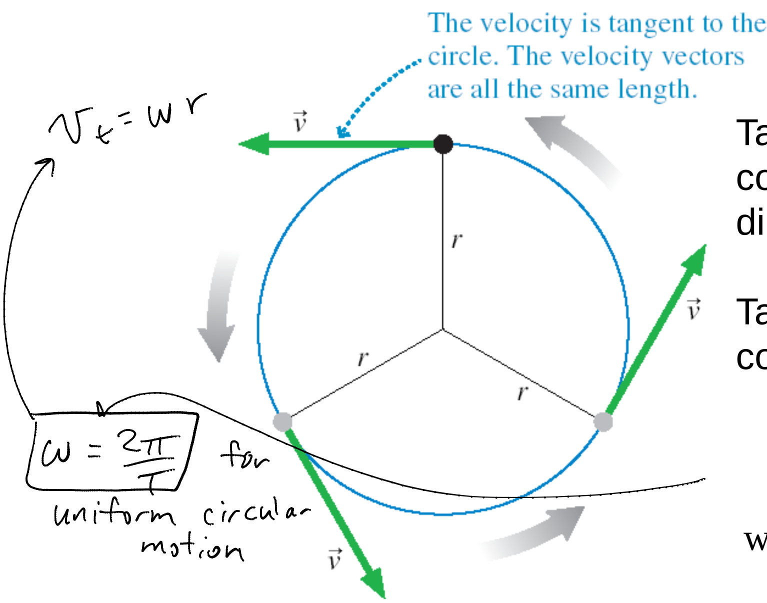
- ω: It's **not** a double-you, it's an omega.

On MasteringPhysics use the pull-down menu with Greek letters:



Special case of circular motion:

Uniform Circular Motion



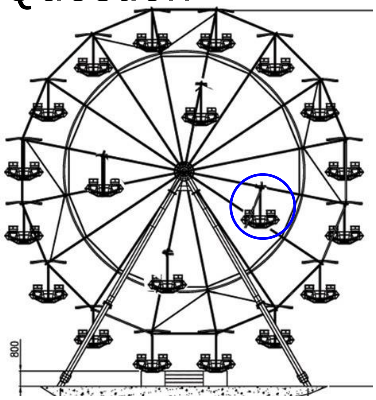
Tangential velocity is constantly changing direction

Tangential speed is constant

$$v_t = \frac{2\pi r}{T} = \frac{\text{circumference}}{\text{Period}}$$

where T = Period [s]

Clicker Question



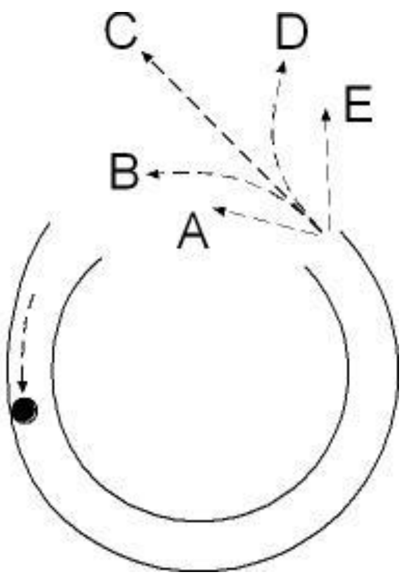
A carnival has a Ferris wheel where some seats are located halfway between the center and the outside rim. Compared with the seats on the outside rim, the inner cars have

A. Smaller angular speed and greater tangential speed
B. Greater angular speed and smaller tangential speed
C. The same angular speed and smaller tangential speed
D. Smaller angular speed and the same tangential speed
E. The same angular speed and the same tangential speed

Clicker Question

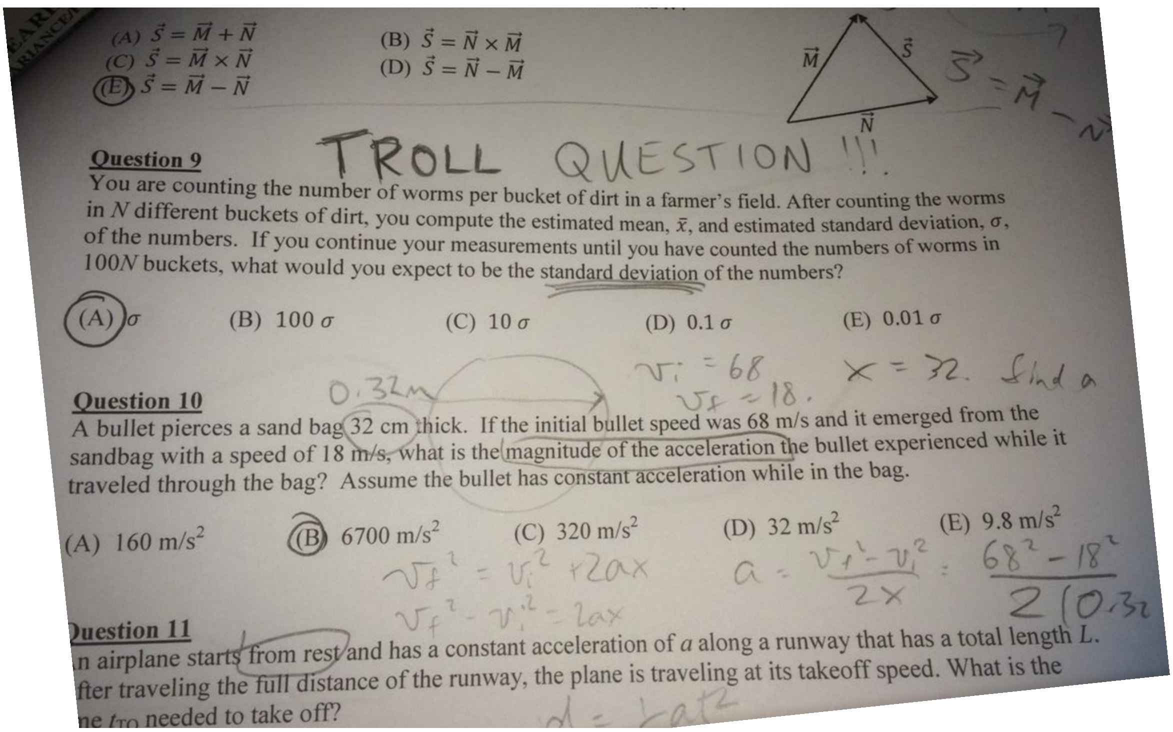
Demo and Discussion Question

A ball rolls in a horizontal circular track (shown from above). Which arrow best represents the ball's path after it leaves the track?



Preparation for Practicals next week:

- Take a ride on the Burton Tower elevators!
- All 4 elevators in the 14-storey tower of McLennan Physical Labs are equipped with a hanging spring-scale.
- It measures the upward force necessary to support a 750 g mass. (a.k.a. "weight")
- You may find that the measured weight of this object changes as you accelerate – check it out!



(A) $\vec{S} = \vec{M} + \vec{N}$ (B) $\vec{S} = \vec{N} \times \vec{M}$
(C) $\vec{S} = \vec{M} \times \vec{N}$ (D) $\vec{S} = \vec{N} - \vec{M}$
(E) $\vec{S} = \vec{M} - \vec{N}$

$\vec{M}$ $\vec{S}$ $\vec{N}$

$\vec{S} = \vec{M} - \vec{N}$

Question 9 TROLL QUESTION !!!

You are counting the number of worms per bucket of dirt in a farmer's field. After counting the worms in N different buckets of dirt, you compute the estimated mean, $\bar{x}$, and estimated standard deviation, σ, of the numbers. If you continue your measurements until you have counted the numbers of worms in $100N$ buckets, what would you expect to be the standard deviation of the numbers?

(A) σ (B) $100\,\sigma$ (C) $10\,\sigma$ (D) $0.1\,\sigma$ (E) $0.01\,\sigma$

0.32m $v_i = 68$ $v_f = 18$. $x = 32$. find a

Question 10

A bullet pierces a sand bag 32 cm thick. If the initial bullet speed was 68 m/s and it emerged from the sandbag with a speed of 18 m/s, what is the magnitude of the acceleration the bullet experienced while it traveled through the bag? Assume the bullet has constant acceleration while in the bag.

(A) 160 m/s^2 (B) 6700 m/s^2 (C) 320 m/s^2 (D) 32 m/s^2 (E) 9.8 m/s^2

$v_f^2 = v_i^2 + 2ax$

$v_f^2 - v_i^2 = 2ax$

$a = \frac{v_f^2 - v_i^2}{2x} = \frac{68^2 - 18^2}{2(0.32}$

uestion 11

n airplane starts from rest and has a constant acceleration of a along a runway that has a total length L. fter traveling the full distance of the runway, the plane is traveling at its takeoff speed. What is the ne t_{TO} needed to take off?

$d = \frac{1}{2}at^2$

The Pointing Game

- There are **three** pairs of orthogonal directions in this room.
- Within a pair, each direction is anti-parallel to the other.
- Each pair is perpendicular to the other two pairs.
- North - South
- West - East
- Up - Down

Survey

- How was length of the test yesterday?

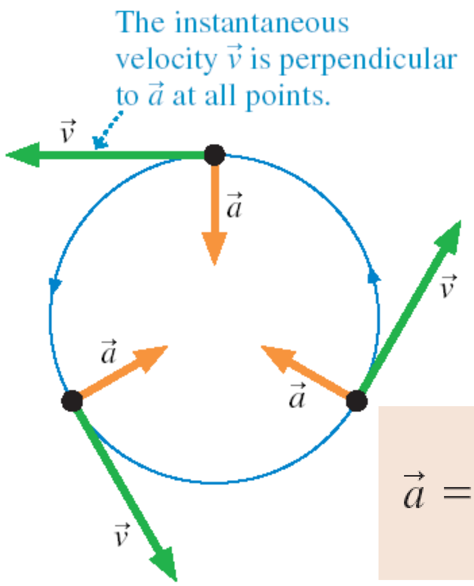
A. 80 minutes was more than enough time; I left early or was tempted to
B. I had enough time to complete the test with a few minutes to spare
C. I felt a little rushed but got the test done
D. I felt very rushed and needed more time
E. The test was FAR TOO LONG – no possible way to do in 80 minutes

Survey

- Given a choice on Test 2, would you prefer

A. 12 multiple choice questions worth 5 points each
B. 8 multiple choice questions worth 7.5 points each
C. Other / I prefer not to answer this question

Centripetal Acceleration



$$\vec{a} = \left(\frac{v^2}{r}, \text{ toward center of circle}\right)$$

Centripetal Acceleration

A bike wheel of diameter 1.0 m turns 20 times per second. What is the magnitude of the centripetal acceleration of a yellow dot on the rim?

Known: diameter = 1.0 m

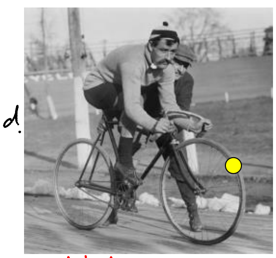
Need a

radius: $r = 0.5$ m

Constant ω, $\omega = \omega_{avg} = \frac{\Delta\theta}{\Delta t}$

$\underbrace{\text{20 revolutions}}_{\Delta\theta}$ in $\underbrace{\text{1 sec.}}_{\Delta t}$

↑ convert "revolutions" to radians.



Use 1 rev = 2π rad

$\Delta\theta = 20 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$

$\Delta\theta = 40\pi$ rad

$\omega = 40\pi \frac{\text{rad}}{s}$

Note: rad unit disappeared

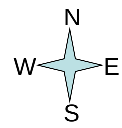
Tangential speed: $v_t = \omega r$
$= 40\pi\ (0.5 m)$
$= 20\pi \frac{m}{s}$

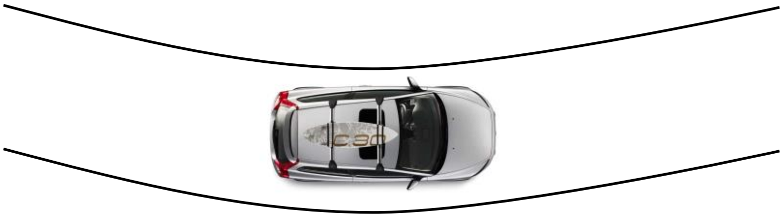
Use $a = \frac{v_t^2}{r} = \frac{(20\pi)^2}{0.5} = 7897 \frac{m}{s^2}$

$\boxed{a = 8000 \frac{m}{s^2}}$ = 800 gees!!

Clicker Question

A car is traveling East at a constant speed of 100 km/hr. Without speeding up of slowing down, it is turning left, following the curve in the highway. What is the **direction** of the acceleration?

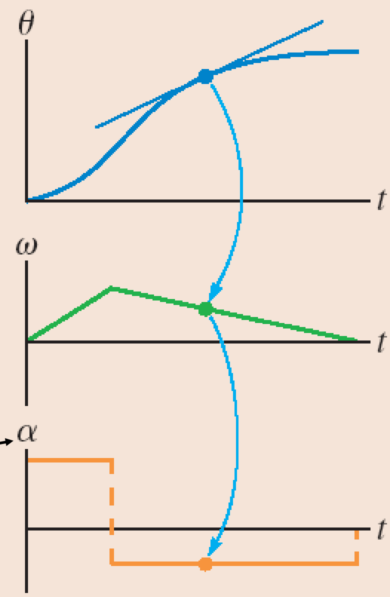




A. North
B. East
C. North-East
D. North-West
E. None; the acceleration is zero.

Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.



$a_{tangential} = \alpha \; r$

$\left[\frac{m}{s^2}\right]$ $\left[\frac{rad}{s^2}\right]$ $[m]$

Determines the *Tangential* acceleration, **NOT** centripetal acceleration!

unrelated:

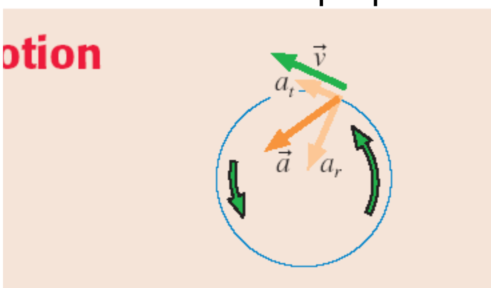
$a_{centripetal} = \frac{v_t^2}{r} = \omega^2 r$

Summary of definitions:

- θ is angular position. The S.I. Unit is radians, where 2π radians = 360°.
- s is the path length along the curve: $s = \theta r$ when θ is in [rad].
- ω is angular velocity. The S.I. Unit is rad/sec.
- v_t is the tangential speed: $v_t = \omega r$ when ω is in [rad/s].
- α is angular acceleration. The S.I. Unit is rad/sec^2.
- a_t is the tangential acceleration: $a_t = \alpha r$ when α is in [rad/s^2].

Nonuniform Circular Motion

- Any object traveling along a curved path has **centripetal acceleration**, equal to v^2/r.
- If, as it is traveling in a circle, it is speeding up or slowing down, it also has **tangential acceleration**, equal to $r\alpha$
- The total acceleration is the vector sum of these two perpendicular components



The 4 Equations of Constant Linear Acceleration, a:

$$v_f = v_i + at$$

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

$$v_f{}^2 = v_i{}^2 + 2a(x_f - x_i)$$

$$x_f = x_i + \left(\frac{v_i + v_f}{2}\right)t$$

The 4 Equations of Constant Angular Acceleration, α:

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f{}^2 = \omega_i{}^2 + 2\alpha(\theta_f - \theta_i)$$

$$\theta_f = \theta_i + \left(\frac{\omega_i + \omega_f}{2}\right)t$$

Clicker Question

Problem: A pebble is dropped from rest off a high balcony, and has an acceleration of 9.8 m/s^2 as it falls. It falls for 2.5 seconds, then hits the ground. How far does it fall in this 2.5 seconds?

Which equation would you use?

A. $v_f = v_i + at$

B. $x_f = x_i + v_i t + \frac{1}{2}at^2$

C. $v_f{}^2 = v_i{}^2 + 2a(x_f - x_i)$

D. $x_f = x_i + \left(\frac{v_i + v_f}{2}\right)t$

Clicker Question

Problem: A centrifuge loaded with two test-tubes starts from rest, and has an angular acceleration of 150 rad/s^2 as it spins up. It speeds up with this angular acceleration for 2.5 seconds, then it has reached its maximum spin rate. How many times has it rotated in this 2.5 seconds?

Which equation would you use?

A. $\omega_f = \omega_i + \alpha t$

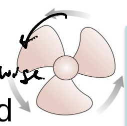
B. $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

C. $\omega_f{}^2 = \omega_i{}^2 + 2\alpha(\theta_f - \theta_i)$

D. $\theta_f = \theta_i + \left(\frac{\omega_i + \omega_f}{2}\right) t$

Example.

Define + = counterclockwise



- A fan is spinning at 30 rad/s, and suddenly starts slowing down.
- It's angular acceleration as it slows is 10 rad/s^2.
- How long does it take to stop spinning?

$\omega_i = +30 \ \frac{rad}{s}$

SIGNS?

slowing down → ω & α have opposite signs.

$\alpha = -10 \ \frac{rad}{s^2}$

find t.

$\omega_f = 0$

Don't care about $\Delta\theta$

Use:

$\omega_f = \omega_i + \alpha t$

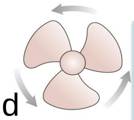
solve for t:

$t = \frac{\omega_f - \omega_i}{\alpha}$

$= \frac{0 - 30}{-10}$

$\boxed{t = 3 \text{ seconds}}$

Example.



- A fan is spinning at 30 rad/s, and suddenly starts slowing down.
- It's maximum angular acceleration as it slows is 10 rad/s^2.
- What is the minimum angle that it must turn as it stops?
- How many revolutions is this?

$\omega_i = +30 \frac{rad}{s}$

Set $\alpha = \alpha_{max}$ for $\Delta\theta_{min}$

$\alpha = -10 \frac{rad}{s^2}$

find $\Delta\theta$, convert to rev.

Don't care about t:

$$\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$$

Solve for $\Delta\theta$:

$$2\alpha\Delta\theta = \omega_f^2 - \omega_i^2$$

$$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha}$$

$$= \frac{0^2 - 30^2}{2(-10)}$$

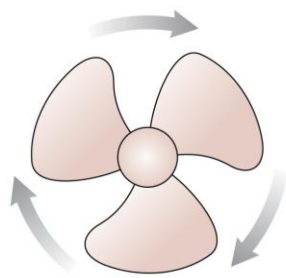
$$\boxed{\Delta\theta = 45 \text{ rad}}$$

Use 2π rad = 1 rev.

$$\Delta\theta = 45 \cancel{\text{rad}}\left(\frac{1 \text{ rev}}{2\pi \cancel{\text{rad}}}\right) = \boxed{7.2 \text{ rev}}$$

Clicker Question

The fan blade is slowing down. What are the signs of ω and α? [Let's define, as Knight often does, positive to be counter-clockwise.]



A. ω is positive and α is positive.
B. ω is negative and α is positive.
C. ω is positive and α is negative.
D. ω is negative and α is negative.

Moving on to Chapters 5 and 6..

- Up until now, we have been studying **kinematics**, a description of HOW things move and how to describe this.

- In Chapter 5 we begin to study WHY things move the way they do: This is **dynamics**, which includes the important concepts of **Force** and **Energy**.

Before Class 9 on Monday

- Please read **Chapter 5** of Knight.
- Don’t forget the pre-class quiz due Mon. at 8am.
- Something to think about: A paperback novel has a mass of 0.3 kg and slides at a constant velocity. A physics textbook has a mass of 3.0 kg, and slides at the **same** constant velocity. How does the net force on the textbook compare to the net force on the novel?