Harlow Solutions. PHY 131 Test 1 Fall 2014. Sep. 30, 2014

MULTIPLE CHOICE [5 points per question × 12 questions = 60 points total]

Possibly helpful information for this test:
$\pi = 3.14159$ is the ratio of the circumference to the diameter of a circle
$g = 9.80$ m/s^2 is the acceleration due to gravity near the Earth's surface.

Common Prefixes:
k = "kilo-" = 10^3 c = "centi-" = 10^{-2} m = "milli-" = 10^{-3} μ = "micro-" = 10^{-6}
n = "nano-" = 10^{-9}

60 seconds = 1 minute; 60 minutes = 1 hour; 24 hours = 1 day; 365.25 days = 1 year
1 litre = 1000 cm^3

The quadratic equation: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Air resistance may be neglected in all questions, unless otherwise stated.

Question 1
You are counting the number of worms per bucket of dirt in a farmer's field. After counting the worms in N different buckets of dirt, you compute the estimated mean, $\bar{x}$, and estimated standard deviation, σ, of the numbers. If you continue your measurements until you have counted the numbers of worms in $100N$ buckets, what would you expect to be the standard deviation of the numbers?

(A) $0.01\,\sigma$ (B) $0.1\,\sigma$ (C) σ (D) $10\,\sigma$ (E) $100\,\sigma$

σ is the width of the distribution, which should not change with N.

Question 2
Your heart pumps approximately 80 cm^3 of blood with each beat. How many litres of blood will your heart pump during your lifetime? [Assume 70 beats/minute and a life span of 90 years.]

(A) 5×10^5 (B) 1×10^7 (C) 4×10^6 (D) 3×10^8 (E) 3×10^{11}

$$80\ \frac{\text{cm}^3}{\text{beat}}\left(\frac{1\ \ell}{1000\ \text{cm}^3}\right)\left(70\ \frac{\text{beats}}{\text{min}}\right)\left(60\ \frac{\text{min}}{\text{hr}}\right)\left(24\ \frac{\text{hr}}{\text{day}}\right)\left(365.25\ \frac{\text{day}}{\text{yr}}\right)\left(90\ \frac{\text{yr}}{\text{life}}\right) = 2.65\times10^8\ \frac{\ell}{\text{life}}$$

Question 3
You wish to calculate the density of a solid sphere of foam by measuring its mass and diameter. Density = mass/volume, and the volume of a sphere = $\frac{\pi}{6}d^3$, where d is the diameter. You can measure the mass of the sphere of foam to a percentage error of 1%, and the diameter to a percentage error of 2%. What will be the percentage error in the density of foam that you compute?

$\left(\frac{\Delta m}{m}\right) = 0.01$ $\left(\frac{\Delta d}{d}\right) = 0.02$

(A) 1% (B) 2% (C) 3% (D) 4% (E) 6%

Set $z = d^3$

Rule #3: $z = x^N$, $\frac{\Delta z}{z} = N\frac{\Delta x}{x}$

$$\frac{\Delta z}{z} = 3\left(\frac{\Delta d}{d}\right) = 3(0.02) = 0.06$$

$V = \frac{\pi}{6}z$ Rule#2.1: $\Delta V = \frac{\pi}{6}\Delta z$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta z}{z} = 0.06$$

Density $= \rho = \frac{m}{V}$

Rule#2: $\frac{\Delta\rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta V}{V}\right)^2}$

$$= \sqrt{0.01^2 + 0.06^2}$$

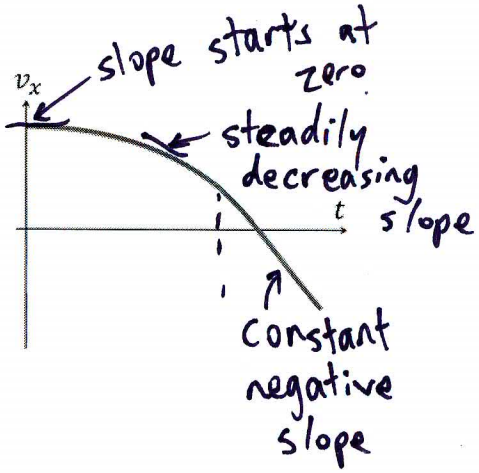
$\frac{\Delta\rho}{\rho} = 0.0608$

$\approx \boxed{6.1\%}$

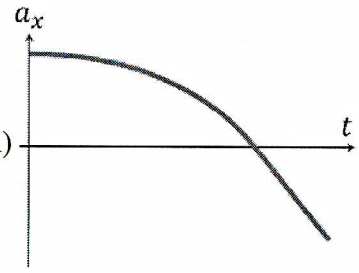
HARLOW SOLUTIONS Fall 2014

Question 4

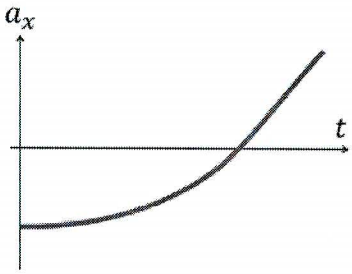
The velocity-versus-time graph for an object is shown to the right. Which figure below best represents the object's acceleration-versus-time graph?



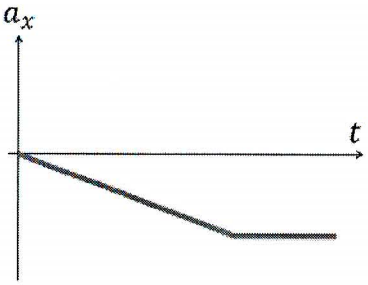
(A)



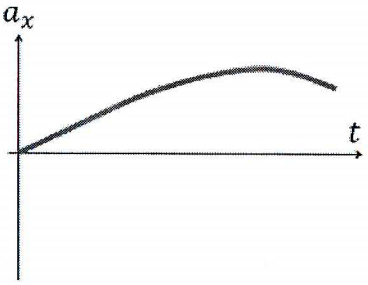
(B)



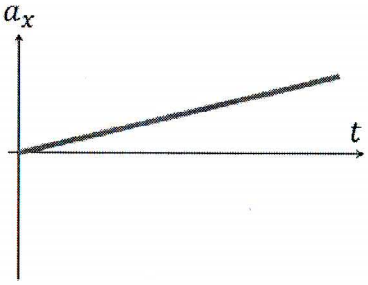
(C)



(D)



(E)



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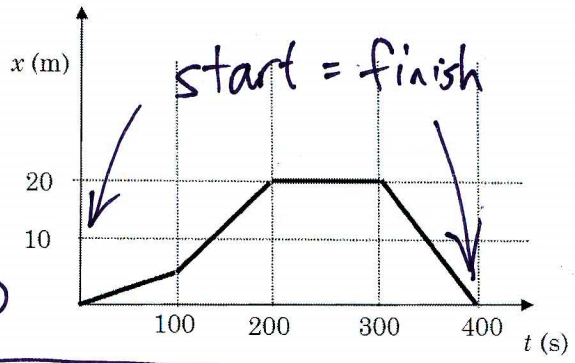
Question 5

Peter goes for a walk along the x-axis. The walk takes him 400 seconds to complete. The graph below shows his position x as a function of time t. What was the average velocity of the walk?



(A) +0.1 m/s (B) −0.2 m/s
(C) +11 m/s (D) +20 m/s
(E) zero

Displacement $\Delta\vec{x} = 0$

$\vec{v}_{avg} = \frac{\Delta\vec{x}}{\Delta t} = 0$

Question 6

A bullet pierces a sand bag 32 cm thick. If the initial bullet speed was 68 m/s and it emerged from the sandbag with a speed of 18 m/s, what is the average magnitude of the acceleration the bullet experienced while it traveled through the bag?

$= 0.32$ m

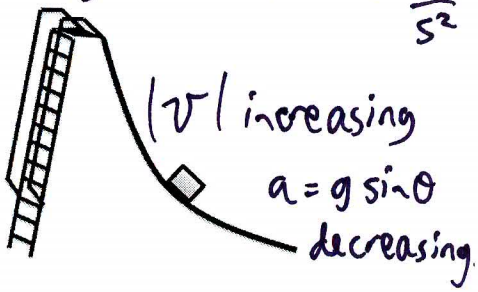
Use $v_f^2 = v_i^2 + 2ax$

(A) 9.8 m/s^2 (B) 32 m/s^2 (C) 160 m/s^2 (D) 320 m/s^2 (E) 6700 m/s^2

$\Rightarrow a = \frac{v_f^2 - v_i^2}{2x} = \frac{18^2 - 68^2}{2(0.32)} = -6719\ m/s^2$ $\quad |a| = 6700\ \frac{m}{s^2}$

Question 7

A box slides down a slippery, curved slide. Air resistance and friction are negligible. When the box is at the point shown in the drawing,



(A) the box is speeding up, and the magnitude of its acceleration is increasing.
(B) the box is speeding up, but the magnitude of its acceleration is constant.
(C) the box is speeding up, but the magnitude of its acceleration is decreasing.
(D) the speed of the box is constant, and the magnitude of its acceleration is also constant.
(E) the speed of the box is constant, but the magnitude of its acceleration is increasing.

Question 8

An airplane starts from rest and has a constant acceleration of a along a runway that has a total length L. After traveling the full distance of the runway, the plane is traveling at its takeoff speed. What is the time t_{TO} needed to take off?

$d = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2d}{a}}$ $\quad d = L$

(A) $\sqrt{\frac{2L}{a}}$ (B) $\sqrt{\frac{L}{a}}$ (C) $\frac{L}{a}$ (D) $\frac{2L}{a}$ (E) La

Question 9

The position of an object as a function of time is given by $x = bt^2 - ct$. What is the instantaneous velocity of the object as a function of time?

(A) $bt^2 - ct$ (B) $2bt - c$ (C) $2bt^2 - c$ (D) $\frac{bt^3}{3} - \frac{ct^2}{2}$ (E) $(b - c)t$

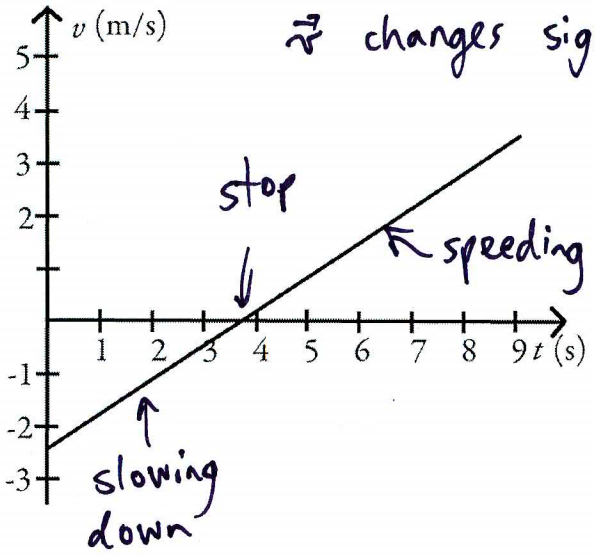
$v = \frac{dx}{dt} = \frac{d}{dt}(bt^2 - ct) = b\frac{d(t^2)}{dt} - c\frac{dt}{dt} = 2bt - c$

HARLOW SOLUTIONS FALL 2014

Question 10

The motion of a particle is described in the velocity versus time graph shown in the figure. Over the time interval $t = 0$ to 9 seconds, we can say that its speed

(A) increases.
(B) decreases.
(C) increases and then decreases.
(D) decreases and then increases.
(E) stays the same.

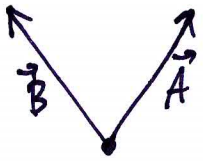


$\vec{a}$ = always positive.
$\vec{v}$ changes sign.

Question 11

The eastward component of vector $\vec{A}$ is not zero, and it is equal to the westward component of vector $\vec{B}$. The northward components of $\vec{A}$ and $\vec{B}$ are equal, and not zero. Given this information, which one of the following statements about these two vectors must be correct?

(A) Vector $\vec{A}$ is parallel to vector $\vec{B}$. No.
(B) Vectors $\vec{A}$ and $\vec{B}$ point in opposite directions. No.
(C) Vector $\vec{A}$ is perpendicular to vector $\vec{B}$. No (Not necessarily)
(D) The magnitude of vector $\vec{A}$ is equal to the magnitude of vector $\vec{B}$. Yes.
(E) The magnitude of vector $\vec{A}$ is twice the magnitude of vector $\vec{B}$. No.



Define:

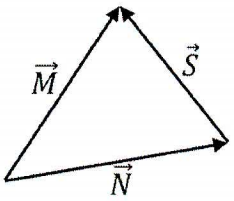
$A_y = B_y$

$-A_x = B_x$

Question 12

For the diagram shown, what is the vector $\vec{S}$ in terms of $\vec{M}$ and $\vec{N}$?

(A) $\vec{S} = \vec{M} - \vec{N}$
(B) $\vec{S} = \vec{N} - \vec{M}$
(C) $\vec{S} = \vec{M} + \vec{N}$
(D) $\vec{S} = \vec{M} \times \vec{N}$
(E) $\vec{S} = \vec{N} \times \vec{M}$



$\vec{N} + \vec{S} = \vec{M}$

$\Rightarrow \vec{S} = \vec{M} - \vec{N}$

FREE-FORM PART (40 points total)
Clearly show your reasoning and work as some part marks may be awarded. Write your final answers in the boxes provided.

PART A
A toy rocket, initially at rest on the ground, accelerates straight upward with constant acceleration of 5.6 m/s^2. The acceleration period lasts for time 2.5 s until the fuel is exhausted. After that, the rocket is in free fall.

2 sig. figs.

1. [**10 points**] What is the maximum height reached by the rocket?

free fall | Seg 2.

Seg. 1 | $\vec{a}$↑

Segment 1: $a_{y1} = +5.6\ m/s^2$ $t_1 = 2.5\ s$

$v_{f1} = 0 + a_{y1} t_1$ $v_{i1} = 0$ $y_{i1} = 0$

$v_{f1} = 14\ m/s$

$y_{f1} = 0 + 0 + \frac{1}{2} a_{y1} t_1^2 = \frac{1}{2}(5.6)2.5^2$

$y_{f1} = 17.5\ m$

Segment 2: $a_{y2} = -g$ (freefall).

$y_{i2} = y_{f1} = 17.5\ m$

$v_{i2} = v_{f1} = 14.\ m/s$

At top of path $v_{f2} = 0$

$v_{f2}^2 = v_{i2}^2 + 2a_2(\Delta y)$

$\Delta y = \frac{v_{i2}^2 - v_{f2}^2}{2g} = \frac{14^2}{2(9.8)} = 10\ m$

$y_{f2} = 17.5\ m + \Delta y = 27.5\ m$

$y_{f2} = 28\ m$

2 sig. figs.

0.5 for correct rounding.

2. [**10 points**] What is the time between the initial launch and when the rocket hits the ground again?

After top of path:

Use: $\Delta y = v_i t_3 + \frac{1}{2} a_y t_3^2$

$\Delta y = 0 - \frac{1}{2} g t_3^2 = -27.5\ m$

$t_3 = \sqrt{\frac{2(-27.5)}{-9.8}} = 2.369\ s$

$\Delta v_2 = a_2 t_2$

$t_2 = \frac{\Delta v_2}{g}$

$t_2 = \frac{-14\ m/s}{9.8}$

$t_2 = 1.42857\ s$

Segment 3. freefall 27.5 m, released from rest.

$t_{total} = t_1 + t_2 + t_3 = (2.5) + (1.42857) + (2.369)$

$t_{total} = 6.2976\ s$

6.3 s

0.5 for correct rounding.

2 sig figs.

PART B

A research student on a farm counts the number of worms per bucket of dirt in a farmer's field. She counts the worms in six different equally-sized buckets, as recorded in the table below.

Number of worms:	70	80	89	68	103	71

1. [**16 points**] Based on these data, what is the mean number of worms per bucket? Be sure to include the error of the mean, and express both the mean and the error to the correct number of significant figures.

$\bar{X} = \frac{(70+80+89+68+103+71)}{6} = 80.1667$ +2

$(70-80.1667)^2 = 103.36$
$(80-80.1667)^2 = 0.03$
$(89-80.1667)^2 = 78.03$
$(68-80.1667)^2 = 148.03$
$(103-80.1667)^2 = 521.36$
$(71-80.1667)^2 = 84.03$

Sum: 186.968

$\sigma = \sqrt{\frac{\text{Sum}}{5}} = 13.673$ +6

Error in the mean $= \frac{\sigma}{\sqrt{N}}$

$\frac{13.673}{\sqrt{6}} = 5.58$ +4

80 ± 6 ← 1 sig fig

or 80.2 ± 5.6 ← 2 sig fig

0.5 for correct rounding.

2. [**4 points**] How many of the six measurements lie within plus-or-minus one standard deviation of the mean?

$\pm 1\sigma$ is $66.5 \rightarrow 93.8$ +2

70 : yes .
80 : yes .
89 : yes .
68 : yes .
103 : NO
71 : yes .

5