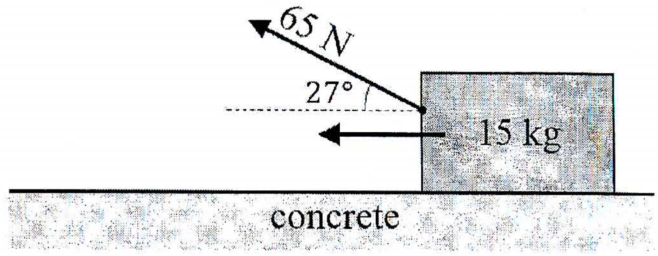
FREE-FORM PART (12 points total)
For full marks, you must clearly show all of your work and reasoning in the space provided. State any assumptions you make, and show all the steps of your calculations. Write your final answers in the boxes provided.
For both problems in the Free-form part, assume:

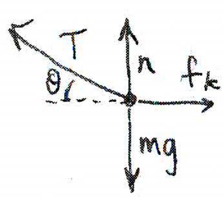
The coefficient of **kinetic** friction between wood and concrete is $\mu_k = 0.15$.
The coefficient of **static** friction between wood and concrete is $\mu_s =$ 0.30

Problem A
A wooden box of mass 15 kg is pulled to the left by a constant tension force of 65 N at an angle of 27° above the horizontal. It starts from rest and slides forward along a smooth, flat, horizontal concrete floor.



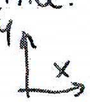
1. (4 points) How much time does it take the box to get up to a speed of 3.0 m/s?

fbd of box:



$f_k = \mu_k n$

Define:



cannot accelerate along y:

$(F_{net})_y = 0 = T\sin\theta + n - mg \Rightarrow \boxed{n = mg - T\sin\theta}$ +1

x-acceleration:

$(F_{net})_x = \boxed{ma_x = -T\cos\theta + \mu_k n}$ +1

$a_x = \frac{1}{m}\left(-T\cos\theta + \mu_k(mg - T\sin\theta)\right)$

$= \frac{1}{15}\left(-65\cdot\cos 27 + 0.15\left(15(9.8) - 65\sin 27\right)\right)$

$\boxed{a_x = -2.6861\ m/s^2}$ +1

$\frac{\Delta v}{\Delta t} = a_x$ +1 $\boxed{\Delta t = \frac{\Delta v}{a_x}} = \frac{-3}{-2.6861} = 1.117\ s$

$t =$ 1.1 s

−0.5 for missing units

2. (2 points) How much thermal energy is generated by friction during this time?

$E_{th} = f_k \cdot d$ $\quad d = \frac{1}{2}at^2, \quad f_k = \mu_k(mg - T\sin\theta)$

$E_{th} = \underbrace{\mu_k(mg - T\sin\theta)}_{+0.5}\underbrace{\frac{at^2}{2}}_{+0.5} = 0.15\left(15(9.8) - 65\sin 27°\right)\cdot\frac{2.6861}{2}\times 1.117^2$

+1 $\boxed{E_{th} = 29.53\ J}$

$E_{th} =$ 29.5 J

(3.0 × 10¹ J) −0.5 for missing units.

No marks deducted for wrong # of sig. figs.

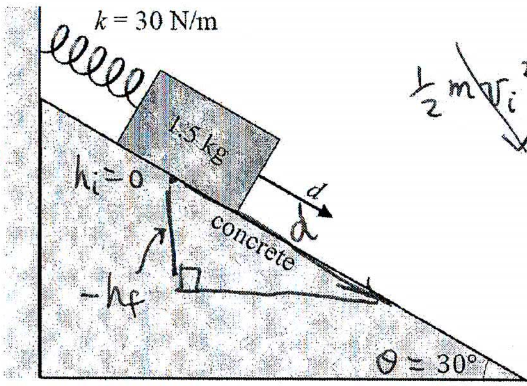
Notes: (0 for force attempt, 1 for any energy/work attempt.)

Problem B (6 points)

1. (5 points) An unstretched spring is attached to a 1.5 kg wooden block on a concrete ramp which makes an angle of 30° with respect to the horizontal. The other end of the spring is fixed. The mass is released and it slides down the ramp and stretches the spring. The spring has a constant of 30 N/m. Find the maximum distance that the block slides down the ramp.

$\mu_k = 0.15$

$\mu_s = 0.30$



$\sin 30° = \frac{-h_f}{d}$

$$\boxed{E_i = E_f} \; +1 \qquad E_{th}$$

$$\tfrac{1}{2}m\underbrace{v_i^2}_{0} + mg\underbrace{h_i}_{0} + \tfrac{1}{2}k\underbrace{x_i^2}_{0} = \tfrac{1}{2}m\underbrace{v_f^2}_{0} + mgh_f + \tfrac{1}{2}kx_f^2 + f_k d$$

$x_f = d$, $h_f = -d\sin 30°$

$+1$

$$0 = mg(-d\sin 30°) + \tfrac{1}{2}kd^2 + \mu_k n d$$

$$\boxed{\frac{kd^2}{2}} + \boxed{\mu_k mg\cos\theta\, d} = \boxed{mgd\sin\theta} \; +1$$

$+1$

$d \neq 0$, so divide both sides by d.

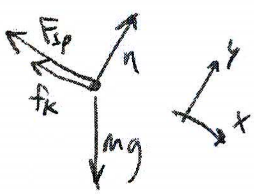
$$\frac{kd}{2} = mg[\sin\theta - \mu_k\cos\theta]$$

$$d = \frac{2mg}{k}[\sin\theta - \mu_k\cos\theta]$$

$$d = \frac{2(1.5)9.8}{30}[\sin 30 - 0.15\cos 30]$$

$$\boxed{d = 0.36269 \text{ m}} \; +1$$

fbd:

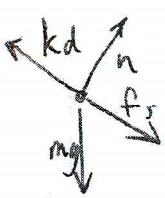


$(F_{net})_y = 0 = n - mg\cos\theta$

$n = mg\cos\theta$

If it stops, f_s holds it there.

fbd:



$(F_{net})_x = 0 = f_s - kd + mg\sin\theta$

$f_s = kd - mg\sin\theta = 3.53$ N

Needed

f_s max: $\mu_s mg\cos\theta = 3.82$ N

Needed < max, so it sticks.

d_{max} = 0.36 m

−0.5 for missing units

2. (1 points) When the block gets to this position, does it stay still, or will it start back up the ramp? (Circle one)

STAY STILL START BACK UP

Multiple choice ONLY

+1 for stays still

0 for starts back up

Special Cases

1. Assume $\mu_k = 0$

$d = 0.49$ m 3/5

2. Assume $\mu_k = 0.3$

$d = 0.235$ m 4/5