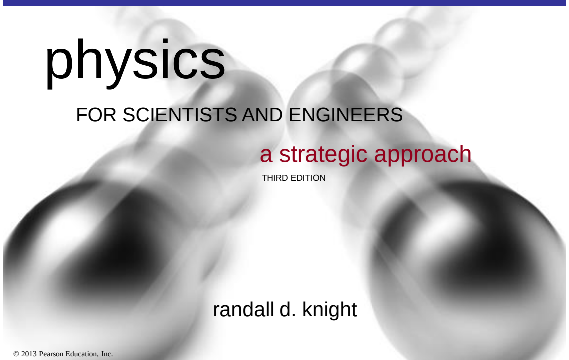
Class 3, Sections 21.1-21.4 Preclass Notes



physics

FOR SCIENTISTS AND ENGINEERS

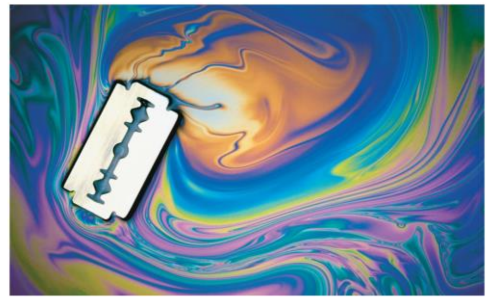
a strategic approach

THIRD EDITION

randall d. knight

ALWAYS LEARNING PEARSON

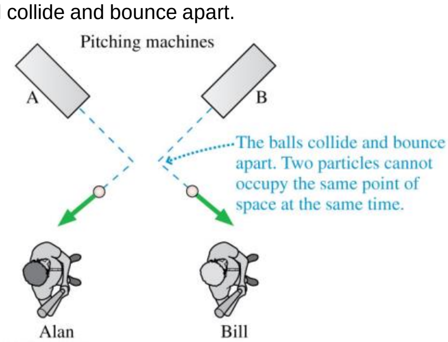
Chapter 21 Superposition



Chapter Goal: To understand and use the idea of superposition.

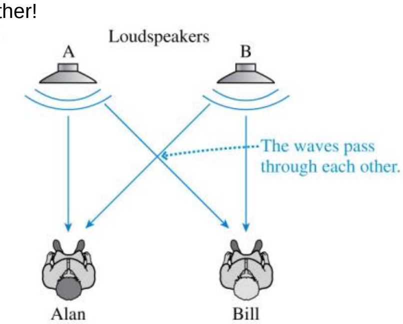
Particles vs. Waves

- Two particles flying through the same point at the same time will collide and bounce apart.



Particles vs. Waves

- But waves, unlike particles, can pass directly through each other!



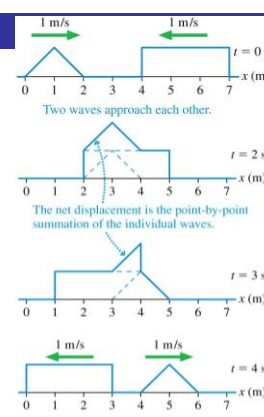
The Principle of Superposition

If wave 1 displaces a particle in the medium by D_1 and wave 2 simultaneously displaces it by D_2, the net displacement of the particle is $D_1 + D_2$.

Principle of superposition When two or more waves are *simultaneously* present at a single point in space, the displacement of the medium at that point is the sum of the displacements due to each individual wave.

The Principle of Superposition

- The figure shows the superposition of two waves on a string as they pass through each other.
- The principle of superposition comes into play wherever the waves overlap.
- The solid line is the sum at each point of the two displacements at that point.

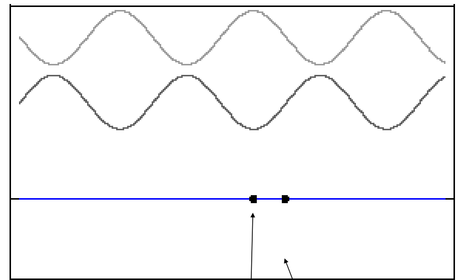


Standing Waves

- Shown is an animation of a *standing wave* on a vibrating string.
- It's not obvious from the animation, but this is actually a superposition of two waves.
- To understand this, consider two sinusoidal waves with the **same frequency, wavelength, and amplitude** traveling in opposite directions.

[animation from http://www.answers.com/topic/wave]

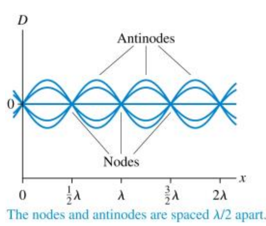
Standing Waves



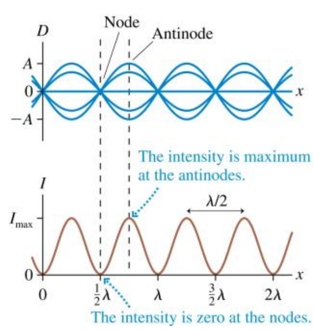
[image from http://www.edu.pe.ca/gray/class_pages/krcutcliffe/physics521/14waves/applets/Standing%20Waves1.htm]

Standing Waves

- The figure has collapsed several graphs into a single graphical representation of a standing wave.
- A striking feature of a standing-wave pattern is the existence of **nodes**, points that *never move!*
- The nodes are spaced $\lambda/2$ apart.
- Halfway between the nodes are the **antinodes** where the particles in the medium oscillate with maximum displacement.



Standing Waves



- In Chapter 20 you learned that the *intensity* of a wave is proportional to the square of the amplitude: $I \propto A^2$.
- Intensity is maximum at points of constructive interference and zero at points of destructive interference.

Standing Waves

- This photograph shows the Tacoma Narrows suspension bridge just before it collapsed.
- Aerodynamic forces caused the amplitude of a particular standing wave of the bridge to increase dramatically.
- The red line shows the original line of the deck of the bridge.

The Mathematics of Standing Waves

- A sinusoidal wave traveling to the right along the *x*-axis with angular frequency $\omega = 2\pi f$, wave number $k = 2\pi/\lambda$ and amplitude a is:

$$D_R = a\sin(kx - \omega t)$$

- An equivalent wave traveling to the left is:

$$D_L = a\sin(kx + \omega t)$$

- We previously used the symbol A for the wave amplitude, but here we will use a lowercase a to represent the amplitude of each individual wave and reserve A for the amplitude of the net wave.

The Mathematics of Standing Waves

- According to the principle of superposition, the net displacement of the medium when both waves are present is the sum of D_R and D_L:

$$D(x, t) = D_R + D_L = a\sin(kx - \omega t) + a\sin(kx + \omega t)$$

- We can simplify this by using a trigonometric identity, and arrive at:

$$D(x, t) = A(x)\cos\omega t$$

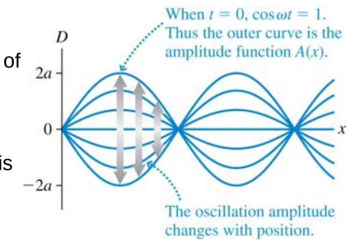
- Where the **amplitude function** $A(x)$ is defined as:

$$A(x) = 2a\sin kx$$

- The amplitude reaches a maximum value of $A_{max} = 2a$ at points where $\sin kx = 1$.

The Mathematics of Standing Waves

- Shown is the graph of $D(x,t)$ at several instants of time.
- The nodes occur at $x_m = m\lambda/2$, where m is an integer.



$$D(x, t) = A(x)\cos\omega t \qquad A(x) = 2a\sin kx$$

Waves on a String with a Discontinuity

- A string with a large linear density is connected to one with a smaller linear density.
- The tension is the same in both strings, so the wave speed is slower on the left, faster on the right.
- When a wave encounters such a discontinuity, some of the wave's energy is transmitted forward and some is reflected.

[animation from http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html]

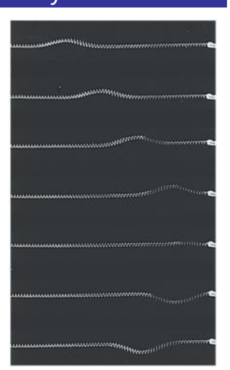
Waves on a String with a Discontinuity

- Below, a wave encounters discontinuity at which the wave speed decreases.
- In this case, the reflected pulse is *inverted.*
- We say that the reflected wave has a *phase change of π upon reflection.*

[animation from http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html]

Waves on a String with a Boundary

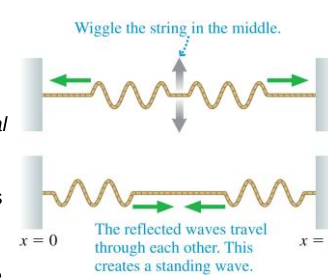
When a wave reflects from a boundary, the reflected wave is inverted, but has the same amplitude.



[animation from http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html]

Creating Standing Waves

- The figure shows a string of length L tied at $x = 0$ and $x = L$.
- Reflections at the ends of the string cause waves of *equal amplitude and wavelength* to travel in opposite directions along the string.
- These are the conditions that cause a standing wave!



Standing Waves on a String

For a string of fixed length L, the boundary conditions can be satisfied only if the wavelength has one of the values:

$$\lambda_m = \frac{2L}{m} \qquad m = 1, 2, 3, 4, \ldots$$

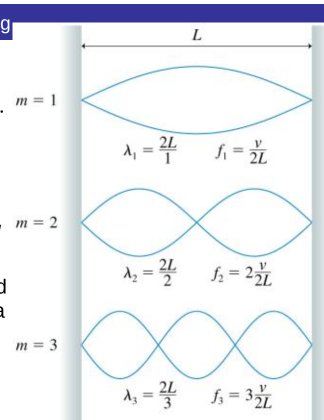
Because $\lambda f = v$ for a sinusoidal wave, the oscillation frequency corresponding to wavelength λ_m is:

$$f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = m\frac{v}{2L} \qquad m = 1, 2, 3, 4, \ldots$$

The lowest allowed frequency is called the **fundamental frequency**: $f_1 = v/2L$.

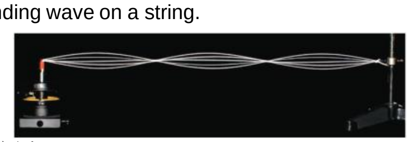
Standing Waves on a String

- Shown are various standing waves on a string of fixed length L.
- These possible standing waves are called the **modes** of the string, or sometimes the *normal modes.*
- Each mode, numbered by the integer m, has a unique wavelength and frequency.



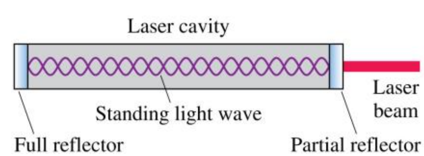
Standing Waves on a String

- m is the number of *antinodes* on the standing wave.
- The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
- The frequencies of the normal modes form a series: $f_1, 2f_1, 3f_1, \ldots$
- The fundamental frequency f_1 can be found as the *difference* between the frequencies of any two adjacent modes: $f_1 = \Delta f = f_{m+1} - f_m$.
- Below is a time-exposure photograph of the $m = 3$ standing wave on a string.



Standing Electromagnetic Waves

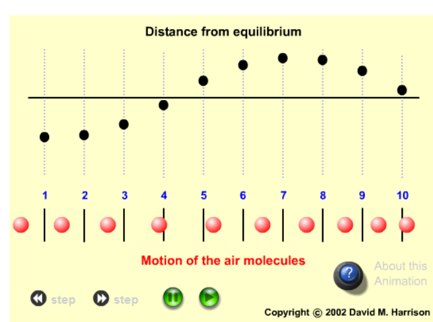
- Standing electromagnetic waves can be established between two parallel mirrors that reflect light back and forth.
- A typical laser cavity has a length $L \approx 30$ cm, and visible light has a wavelength $\lambda \approx 600$ nm.
- The standing light wave in a typical laser cavity has a mode number m that is $2L/\lambda \approx 1{,}000{,}000$!



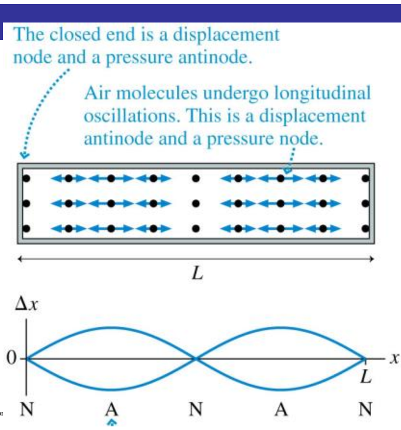
Standing Sound Waves

- A long, narrow column of air, such as the air in a tube or pipe, can support a longitudinal standing sound wave.
- A closed end of a column of air must be a displacement node, thus the boundary conditions—nodes at the ends—are the same as for a standing wave on a string.
- It is often useful to think of sound as a pressure wave rather than a displacement wave: The pressure oscillates around its equilibrium value.
- The nodes and antinodes of the pressure wave are interchanged with those of the displacement wave.

Standing Sound Wave



http://faraday.physics.utoronto.ca/IYearLab/Intros/StandingWaves/Flash/long_wave.html



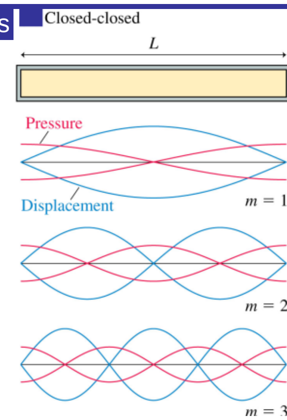
Standing Sound Waves

Shown are displacement and pressure graphs for the first three standing-wave modes of a tube closed at both ends:

$$\lambda_m = \frac{2L}{m}$$

$$f_m = m\frac{v}{2L}$$

$$m = 1, 2, 3, 4, \ldots$$



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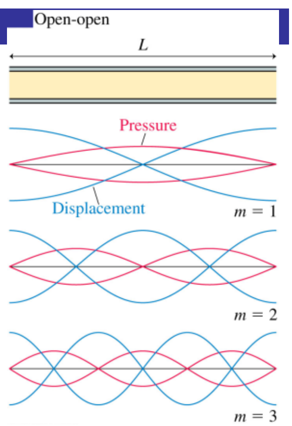


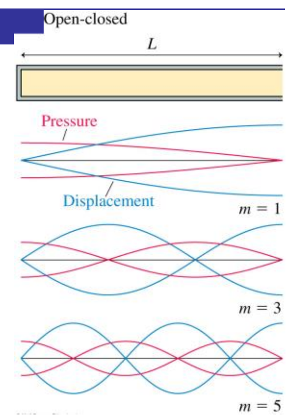
Standing Sound Waves

Shown are displacement and pressure graphs for the first three standing-wave modes of a tube open at one end but closed at the other:

$$\lambda_m = \frac{4L}{m}$$

$$f_m = m\frac{v}{4L}$$

$$m = 1, 3, 5, 7, \ldots$$



Musical Instruments

- Instruments such as the harp, the piano, and the violin have strings fixed at the ends and tightened to create tension.
- A disturbance generated on the string by plucking, striking, or bowing it creates a **standing wave** on the string.
- The fundamental frequency is the musical note you hear when the string is sounded:

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T_s}{\mu}}$$

where T_s is the tension in the string and μ is its linear density.

Musical Instruments

- With a wind instrument, blowing into the mouthpiece creates a standing sound wave inside a tube of air.
- The player changes the notes by using her fingers to cover holes or open valves, changing the length of the tube and thus its fundamental frequency:

$$f_1 = \frac{v}{2L}$$ for an open-open tube instrument, such as a flute

$$f_1 = \frac{v}{4L}$$ for an open-closed tube instrument, such as a clarinet

- In both of these equations, v is the speed of sound in the air *inside* the tube.
- Overblowing wind instruments can sometimes produce higher harmonics such as $f_2 = 2f_1$ and $f_3 = 3f_1$.