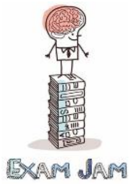
Jason Harlow
Monday Apr. 7
11:00-12:00
SS2118


- Position, Velocity, Acceleration
- Significant Figures, Measurements, Errors
- Vectors, Relative Motion
- Studying Tips
- Equilibrium and Non-equilibrium Problems
- Circular Motion, Centripetal Force

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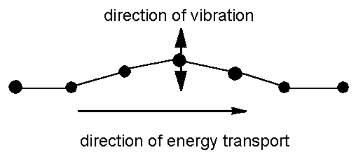
## Transverse waves

- Medium vibrates perpendicularly to direction of energy transfer
- Side-to-side movement

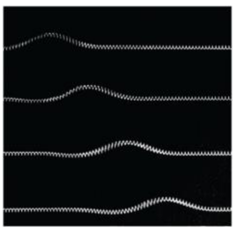
  Example:
  - Vibrations in stretched strings of musical instruments




## Transverse waves

The speed of transverse waves on a string stretched with tension $T_s$ is:

$$v_{\text{string}} = \sqrt{\frac{T_s}{\mu}}$$



Where $\mu$ is the string's mass-to-length ratio, also called the **linear density**:

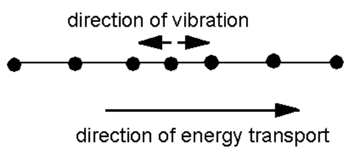
$$\mu = \frac{m}{L}$$

Units: [kg/m]

## Longitudinal waves

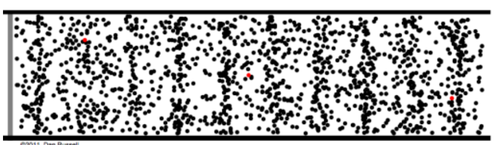
- Medium vibrates parallel to direction of energy transfer
- Backward and forward movement consists of
  - compressions (wave compressed)
  - rarefactions (stretched region between compressions)

  Example: sound waves in solid, liquid, gas




## Longitudinal Waves

- Sound is a longitudinal wave.
- Compression regions travel at the speed of sound.
- In a compression region, the density and pressure of the air is higher than the average density and pressure.




## The Mathematics of Sinusoidal Waves

$$D(x, t) = A \sin(kx - \omega t + \phi_0)$$
(sinusoidal wave traveling in the positive $x$-direction)

- The *angular frequency* of the wave is omega:

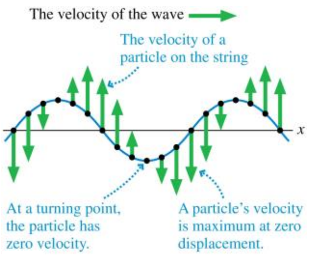
$$\omega = 2\pi f = \frac{2\pi}{T}$$

- The *wave number* of the wave is k:

$$k = \frac{2\pi}{\lambda}$$

This wave travels at a speed: $v = \frac{\omega}{k}$.

## Sinusoidal Waves

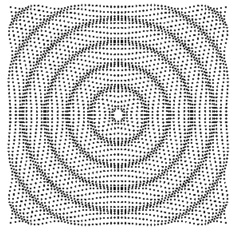



As the wave moves along $x$, the velocity of a particle on the string is in the $y$-direction.

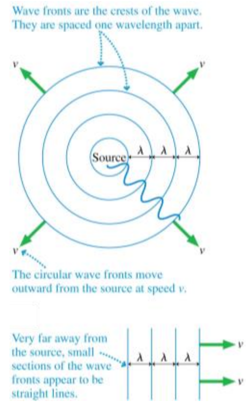
$$v_y = \frac{dy}{dt} = -\omega A \cos(kx - \omega t + \phi_0)$$

$$a_y = \frac{dv_y}{dt} = -\omega^2 A \sin(kx - \omega t + \phi_0)$$

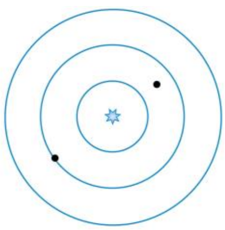
## Waves in Two and Three Dimensions



[Animation courtesy of Dan Russell, Penn State]




A spherical wave travels outward from a point source. What is the phase difference between the two points on the wave marked with dots?



A. $\pi/4$ radians.
B. $\pi/2$ radians.
C. $\pi$ radians.
D. $7\pi/2$ radians.
E. $7\pi$ radians.

## The Index of Refraction

- Light waves travel with speed $c$ in a vacuum, but they slow down as they pass through transparent materials such as water or glass or even, to a very slight extent, air.
- The speed of light in a material is characterized by the material's index of refraction $n$, defined as

$$n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in the material}} = \frac{c}{v}$$

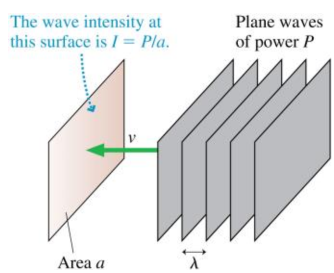
## The Index of Refraction

**TABLE 20.2** Typical indices of refraction

| Material | Index of refraction |
|---|---|
| Vacuum | 1 exactly |
| Air | 1.0003 |
| Water | 1.33 |
| Glass | 1.50 |
| Diamond | 2.42 |

## Power and Intensity

- The *power* of a wave is the rate, in joules per second, at which the wave transfers energy.
- When plane waves of power $P$ impinge on area $a$, we define the **intensity** $I$ to be:

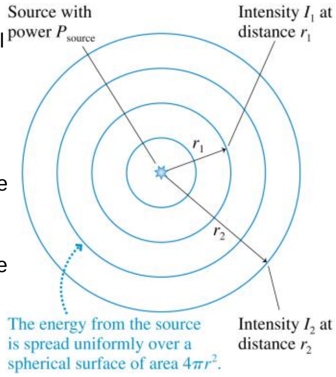



$$I = \frac{P}{a} = \text{power-to-area ratio}$$

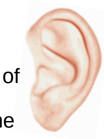
## Intensity of Spherical Waves

- If a source of spherical waves radiates uniformly in all directions, then the power at distance *r* is spread uniformly over the surface of a sphere of radius *r.*
- The intensity of a uniform spherical wave is:

$$I = \frac{P_{source}}{4\pi r^2}$$




## Intensity and Decibels



- Human hearing spans an extremely wide range of intensities, from the *threshold of hearing* at $\approx 1 \times 10^{-12}$ W/m$^2$ (at midrange frequencies) to the *threshold of pain* at $\approx 10$ W/m$^2$.
- If we want to make a scale of loudness, it's convenient and logical to place the zero of our scale at the threshold of hearing.
- To do so, we define the **sound intensity level**, expressed in **decibels** (dB), as:

$$\beta = (10\ \text{dB}) \log_{10}\left(\frac{I}{I_0}\right)$$

where $I_0 = 1 \times 10^{-12}$ W/m$^2$.

**TABLE 20.3** Sound intensity levels of common sounds

| Sound | $\beta$ (dB) |
|---|---|
| Threshold of hearing | 0 |
| Person breathing, at 3 m | 10 |
| A whisper, at 1 m | 20 |
| Quiet room | 30 |
| Outdoors, no traffic | 40 |
| Quiet restaurant | 50 |
| Normal conversation, at 1 m | 60 |
| Busy traffic | 70 |
| Vacuum cleaner, for user | 80 |
| Niagara Falls, at viewpoint | 90 |
| Snowblower, at 2 m | 100 |
| Stereo, at maximum volume | 110 |
| Rock concert | 120 |
| Threshold of pain | 130 |

- When you turn up the volume on your ipod, the sound originally entering your ears at 50 decibels is boosted to 80 decibels. By what factor is the intensity of the sound has increased?

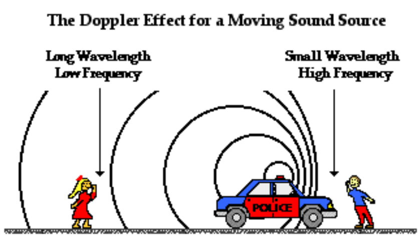
A. 1 (no increase)
B. 30
C. 100
D. 300
E. 1000

## Doppler Effect

- If a sound source is not moving relative to you, you hear the "rest frequency" of the emitted sound.
- If the source is moving toward you, you will hear a frequency that is higher than the rest frequency.
- If the source is moving away from you, you will hear a frequency that is lower than the rest frequency.

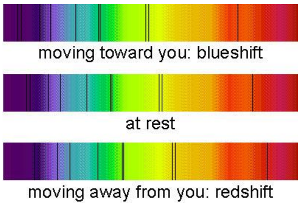
- By measuring the difference between the observed and known rest frequencies, you can determine the speed of the source.


## Doppler Effect




[image from http://www.physicsclassroom.com/class/waves/u10l3d.cfm ]

Doppler Shift for Light




- The Doppler shift can be observed with carefully obtained spectra of very fast moving objects like stars
- There is a slight shift in “absorption lines”

## Particles and Waves

- Particles cannot occupy the same space. They **collide**.

- Waves pass right through each other. They **interfere**.

[Animations from http://www.physicsclassroom.com/mmedia/newtlaws/mb.cfm and http://www.acs.psu.edu/drussell/demos/superposition/superposition.html ]

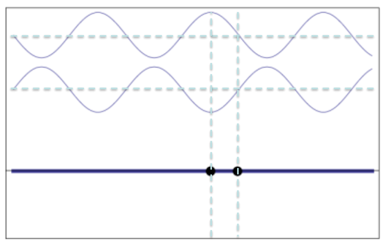
## The Principle of Superposition

If two or more waves combine at a given point, the resulting disturbance is the *sum* of the disturbances of the individual waves.

$$D = D_1 + D_2$$

## Standing Wave:

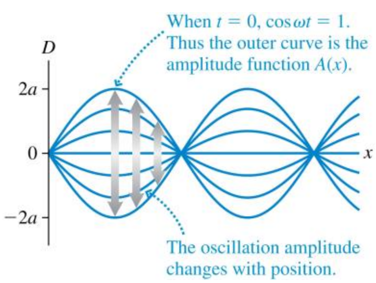
The superposition of two 1-D sinusoidal waves traveling in opposite directions.



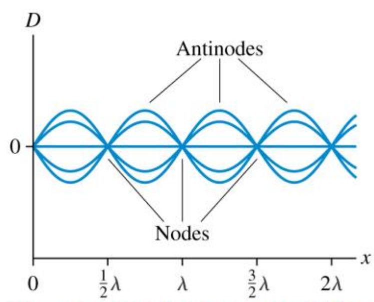
[Animation courtesy of Dan Russell, Penn State]

## The Mathematics of Standing Waves

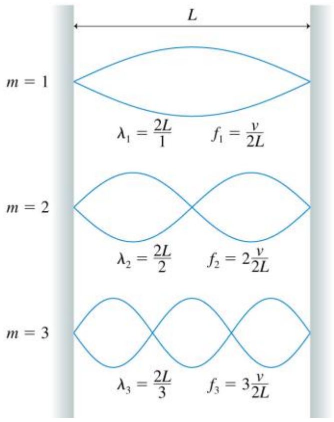
The amplitude reaches a maximum value of $A_{max} = 2a$ at points where $\sin(kx) = 1$.




## Node Spacing on a String




The nodes and antinodes are spaced $\lambda/2$ apart.




On a string of length $L$ with fixed end points,

$D(0,t) = 0$ and $D(L,t) = 0$

Only oscillations with specific wavelengths are allowed.

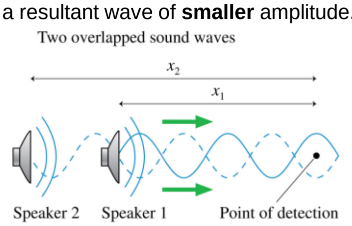
## Standing Waves on a String

There are three things to note about the normal modes of a string.

1. $m$ is the number of *antinodes* on the standing wave.
2. The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
3. The frequencies of the normal modes form a series: $f_1$, $2f_1$, $3f_1$, …These are also called **harmonics**. $2f_1$ is the "second harmonic", $3f_1$ is the "third harmonic", etc.

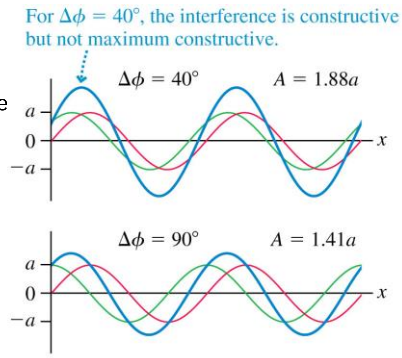
## Wave Interference

- The pattern resulting from the superposition of two waves is called interference. Interference can be
- ***constructive***, meaning the disturbances **add** to make a resultant wave of **larger** amplitude, or
- ***destructive***, meaning the disturbances **cancel**, making a resultant wave of **smaller** amplitude.

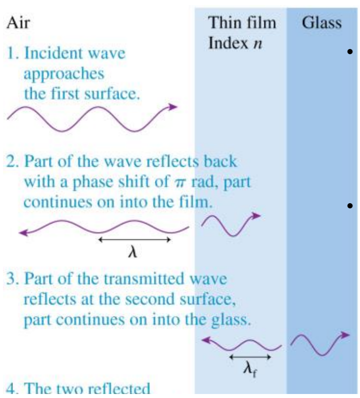



## The Mathematics of Interference

- It is entirely possible, of course, that the two waves are neither exactly in phase nor exactly out of phase.




## Thin-Film Optical Coatings




- Thin transparent films, placed on glass surfaces, such as lenses, can control reflections from the glass.
- Antireflection coatings on the lenses in cameras, microscopes, and other optical equipment are examples of thin-film coatings.

## Application: Thin-Film Optical Coatings

- The phase difference between the two reflected waves is:

$$\Delta\phi = 2\pi\frac{2d}{\lambda/n} = 2\pi\frac{2nd}{\lambda}$$

where $n$ is the index of refraction of the coating, $d$ is the thickness, and $\lambda$ is the wavelength of the light in vacuum or air.

- For a particular thin-film, constructive or destructive interference depends on the wavelength of the light:

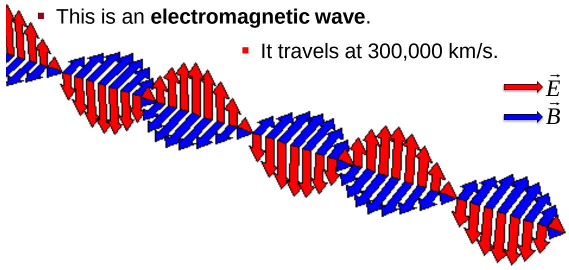
$$\lambda_C = \frac{2nd}{m} \qquad m = 1, 2, 3, \ldots \qquad \text{(constructive interference)}$$

$$\lambda_D = \frac{2nd}{m - \frac{1}{2}} \qquad m = 1, 2, 3, \ldots \qquad \text{(destructive interference)}$$

- What is light?

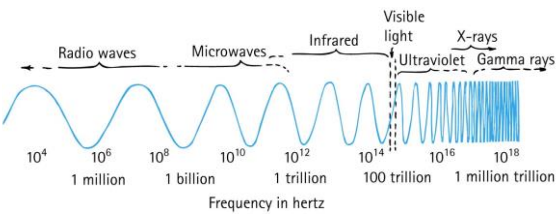
- Light is an electromagnetic wave – and is highly useful in our everyday life!

- A changing electric field creates a magnetic field, which then changes in just the right way to recreate the electric field, which then changes in just the right way to again recreate the magnetic field, and so on.
- This is an **electromagnetic wave**.
  - It travels at 300,000 km/s.




## Electromagnetic Spectrum

- In a vacuum, all electromagnetic waves move at the same speed
- We classify electromagnetic waves according to their frequency (or wavelength)
- Light is one kind of electromagnetic wave

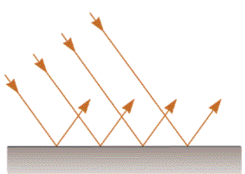



## Reflection
## CHECK YOUR NEIGHBOUR

Which reflects more light, a white piece of paper or a mirror?

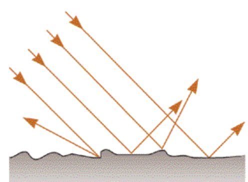
A. White Paper
B. Mirror
C. About the same

## Specular Reflection



**Mirrors**
• The surface is *flat* at distance scales near or above the wavelength of light
• It looks "shiny", and you can see images in it.

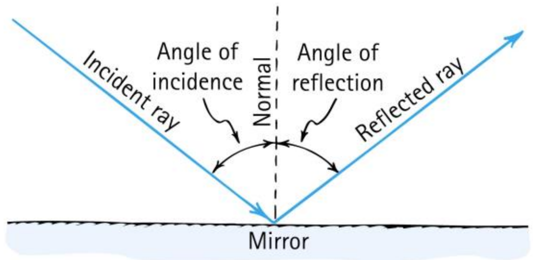
## Diffuse Reflection



**White Paper**
• The surface is *rough* at distance scales near or above the wavelength of light
• Almost ***all*** surfaces reflect in this way!

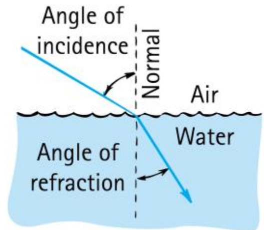
## Law of Specular Reflection

The angle of reflection equals the angle of incidence.



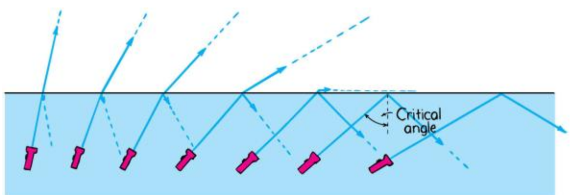

## Refraction

When light bends in going obliquely from one medium to another, we call this process refraction.

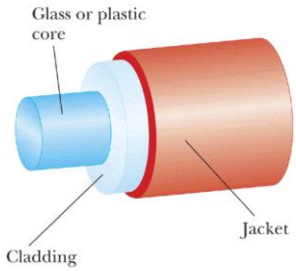



## Total Internal Reflection

- Total reflection of light traveling within a medium that strikes the boundary of another medium at an angle at, or greater than, the critical angle

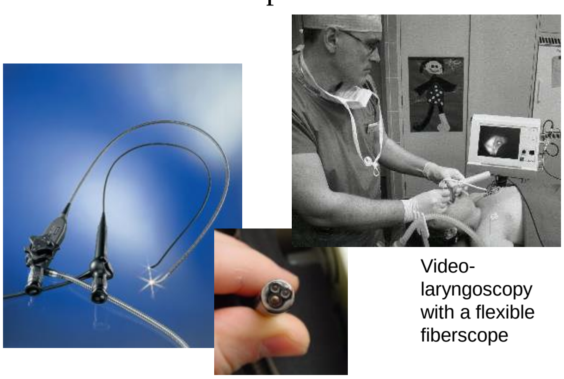



## An Optical Fibre

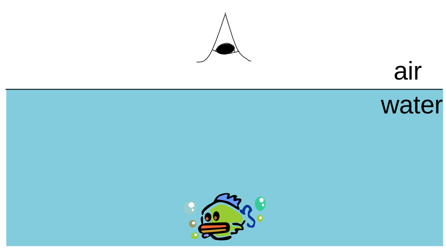



Speed of light in cladding is *higher* than speed of light in core.

## Medical Fibrescopes



Video-laryngoscopy with a flexible fiberscope



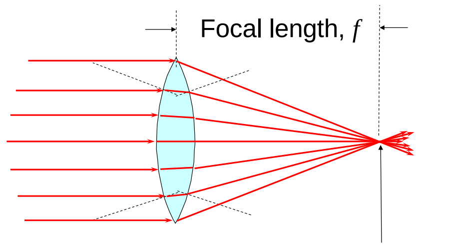

A fish swims *directly* below the surface of the water. An observer sees the fish at:

A. a greater depth than it really is.

B. its true depth.

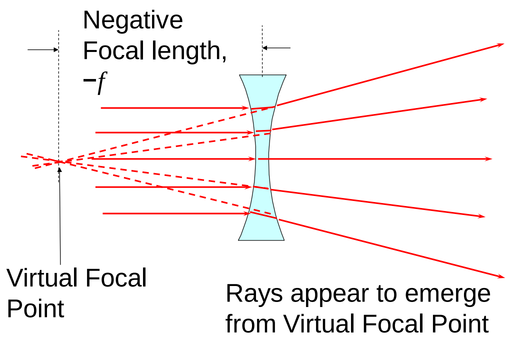
C. a smaller depth than it really is.

## Converging Lens




NOTE: Focal length is defined for initially *parallel* rays.

## Diverging Lens




Rays appear to emerge from Virtual Focal Point

## Focusing Power

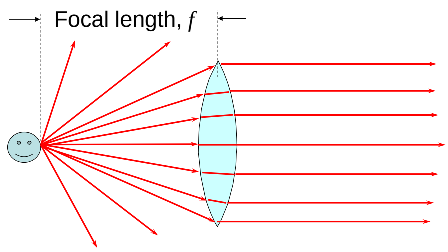
- Traditionally, lenses are specified not by their focal length, but by the inverse of their focal length.
- This is called "focusing power"

$$P = \frac{1}{f}$$

- The S.I. unit of focusing power is $\mathrm{m}^{-1}$
- Traditionally, this unit is called the "diopter," abbreviated D.

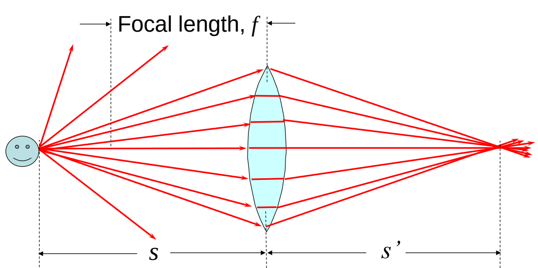
$$1\ \mathrm{D} = 1\ \mathrm{m}^{-1}$$

## Diverging rays through a Converging Lens



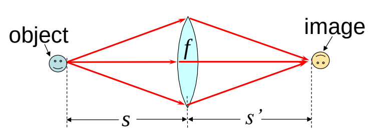

If an object emits rays at the focal point, they end up being parallel on the other side of the converging lens.

## Diverging rays through a Converging Lens




Thin Lens Equation: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

## Thin Lens Equation: sign conventions




$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$s$ is positive for objects to the left of lens, negative for objects to the right of lens (virtual objects).

$s'$ is positive for images to the right of lens, negative for images to the left of lens (virtual images).

$f$ is positive for converging lenses, negative for diverging lenses.

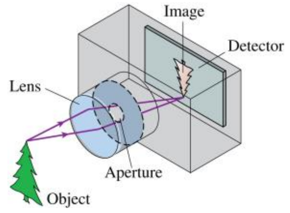
## Lateral Magnification

$$|M| \equiv \frac{h'}{h} \qquad M = -\frac{s'}{s}$$

- The absolute magnitude of the magnification $|M|$ is defined to be the ratio of image height to object height.
- A positive value of $M$ indicates that the image is upright relative to the object. A negative value of $M$ indicates the image is inverted relative to the object.
- Note that when $s$ and $s'$ are both positive, $M$ is negative.

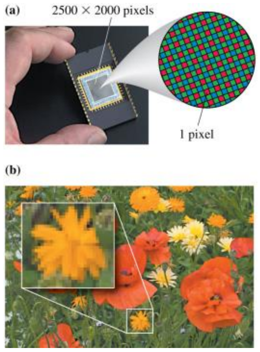
## The Camera

- A **camera** "takes a picture" by using a lens to form a real, inverted image on a light-sensitive detector in a light-tight box.
- We can model a combination lens as a single lens with an **effective focal length** (usually called simply "the focal length").
- A *zoom lens* changes the effective focal length by varying the spacing between the converging lens and the diverging lens.



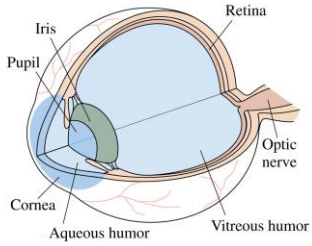

## The Detector

- Figure (a) shows a CCD "chip."
- To record color information, different pixels are covered by red, green, or blue filters.
- The pixels are so small that the picture looks "smooth" even after some enlargement.
- As you can see in figure (b), sufficient magnification reveals the individual pixels.



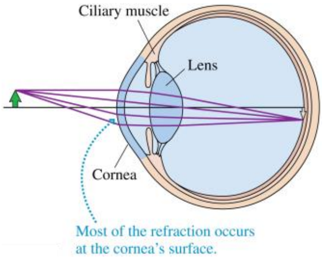

## Vision

- The human eye is roughly spherical, about 2.4 cm in diameter.
- The transparent **cornea** and the *lens* are the eye's refractive elements.
- The eye is filled with a clear, jellylike fluid called the *aqueous humor* and the *vitreous humor.*



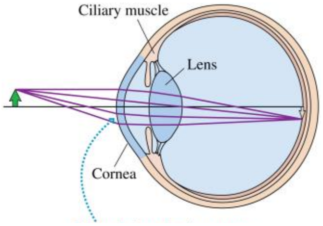

## Vision

- The indices of refraction of the aqueous and vitreous humors are 1.34, only slightly different from water.
- The lens has an average index of 1.44.
- The **pupil,** a variable-diameter aperture in the **iris,** automatically opens and closes to control the light intensity.



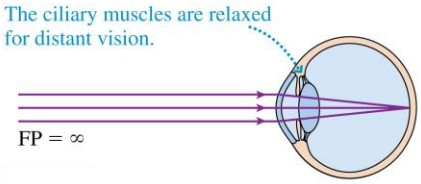

## Focusing and Accommodation

- The eye focuses by changing the focal length of the lens by using the *ciliary muscles* to change the curvature of the lens surface.
- Tensing the ciliary muscles causes **accommodation,** which decreases the lens's radius of curvature and thus decreases its focal length.



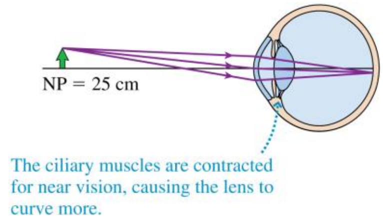
## Focusing and Accommodation

- The farthest distance at which a relaxed eye can focus is called the eye's **far point** (FP).
- The far point of a normal eye is infinity; that is, the eye can focus on objects extremely far away.



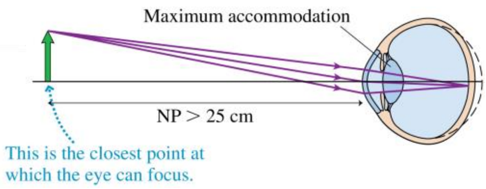

## Focusing and Accommodation

- The closest distance at which an eye can focus, using maximum accommodation, is the eye's **near point** (NP).



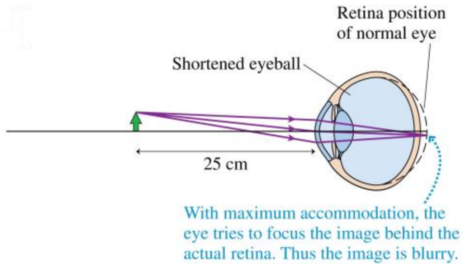

## Hyperopia

A person who is *farsighted* can see faraway objects (but even then must use some accommodation rather than a relaxed eye), but his near point is larger than 25 cm, often much larger, so he cannot focus on nearby objects.



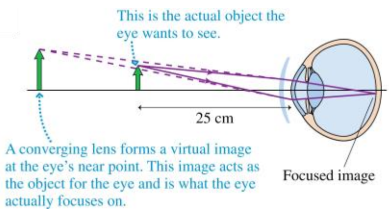

## Hyperopia

The cause of farsightedness — called **hyperopia** — is an eyeball that is too short for the refractive power of the cornea and lens.



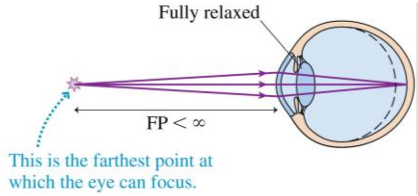

## Hyperopia

- With hyperopia, the eye needs assistance to focus the rays from a near object onto the closer-than-normal retina.
- This assistance is obtained by adding refractive power with the positive (i.e., converging) lens.



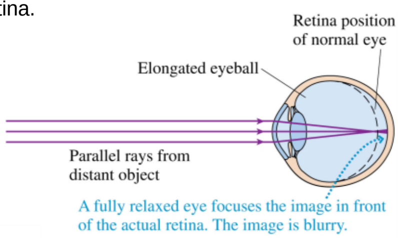

## Myopia

A person who is *nearsighted* can clearly see nearby objects when the eye is relaxed (and extremely close objects by using accommodation), but no amount of relaxation allows her to see distant objects.



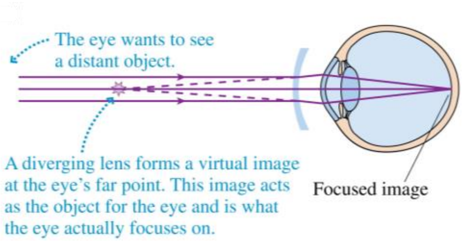
## Myopia

- Nearsightedness—called **myopia**—is caused by an eyeball that is too long.
- Rays from a distant object come to a focus in front of the retina and have begun to diverge by the time they reach the retina.



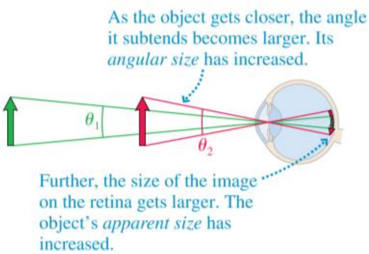

## Myopia

To correct myopia, we needed a diverging lens to slightly defocus the rays and move the image point back to the retina.




## Optical Systems That Magnify

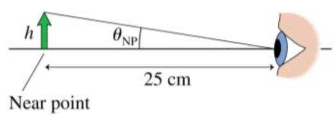
- The easiest way to magnify an object requires no extra optics at all; simply get closer!
- Closer objects look larger because they subtend a larger angle $\theta$, called the **angular size** of the object.




## Optical Systems That Magnify

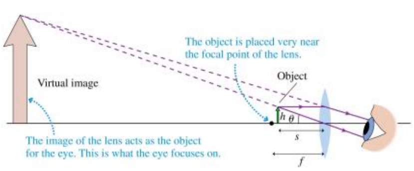
- You can't keep increasing an object's angular size because you can't focus on the object if it's closer than your near point, which is $\approx 25$ cm.
- The maximum angular size viewable by your unaided eye is:

$$\theta_{NP} = \frac{h}{25\ \text{cm}}$$




## The Magnifier

- Suppose we view an object of height *h* through a single converging lens.
- If the object's distance from the lens is less than the lens's focal length, we'll see an enlarged, upright image.
- Used in this way, the lens is called a **magnifier**.




## The Magnifier

- When using a magnifier, your eye sees a virtual image subtending an angle $\theta = h/s$.
- If we place the image at a distance $s' \approx \infty$ the object distance is $s \approx f$, so:

$$\theta = \frac{h}{s} \approx \frac{h}{f}$$

- Angular magnification is the ratio of the apparent size of the object when using a magnifying lens rather than simply holding the object at your near point:

$$M = \theta/\theta_{NP}$$

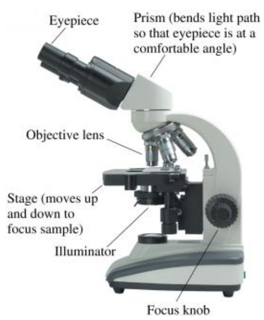
- Combining these equations, we find the angular magnification of a magnifying glass is:

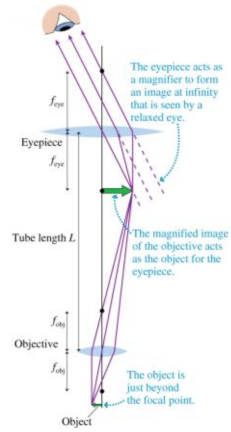
$$M = \frac{25\ \text{cm}}{f}$$

## The Microscope

- A microscope, whose major parts are shown in the next slide, can attain a magnification of up to 1000× by a *two-step* magnification process.
- A specimen to be observed is placed on the *stage* of the microscope, directly beneath the **objective**, a converging lens with a relatively short focal length.
- The objective creates a magnified real image that is further enlarged by the **eyepiece.**

## The Microscope




## The Microscope

- The lateral magnification of the objective is:

$$m_{\text{obj}} = -\frac{s'}{s} \approx -\frac{L}{f_{\text{obj}}}$$

- Together, the objective and eyepiece produce a total angular magnification:

$$M = m_{\text{obj}} M_{\text{eye}} = -\frac{L}{f_{\text{obj}}} \frac{25\text{ cm}}{f_{\text{eye}}}$$

- The minus sign shows that the image seen in a microscope is inverted.
- Most biological microscopes are standardized with a tube length $L$ = 160 mm.

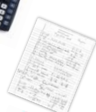
## The Final Exam Cometh...

| Course | Last Name | Date | Time | Location |
|---|---|---|---|---|
| PHY132H1S | A - KH | MON 14 APR | EV 7:00 - 9:00 | EX 100 |
| PHY132H1S | KI - WA | MON 14 APR | EV 7:00 - 9:00 | EX 200 |
| PHY132H1S | WE - Z | MON 14 APR | EV 7:00 - 9:00 | EX 310 |

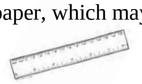
- EX is Central Exams Facility, 255 McCaul St. (just south of College St.)

## Aids Allowed on the Final Exam

- Any calculator without communication capability.
- Aid sheet: one single, original, handwritten 8 1/2 × 11 inch sheet of paper, which may be written on both sides.



- A ruler.



- A **paper** copy of an English translation dictionary.

- Also:

## Sun. Apr. 13 evening: Go see a movie!


- The evening before a test is NOT the best time to study (it is just the most popular)
- Don't worry – you have been studying since the 1st week of classes!
- If you can, sleep in Monday morning – you want to still be wide awake at 9pm!

## During the Exam

- Exam begins at **7:00pm SHARP**!!!
- Skim over the entire exam from front to back **before** you begin. Look for problems that you have confidence to solve first.
- If you start a problem but can't finish it, leave it, make a mark on the edge of the paper beside it, and come back to it after you have solved all the easy problems.
- When you are in a hurry and your hand is not steady, you can make little mistakes; if there is time, do the calculation twice and obtain agreement.
- Bring a snack or drink.
- *Don't leave a test early!*

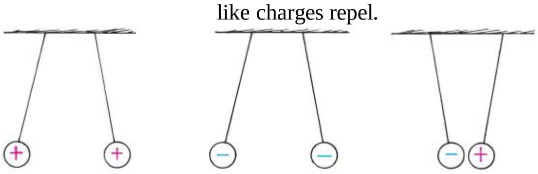
## Benjamin Franklin (1706-1790)

- Recognized that there were two types of electric charge.
- When a glass rod was rubbed with silk, it became charged in one way; Franklin called this "positive"
- When a piece of amber was rubbed with animal fur, it became charged in the opposite way; Franklin called this "negative".

## Electric Force

- When two objects have electric charges, there is a long-range force between them called the **electric force**.
- The rule for the electric force is:

Opposite charges attract one another;
like charges repel.

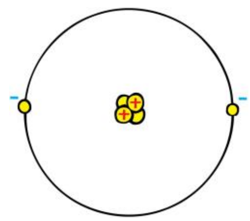


## 20th Century Discovery: Atomic Structure

Protons
- Positive electric charges
- Repel positives, but attract negatives

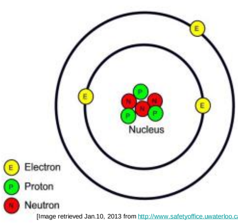
Electrons
- Negative electric charges
- Repel negatives, but attract positives

Neutrons
- No electric charge
- "neutral"



## Fundamental facts about atoms

1. Every atom is composed of a positively charged nucleus surrounded by negatively charged electrons.
2. Each of the electrons in any atom has the same quantity of negative charge and the same mass.

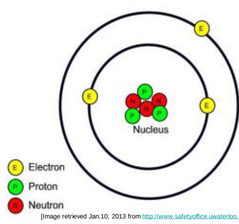



Lithium Atom
Net charge = 0

[Image retrieved Jan.10, 2013 from http://www.safetyoffice.uwaterloo.ca/hse/radiation/rad_sealed/matter/atom_structure.htm ]

## Fundamental facts about atoms

3. Protons and neutrons compose the nucleus. Protons are about 1800 times more massive than electrons, but each one carries an amount of positive charge equal to the negative charge of electrons. Neutrons have slightly more mass than protons and have no net charge.

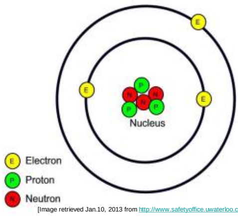



Lithium Atom
Net charge = 0

[Image retrieved Jan.10, 2013 from http://www.safetyoffice.uwaterloo.ca/hse/radiation/rad_sealed/matter/atom_structure.htm ]

## Fundamental facts about atoms

4. Atoms usually have as many electrons as protons, so the atom has zero net charge.




Lithium Atom
Net charge = 0

[Image retrieved Jan.10, 2013 from http://www.safetyoffice.uwaterloo.ca/hse/radiation/rad_sealed/matter/atom_structure.htm ]

## An "Ion" is a charged atom

- Positive ion — an atom which has lost one or more of its electrons, and so has a positive net charge.
- Negative ion — an atom which has gained one or more electrons, and so has a negative net charge.

## Coulomb's Law

$q_1$ ● ←— $d$ —→ ● $q_2$

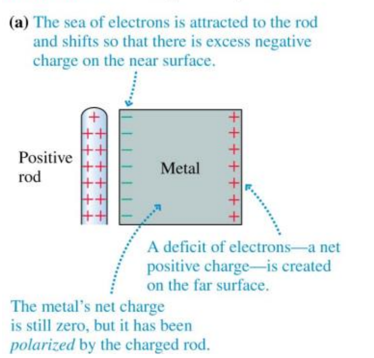
- The magnitude of the force, $F$, between two point charges depends on the product of their charges, and the distance between them.
- In equation form:

$$F = k\frac{q_1 q_2}{d^2} \qquad k = 9\times10^9 \text{ Nm}^2/\text{C}^2$$

- Unit of charge is **coulomb**, C
- Similar to Newton's law of gravitation for masses
- Underlies the bonding forces between molecules
- Electrical forces may be either attractive or repulsive.
- Gravitational forces are only attractive.

## Charge Polarization

FIGURE 26.12 A charged rod polarizes a metal.




## Charge Polarization

- When two small electrically charged objects are brought together, opposites attract and sames repel.
- When the objects have finite size and one of them is neutral or has very little charge on it, it will become *polarized*.
- The resulting force is *always attractive*. Both positive and negative objects tend to attract neutral objects due to *charge polarization*.

## Coulomb's Law, and The Permittivity Constant

- We can make many future equations easier to use if we rewrite Coulomb's law in a somewhat more complicated way.
- Let's define a new constant, called the **permittivity constant** $\epsilon_0$:

$$\epsilon_0 = \frac{1}{4\pi K} = 8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2$$

- Rewriting Coulomb's law in terms of $\epsilon_0$ gives us:

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}$$

## The Electric Field

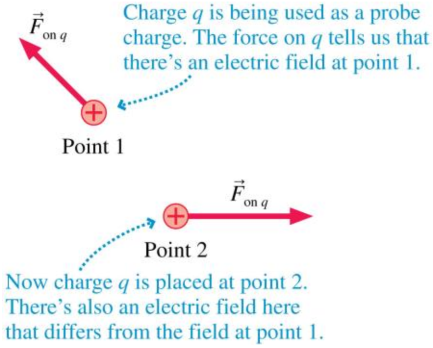
A charged particle with charge $q$ at a point in space where the electric field is $\vec{E}$ experiences an electric force:

$$\vec{F}_{\text{on } q} = q\vec{E}$$

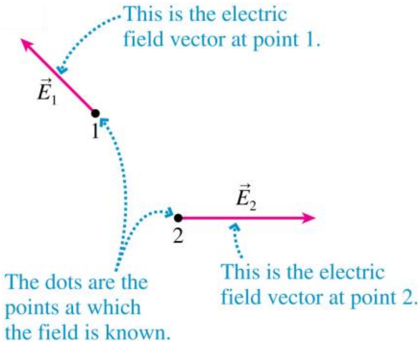
- If $q$ is positive, the force on the particle is in the direction of $\vec{E}$.
- The force on a negative charge is *opposite* the direction of $\vec{E}$.

The units of the electric field are N/C. The magnitude $E$ of the electric field is called the **electric field strength.**

## The Electric Force




## The Electric Field

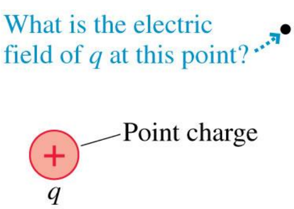



## The Electric Field of a Point Charge

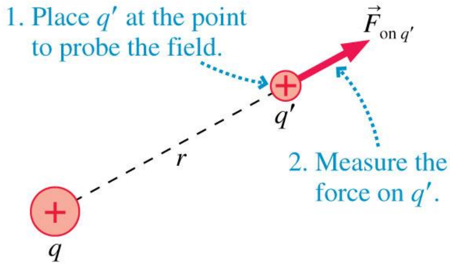
- The electric field at a distance $r$ away from a point charge, $q$, is given by:

$$\vec{E} = \frac{\vec{F}_{\text{on } q'}}{q'} = \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}, \text{away from } q\right)$$

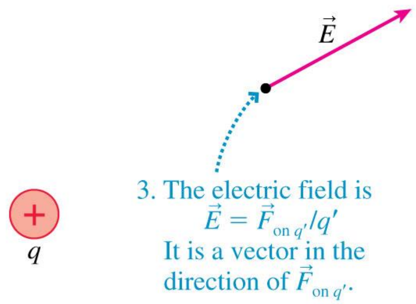
## The Electric Field of a Point Charge




## The Electric Field of a Point Charge

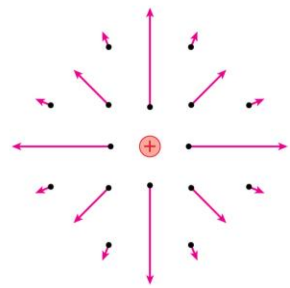


## The Electric Field of a Point Charge




## The Electric Field of a Point Charge

- If we calculate the field at a sufficient number of points in space, we can draw a **field diagram**.
- Notice that the field vectors all point straight away from charge $q$.
- Also notice how quickly the arrows decrease in length due to the inverse-square dependence on $r$.

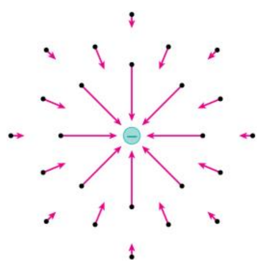


## The Electric Field of a Point Charge

- Using unit vector notation, the electric field at a distance $r$ from a point charge $q$ is:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

- A negative sign in front of a vector simply reverses its direction.
- The figure shows the electric field of a negative point charge.



## The Electric Field of Multiple Point Charges

- Suppose the source of an electric field is a group of point charges $q_1$, $q_2$, …
- The net electric field $\vec{E}_{\text{net}}$ at each point in space is a superposition of the electric fields due to each individual charge:

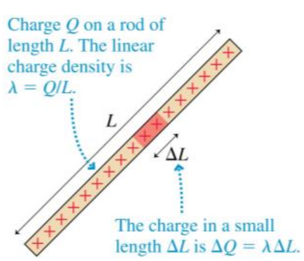
$$(E_{\text{net}})_x = (E_1)_x + (E_2)_x + \cdots = \sum (E_i)_x$$
$$(E_{\text{net}})_y = (E_1)_y + (E_2)_y + \cdots = \sum (E_i)_y$$
$$(E_{\text{net}})_z = (E_1)_z + (E_2)_z + \cdots = \sum (E_i)_z$$

## Continuous Charge Distributions

The linear charge density of an object of length $L$ and charge $Q$ is defined as

$$\lambda = \frac{Q}{L}$$

Linear charge density, which has units of C/m, is the amount of charge *per meter* of length.

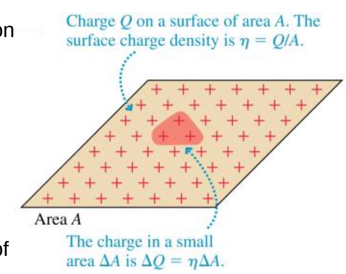



## Continuous Charge Distributions

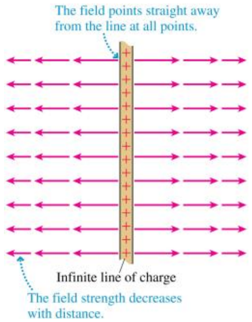
The surface charge density of a two-dimensional distribution of charge across a surface of area $A$ is defined as:

$$\eta = \frac{Q}{A}$$

Surface charge density, with units $C/m^2$, is the amount of charge *per square meter*.



## An Infinite Line of Charge




The electric field of a thin, uniformly charged rod may be written:

$$E_{\text{rod}} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}\frac{1}{\sqrt{1+4r^2/L^2}}$$

If we now let $L \to \infty$, the last term becomes simply $1$ and we're left with:

$$E_{\text{line}} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}$$

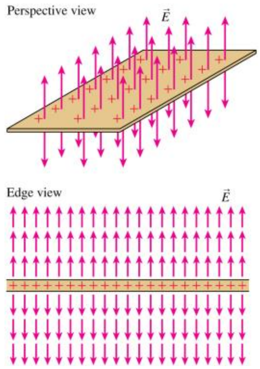
## A Plane of Charge

- The electric field of a plane of charge is found from the on-axis field of a charged disk by letting the radius $R \to \infty$.
- The electric field of an infinite plane of charge with surface charge density $\eta$ is:

$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant}$$

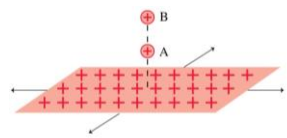
- For a positively charged plane, with $\eta > 0$, the electric field points *away from* the plane on both sides of the plane.
- For a negatively charged plane, with $\eta < 0$, the electric field points *towards* the plane on both sides of the plane.

## A Plane of Charge




$$(E_{\text{plane}})_z = \begin{cases} +\dfrac{\eta}{2\epsilon_0} & z > 0 \\ -\dfrac{\eta}{2\epsilon_0} & z < 0 \end{cases}$$

Two protons, A and B, are next to an infinite plane of positive charge. Proton B is twice as far from the plane as proton A. Which proton has the larger acceleration?



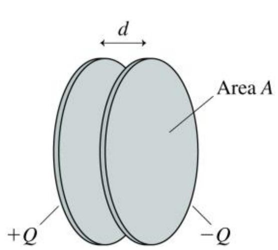

A. Proton A.

B. Proton B.

C. Both have the same acceleration.

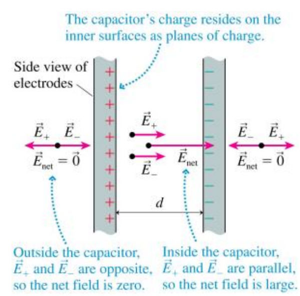
## The Parallel-Plate Capacitor

- The figure shows two electrodes, one with charge $+Q$ and the other with $-Q$ placed face-to-face a distance $d$ apart.
- This arrangement of two electrodes, charged equally but oppositely, is called a **parallel-plate capacitor.**
- Capacitors play important roles in many electric circuits.




## The Parallel-Plate Capacitor

- The figure shows two capacitor plates, seen from the side.
- Because opposite charges attract, all of the charge is on the *inner* surfaces of the two plates.
- Inside the capacitor, the net field points toward the negative plate.
- Outside the capacitor, the net field is zero.



## The Parallel-Plate Capacitor

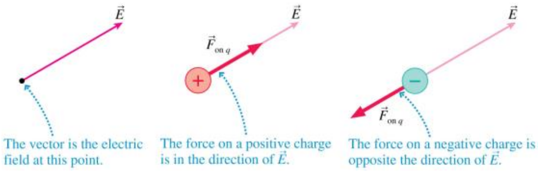
The electric field inside a capacitor is

$$\vec{E}_{\text{capacitor}} = \vec{E}_+ + \vec{E}_- = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$
$$= \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right)$$

where $A$ is the surface area of each electrode. Outside the capacitor plates, where $E_+$ and $E_-$ have equal magnitudes but *opposite* directions, the electric field is zero.

## Motion of a Charged Particle in an Electric Field

- Consider a particle of charge $q$ and mass $m$ at a point where an electric field $\vec{E}$ has been produced by *other* charges, the source charges.
- The electric field exerts a force $\vec{F}_{\text{on } q} = q\vec{E}$.




## Motion of a Charged Particle in an Electric Field

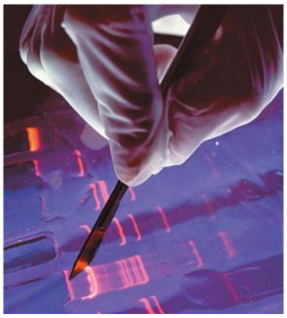
- The electric field exerts a force $\vec{F}_{\text{on } q} = q\vec{E}$ on a charged particle.
- If this is the only force acting on $q$, it causes the charged particle to accelerate with

$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E}$$

- In a uniform field, the acceleration is constant:

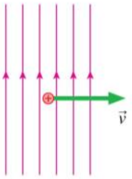
$$a = \frac{qE}{m} = \text{constant}$$

## Motion of a Charged Particle in an Electric Field

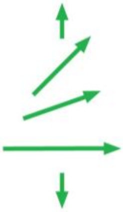


- "DNA fingerprints" are measured with the technique of *gel electrophoresis.*
- A solution of negatively charged DNA fragments migrate through the gel when placed in a uniform electric field.
- Because the gel exerts a drag force, the fragments move at a terminal speed inversely proportional to their size.

A proton is moving to the right in a vertical electric field. A very short time later, the proton's velocity is



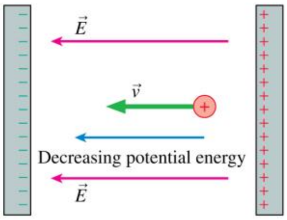

A.
B.
C.
D.
E.



## Electric Potential Energy in a Uniform Field

$$U_{\text{elec}} = U_0 + qEs$$

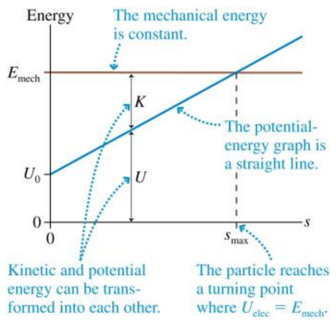
A positively charged particle gains kinetic energy as it moves in the direction of decreasing potential energy.




The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.

## Electric Potential Energy in a Uniform Field

- The figure shows the **energy diagram** for a positively charged particle in a uniform electric field.
- The potential energy increases linearly with distance, but the total mechanical energy $E_{mech}$ is fixed.




## The Potential Energy of Two Point Charges

Consider two point charges, $q_1$ and $q_2$, separated by a distance $r$. The electric potential energy is

$$U_{elec} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

- This is explicitly the energy of *the system*, not the energy of just $q_1$ or $q_2$.
- Note that the potential energy of two charged particles approaches zero as $r \to \infty$.

## The Potential Energy of Multiple Point Charges

Consider more than two point charges, the potential energy is the sum of the potential energies due to all pairs of charges:

$$U_{elec} = \sum_{i<j} \frac{Kq_iq_j}{r_{ij}}$$

where $r_{ij}$ is the distance between $q_i$ and $q_j$.
The summation contains the $i < j$ restriction to ensure that each pair of charges is counted only once.

## The Electric Potential

- We define the electric potential $V$ (or, for brevity, just the potential) as

$$V \equiv \frac{U_{q+\text{sources}}}{q}$$

- This is NOT the same as electric potential energy. (different units, for one thing).
- The unit of electric potential is the joule per coulomb, which is called the volt V:

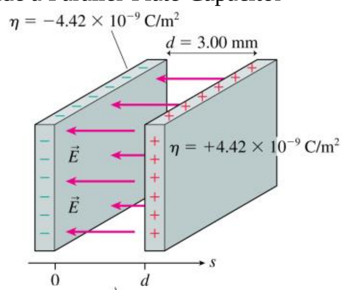
$$1 \text{ volt} = 1\text{ V} \equiv 1 \text{ J/C}$$

A proton is released from rest at the dot. Afterward, the proton


A. Remains at the dot.
B. Moves upward with steady speed.
C. Moves upward with an increasing speed.
D. Moves downward with a steady speed.
E. Moves downward with an increasing speed.

## The Electric Field Inside a Parallel-Plate Capacitor

This is a review of Chapter 26.




$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive toward negative}\right)$$
$$= (500 \text{ N/C, from right to left})$$

## The Electric Potential Inside a Parallel-Plate Capacitor

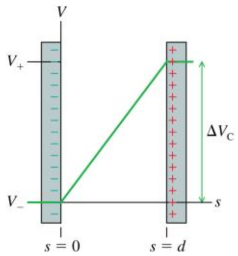
- The electric potential inside a parallel-plate capacitor is

$V = Es$ (electric potential inside a parallel-plate capacitor)

where $s$ **is the distance from the *negative* electrode.**

- The *potential difference* $\Delta V_C$, or "voltage" between the two capacitor plates is

$$\Delta V_C = V_+ - V_- = Ed$$




## Units of Electric Field

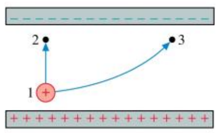
- If we know a capacitor's voltage $\Delta V$ and the distance between the plates $d$, then the electric field strength within the capacitor is:

$$E = \frac{\Delta V_C}{d}$$

- This implies that the units of electric field are volts per meter, or V/m.
- Previously, we have been using electric field units of newtons per coulomb.
- In fact, these units are equivalent to each other:

$$1 \text{ N/C} = 1 \text{ V/m}$$

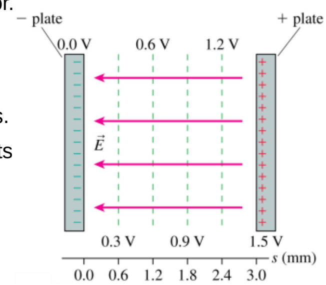
Two protons, one after the other, are launched from point 1 with the same speed. They follow the two trajectories shown. The protons' speeds at points 2 and 3 are related by




A. $v_2 > v_3$.
B. $v_2 = v_3$.
C. $v_2 < v_3$.
D. Not enough information to compare their speeds.

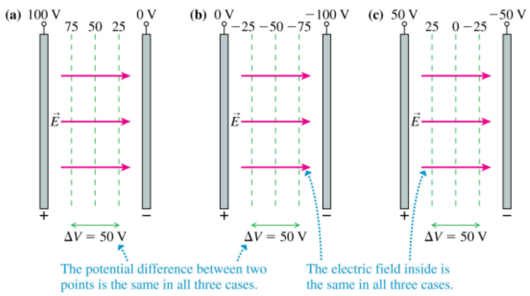
## The Parallel-Plate Capacitor

- The figure shows the contour lines of the electric potential and the electric field vectors inside a parallel-plate capacitor.
- The electric field vectors are *perpendicular* to the equipotential surfaces.
- The electric field points in the direction of *decreasing* potential.




## The Zero Point of Electric Potential

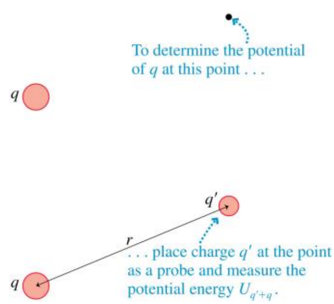
Where you choose $V = 0$ is arbitrary. The three contour maps below represent the *same physical situation*.




## The Electric Potential of a Point Charge

- Let $q$ in the figure be the source charge, and let a second charge $q'$, a distance $r$ away, probe the electric potential of $q$.
- The potential energy of the two point charges is

$$U_{q'+q} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r}$$



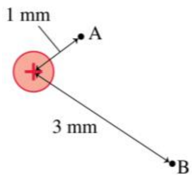
## The Electric Potential of a Point Charge

- The electric potential due to a point charge $q$ is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad \text{(electric potential of a point charge)}$$

- The potential extends through all of space, showing the influence of charge $q$, but it weakens with distance as $1/r$.
- This expression for $V$ assumes that we have chosen $V = 0$ to be at $r = \infty$.

What is the ratio $V_B/V_A$ of the electric potentials at the two points?




A. 9.
B. 3.
C. 1/3.
D. 1/9.
E. Undefined without knowing the charge.

## The Electric Potential of Many Charges

- The electric potential $V$ at a point in space is the sum of the potentials due to each charge:

$$V = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i}$$

where $r_i$ is the distance from charge $q_i$ to the point in space where the potential is being calculated.

- **The electric potential, like the electric field, obeys the principle of superposition.**

Good Luck!!

- I'll see you on Monday, Apr. 14 at 7:00pm
- Then I hope to see you again in the future – please say hi if you see me around campus, and feel free to stop by my office any time you see my open door.
- It's been a lot of fun teaching you physics this year – have a fantastic rest of your life!!!!
- Next up: Andrew at 12:00pm

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