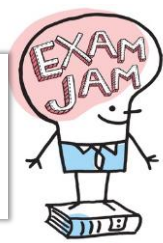




Jason Harlow  
 Monday Apr. 6  
 2:00-4:00  
 SS2117



Velma's moves p at a spec 0.999,999

The field emerges from the center of the loop's

The field returns around the outside of the loop.

**PHY132 Exam:**  
 Thursday Apr.9 at 2:00pm  
**Comprehensive** (covers the entire course)

# THE FINAL EXAM

Course	Last Name	Date	Time	Location
PHY132H1S	A - LE	THU 09 APR	PM 2:00 - 4:00	EX 100
PHY132H1S	LI - W	THU 09 APR	PM 2:00 - 4:00	EX 200
PHY132H1S	X - Z	THU 09 APR	PM 2:00 - 4:00	EX 320

- EX is Central Exams Facility, 255 McCaul St. (just south of College St.)

## Aids Allowed on the Final Exam

- Any calculator without communication capability.
- Aid sheet: one single, original, handwritten 8 1/2 × 11 inch sheet of paper, which may be written on both sides.
- A ruler.
- A **paper** copy of an English translation dictionary.
- Also:



## During the Exam

- Exam begins at **2:00pm SHARP!!!**
- Skim over the entire exam from front to back **before** you begin. Look for problems that you have confidence to solve first.
- If you start a problem but can't finish it, leave it, make a mark on the edge of the paper beside it, and come back to it after you have solved all the easy problems.
- When you are in a hurry and your hand is not steady, you can make little mistakes; if there is time, do the calculation twice and obtain agreement.
- Bring a snack or drink.
- *Don't leave an exam early!*



No phones!



No Apple Watch!

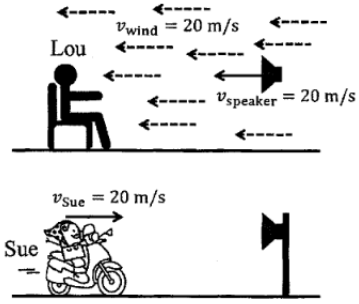


No Google Glasses!

From April 2014 Final Exam

**Question 4**

Lou is sitting facing a strong wind that is blowing at a constant 20 m/s (72 km/hr). A loudspeaker is oscillating with a frequency  $f_0$ , and is approaching Lou also at 20 m/s relative to him. A second loudspeaker is also oscillating at  $f_0$  and is stationary relative to the ground. There is no wind blowing. Sue is riding her motor scooter towards the second speaker at 20 m/s. How do the frequencies that Lou and Sue hear compare?



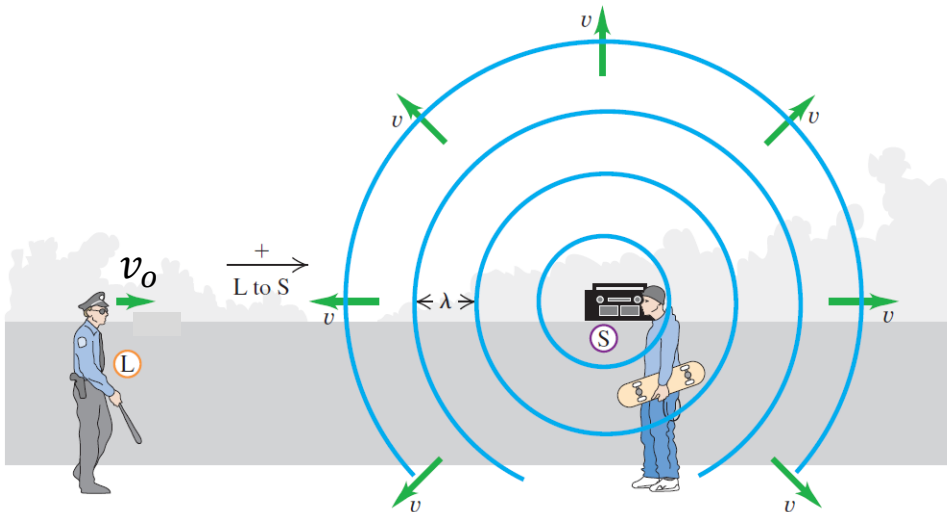
- (A)  $f_{\text{Lou}} > f_{\text{Sue}}$
- (B)  $f_{\text{Lou}} = f_{\text{Sue}}$
- (C)  $f_{\text{Lou}} < f_{\text{Sue}}$
- (D) It depends on the actual value of the speed of sound, which is not given.



Let's review.

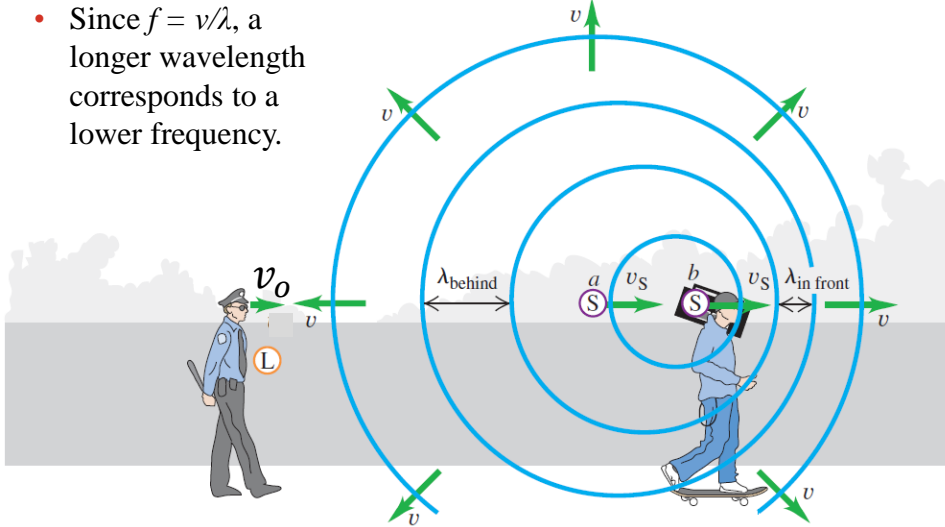
The Doppler effect: Moving listener

- An **observer** moving *toward* a stationary **source** hears a frequency that is *higher* than the at-rest frequency  $f_0$ .



## The Doppler effect: Moving source

- When a **source** is moving *away* from an **observer**, the waves behind the source are stretched to a *longer* wavelength.
- Since  $f = v/\lambda$ , a longer wavelength corresponds to a lower frequency.



## The Doppler Effect

The frequencies heard by a stationary observer when the sound source is moving at speed  $v_o$  are

$$f_+ = \frac{f_0}{1 - v_s/v} \quad (\text{Doppler effect for an approaching source}) \quad (20.39)$$

$$f_- = \frac{f_0}{1 + v_s/v} \quad (\text{Doppler effect for a receding source})$$

son Education, Inc.

The frequencies heard by an observer moving at speed  $v_o$  relative to a stationary sound source emitting frequency  $f_0$  are

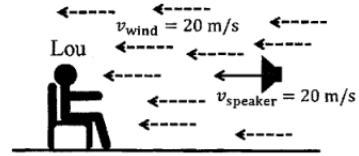
$$f_+ = (1 + v_o/v)f_0 \quad (\text{observer approaching a source}) \quad (20.40)$$

$$f_- = (1 - v_o/v)f_0 \quad (\text{observer receding from a source})$$

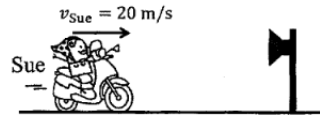
From April 2014 Final Exam

**Question 4**

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- (A)  $f_{\text{Lou}} > f_{\text{Sue}}$
- (B)  $f_{\text{Lou}} = f_{\text{Sue}}$
- (C)  $f_{\text{Lou}} < f_{\text{Sue}}$
- (D) It depends on the actual value of the speed of sound, which is not given.



All Doppler velocities are relative to the air.

Lou:  $v_o = 20 \text{ m/s}$   $v_s = 0$

Sue:  $v_o = 20 \text{ m/s}$   $v_s = 0$

→ Same situation relative to air  
 ⇒ same frequency shift.

From April 2014 Final Exam

**Question 1**

Oil leaks out of the engine of a boat and forms a thin film floating on top of the water. You look straight down and see a bright reflection of sunlight at a wavelength of  $\lambda$  in the air. You know the index of refraction of the oil is  $n_{\text{oil}}$ , and the index of refraction of water is  $n_{\text{water}}$ , where  $n_{\text{water}} < n_{\text{oil}}$ . Both the water and the oil have indices of refraction greater than that of air, which you can assume to be  $n_{\text{air}} = 1$ . What is the minimum thickness  $t$  of the oil slick at the spot where you see the bright reflection?

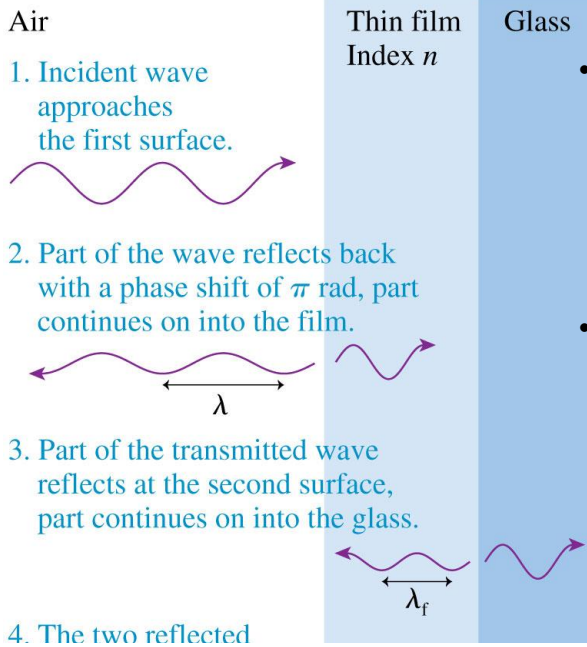
- (A)  $\frac{\lambda}{2n_{\text{oil}}}$
- (B)  $\frac{\lambda}{3n_{\text{oil}}}$
- (C)  $\frac{\lambda}{4n_{\text{oil}}}$
- (D)  $\frac{\lambda}{2n_{\text{water}}}$
- (E)  $\frac{\lambda}{4n_{\text{water}}}$



Let's review.

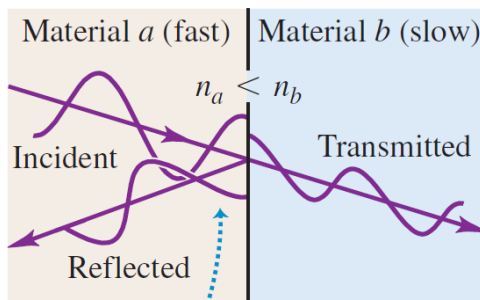
## Thin-Film

## Optical Coatings

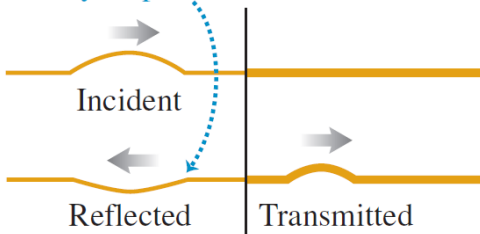


- Thin transparent films, placed on glass surfaces, such as lenses, can control reflections from the glass.
- Antireflection coatings on the lenses in cameras, microscopes, and other optical equipment are examples of thin-film coatings.

If the transmitted wave moves slower than the incident wave ...



... the reflected wave undergoes a half-cycle phase shift.

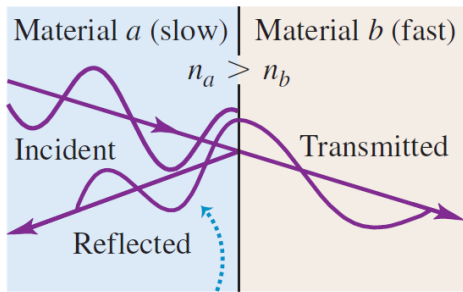


- When the index of a film is less than the index of the material beyond it (ie glass), then the half-cycle phase shift occurs for both reflections, and the wavelengths of constructive and destructive interference were as mentioned in class:

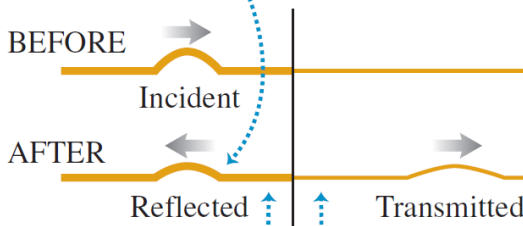
$$\lambda_C = \frac{2n_{\text{film}}d}{m} \quad m = 1, 2, 3, \dots$$

$$\lambda_D = \frac{2n_{\text{film}}d}{m - \frac{1}{2}} \quad m = 1, 2, 3, \dots$$

If the transmitted wave moves faster than the incident wave ...



... the reflected wave undergoes no phase change.



- When the index of a film is greater than the index of the material beyond it (ie glass), then the half-cycle phase shift occurs for the front surface, but not the back-surface of the film.
- This flips the constructive vs destructive equations:

$$\lambda_C = \frac{2n_{\text{film}}d}{m - \frac{1}{2}} \quad m = 1, 2, 3, \dots$$

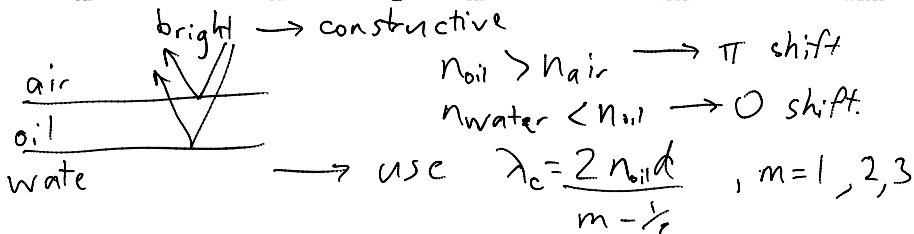
$$\lambda_D = \frac{2n_{\text{film}}d}{m} \quad m = 1, 2, 3, \dots$$

From April 2014 Final Exam

**Question 1**

Oil leaks out of the engine of a boat and forms a thin film floating on top of the water. You look straight down and see a bright reflection of sunlight at a wavelength of  $\lambda$  in the air. You know the index of refraction of the oil is  $n_{\text{oil}}$ , and the index of refraction of water is  $n_{\text{water}}$ , where  $n_{\text{water}} < n_{\text{oil}}$ . Both the water and the oil have indices of refraction greater than that of air, which you can assume to be  $n_{\text{air}} = 1$ . What is the minimum thickness  $t$  of the oil slick at the spot where you see the bright reflection?

- (A)  $\frac{\lambda}{2n_{\text{oil}}}$       (B)  $\frac{\lambda}{3n_{\text{oil}}}$       (C)  $\frac{\lambda}{4n_{\text{oil}}}$       (D)  $\frac{\lambda}{2n_{\text{water}}}$       (E)  $\frac{\lambda}{4n_{\text{water}}}$



Solve for  $d$ :  $d = \frac{\lambda(m - \frac{1}{2})}{2n_{\text{oil}}} \leftarrow \text{min. when } m = 1$   
 $d = \frac{\lambda \frac{1}{2}}{2n_{\text{oil}}} = \frac{\lambda}{4n_{\text{oil}}}$



- A fish swims *directly* below the surface of the water. An observer sees the fish at:
- A. a greater depth than it really is.
  - B. its true depth.
  - C. a smaller depth than it really is.

## The Electric Field

A charged particle with charge  $q$  at a point in space where the electric field is  $\vec{E}$  experiences an electric force:

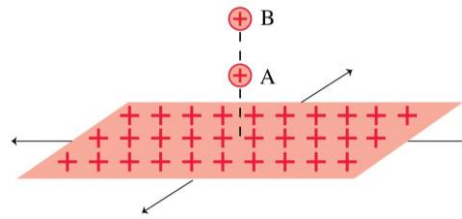
$$\vec{F}_{\text{on } q} = q\vec{E}$$

- If  $q$  is positive, the force on the particle is in the direction of  $\vec{E}$ .
- The force on a negative charge is *opposite* the direction of  $\vec{E}$ .

The units of the electric field are N/C. The magnitude  $E$  of the electric field is called the **electric field strength**.



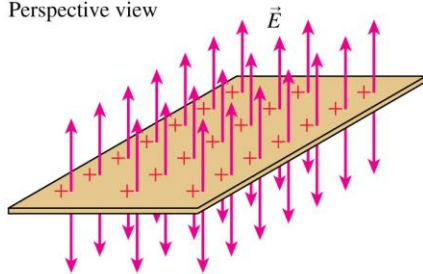
Two protons, A and B, are next to an infinite plane of positive charge. Proton B is twice as far from the plane as proton A. Which proton has the larger acceleration?



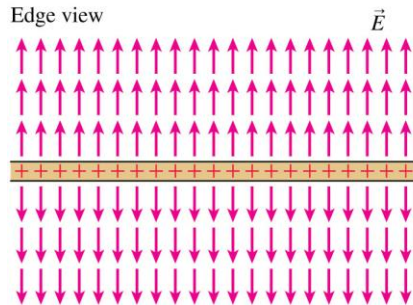
- A. Proton A.
- B. Proton B.
- C. Both have the same acceleration.

## A Plane of Charge

Perspective view



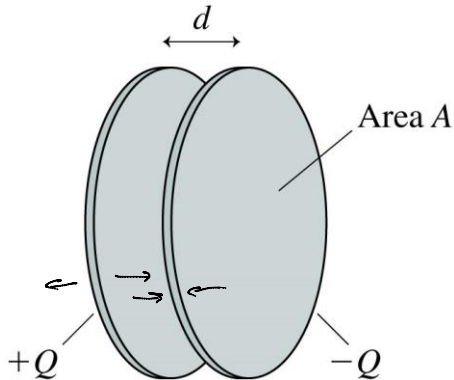
Edge view



$$(E_{\text{plane}})_z = \begin{cases} +\frac{\eta}{2\epsilon_0} & z > 0 \\ -\frac{\eta}{2\epsilon_0} & z < 0 \end{cases}$$

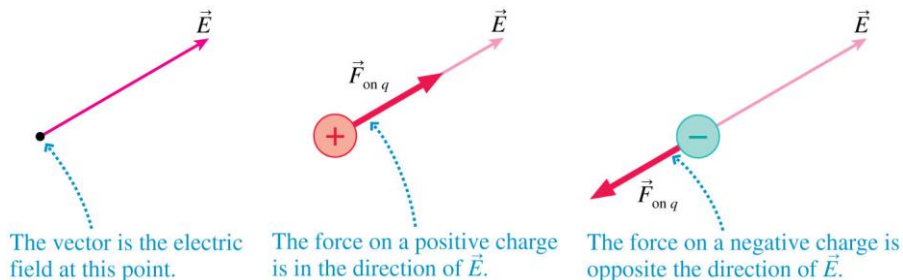
## Capacitors

- The figure shows two electrodes, one with charge  $+Q$  and the other with  $-Q$  placed face-to-face a distance  $d$  apart.
- This arrangement of two electrodes, charged equally but oppositely, is called a **capacitor**.
- Capacitors play important roles in many electric circuits.



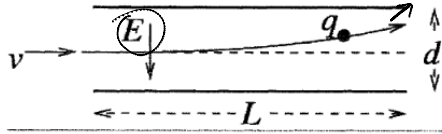
## Motion of a Charged Particle in an Electric Field

- Consider a particle of charge  $q$  and mass  $m$  at a point where an electric field  $\vec{E}$  has been produced by *other* charges, the source charges.
- The electric field exerts a force  $\vec{F}_{\text{on } q} = q\vec{E}$ .



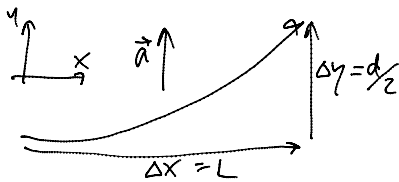
**Question 7**

In an ink-jet printer, an ink droplet of mass  $m$  is given a negative charge  $q$  by a computer-controlled charging unit. It then enters at speed  $v$  (see figure below) the region half-way between two deflecting parallel plates of length  $L$  separated by distance  $d$ . Throughout this region a uniform downward electric field exists. Neglecting the gravitational force on the droplet, what is the maximum charge that can be given to that droplet so that it does not hit a plate?



- (A)  $\frac{mv^2 E}{dL^2}$       (B)  $\frac{mv^2 d}{EL^2}$       (C)  $\frac{md}{E(vL)^2}$       (D)  $\frac{mv^2}{dL}$       (E)  $\frac{mv^2 L}{Ed^2}$

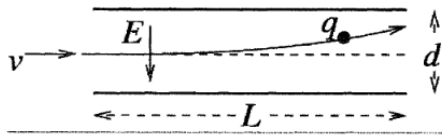
max charge  $\rightarrow$  just hits corner of plate.



$$a_y = \frac{F_{net}}{m} \quad v_{yi} = 0$$

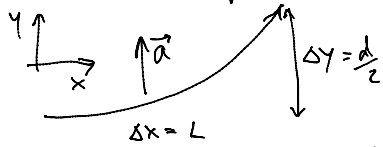
$$\Delta y = \frac{1}{2} a_y t^2$$

$$\frac{d}{2} = \frac{F}{2m} t^2, \quad F = Eq$$



- (A)  $\frac{mv^2 E}{dL^2}$       (B)  $\frac{mv^2 d}{EL^2}$       (C)  $\frac{md}{E(vL)^2}$       (D)  $\frac{mv^2}{dL}$       (E)  $\frac{mv^2 L}{Ed^2}$

max charge  $\rightarrow$  just hits corner of plate.



$$a_y = \frac{F}{m} \quad v_{yi} = 0$$

$$\Rightarrow \Delta y = \frac{d}{2} = \frac{1}{2} a_y t^2$$

$$\frac{d}{2} = \frac{F}{2m} t^2, \quad F = Eq$$

$$\frac{d}{2} = \frac{Eq}{2m} t^2 \quad (1)$$

$$\frac{d}{2} = \frac{Eq}{2m} \left(\frac{L}{v}\right)^2, \text{ solve for } q.$$

$$q = \frac{d v^2 m}{L^2 E}$$

$x$ -motion:  $\Delta x = v_x t$

$$L = vt$$

$$\Rightarrow t = \frac{L}{v}$$

## The Electric Potential Inside a Capacitor

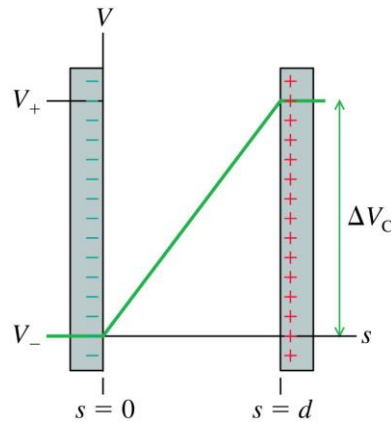
- The electric potential inside a capacitor is

$$V = Es \quad (\text{electric potential inside a parallel-plate capacitor})$$

where  $s$  is the distance from the *negative* electrode.

- The *potential difference*  $\Delta V_C$ , or “voltage” between the two capacitor plates is

$$\Delta V_C = V_+ - V_- = Ed$$



## Units of Electric Field

- If we know a capacitor’s voltage  $\Delta V$  and the distance between the plates  $d$ , then the electric field strength within the capacitor is:

$$E = \frac{\Delta V_C}{d}$$

- This implies that the units of electric field are volts per meter, or V/m.
- Previously, we have been using electric field units of newtons per coulomb.
- In fact, these units are equivalent to each other:

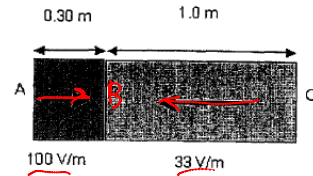
$$1 \text{ N/C} = 1 \text{ V/m}$$

From April 2013 Final Exam

**Question 7**

The image below shows two regions with electric fields as labeled. Which point has the highest electric potential? Assume there are no other fields.

- (A) A    (B) ~~B~~    (C) C  
 (D) This cannot be determined from the information given.



$\vec{E}$  points in direction of decreasing  $V$ .  
 $\rightarrow$  B must have lowest  $V$ .

$|\vec{E}| = \frac{\Delta V}{\Delta x}$  , Assume  $V_B = 0$  ← arbitrary zero point.

$V_A = E \Delta x = (100)(0.30)$   
 $V_A = 30 \text{ V}$

$V_C = E \Delta x = 33(1) = 33 \text{ V}$

In class discussion question

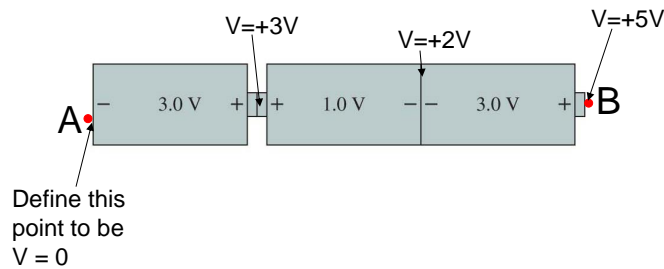
**What total potential difference ( $V_B - V_A$ ) is created by these three batteries?**



- A. 1.0 V  
 B. 2.0 V  
 C. 5.0 V  
 D. 6.0 V  
 E. 7.0 V

The point:

- Electric Potential is a **property of space**.
- Every point has a certain value, and it must change from one side of a battery to another by its emf.
- Also,  $V$  is higher on the  $+$  side of a battery.



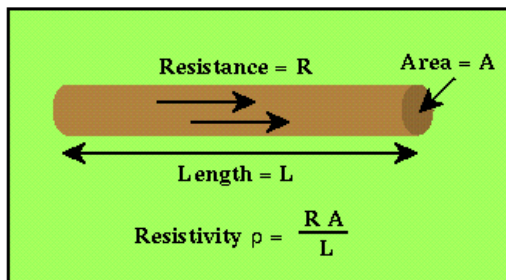
## Definition of Current

If  $Q$  is the total amount of charge that has moved past a point in a wire, we define the current  $I$  in the wire to be the rate of charge flow:

$$I \equiv \frac{dQ}{dt} \quad \text{current is the } \textit{rate} \text{ at which charge flows}$$

The SI unit for current is the coulomb per second, which is called the **ampere**.  $1 \text{ ampere} = 1 \text{ A} = 1 \text{ C/s}$ .

## Resistance and Ohm's Law



The SI unit of **resistance** is the ohm.

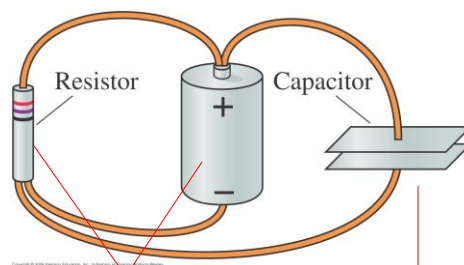
1 ohm = 1  $\Omega$  = 1 V/A.

The current through a conductor is determined by the potential difference  $\Delta V$  along its length:

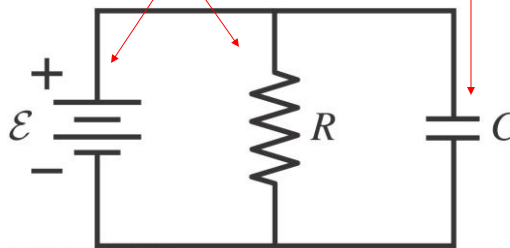
$$I = \frac{\Delta V}{R} \quad (\text{Ohm's law})$$

## Circuit Diagrams

Real life:



A circuit diagram:



## Series Resistors

- Resistors that are aligned end to end, *with no junctions between them*, are called **series resistors** or, sometimes, resistors “in series.”

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_N \quad (\text{series resistors})$$

## Parallel Resistors

- Resistors connected *at both ends* are called **parallel resistors** or, sometimes, resistors “in parallel.”

$$R_{\text{eq}} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \right)^{-1} \quad (\text{parallel resistors})$$

From April 2014 Final Exam

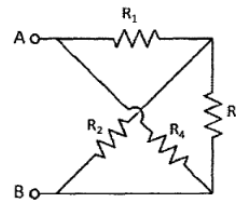
**Question 10**

Compute the equivalent resistance between point A and point B in the diagram on the right.

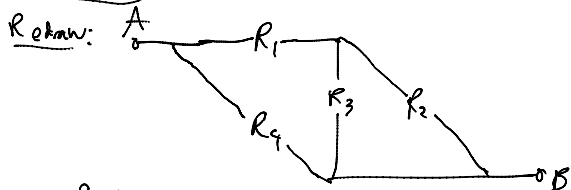
$R_1 = R_2 = R_3 = R_4 = 4 \Omega$ .

- (A) 1.7  $\Omega$                       (B) 1.5  $\Omega$   
 (D) 8.0  $\Omega$                       (E) 10.2  $\Omega$

(C) 2.4  $\Omega$

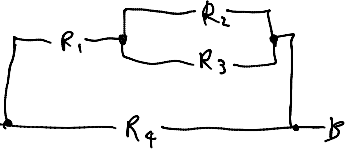


Parallel:  $R_{\text{eq}23} = \left[ \frac{1}{4} + \frac{1}{4} \right]^{-1} = \frac{1}{2}^{-1} = 2 \Omega$



Series:  $R_{\text{eq}123} = 4 + 2 = 6 \Omega$       Redraw:

Parallel:  $R_{\text{eq}1234} = \left( \frac{1}{6} + \frac{1}{4} \right)^{-1} = \left( \frac{4 + 6}{24} \right)^{-1} = \frac{24}{10} = 2.4 \Omega$





## Some quick notes about circuits:

- “Electric Potential Difference” and “Voltage” mean the same thing.
- We speak of voltage “across” a resistor, and current “through” a resistor.
- Two points connected by an ideal wire always have the same potential. (A wire is an equipotential.)

Variation on April 2014 Final Exam

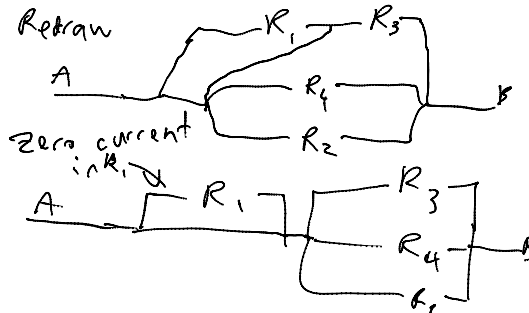
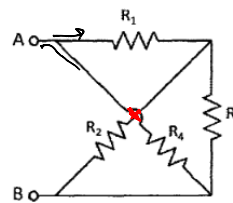
### Question 10

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$$R_1 = R_2 = R_3 = R_4 = 4 \Omega.$$

- (A) 1.7  $\Omega$       (B) 1.5  $\Omega$   
 (D) 8.0  $\Omega$       (E) 10.2  $\Omega$

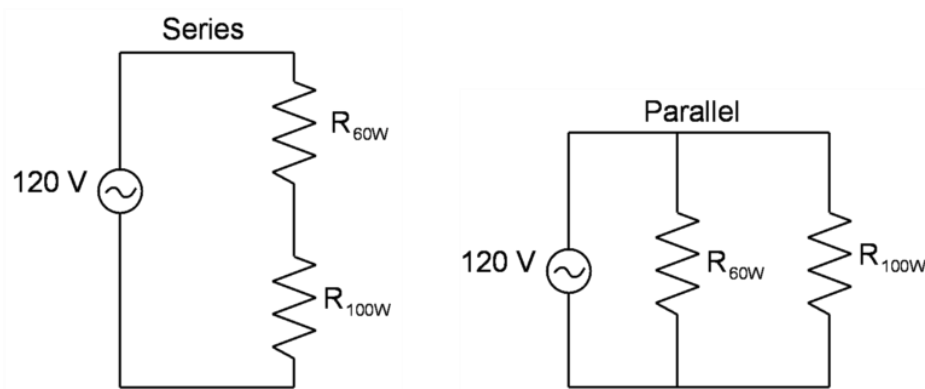
(C) 2.4  $\Omega$



$$R_{eq} = \left( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \right)^{-1}$$

$$= \frac{4}{3} \Omega$$

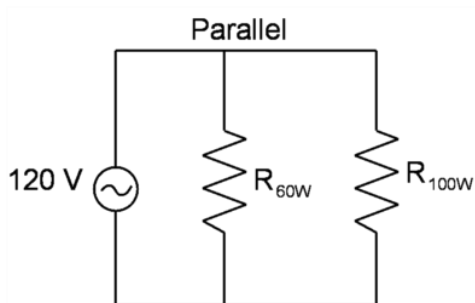
**Demonstration.** Two ways of wiring two different light bulbs.  
Note: A circle with a wavy line in it represents an Alternating Current (AC) power supply. It is like a battery, except the voltage flips direction 60 times per second.



**Demonstration.** In Class Discussion Question

If the bulbs are wired in parallel, which bulb will consume more power?

- A. The 60 W bulb.
- B. The 100 W bulb.
- C. both will consume the same power.



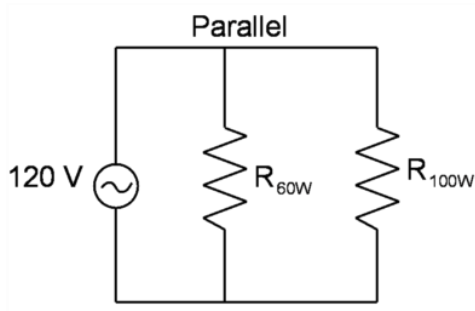
**Demonstration.** In Class Discussion Question

If the bulbs are wired in parallel, which bulb will consume more power?

A. The 60 W bulb.

**B. The 100 W bulb.**

C. both will consume the same power.



- The labels on the bulbs advertise the power *assuming* 120 V.
- Voltage across each bulb is the same: 120 V
- $P = V^2/R$
- So the larger the resistance, the lower the power.
- This means  $R_{60W} > R_{100W}$

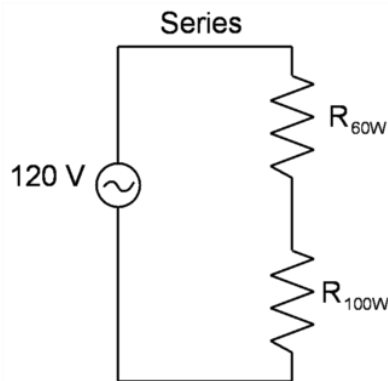
**Demonstration.** In Class Discussion Question

If the bulbs are wired in series, which bulb will consume more power?

A. The 60 W bulb.

B. The 100 W bulb.

C. both will consume the same power.



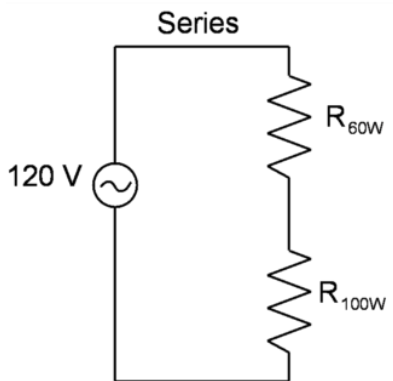
**Demonstration.** In Class Discussion Question

If the bulbs are wired in series, which bulb will consume more power?

**A. The 60 W bulb.**

B. The 100 W bulb.

C. both will consume the same power.



- Current through each bulb is the same.
- $P = I^2 R$
- So the larger the resistance, the larger the power.
- Recall:  $R_{60W} > R_{100W}$
- So this means:  $P_{60W} > P_{100W}$

**Demonstration.** The moral:

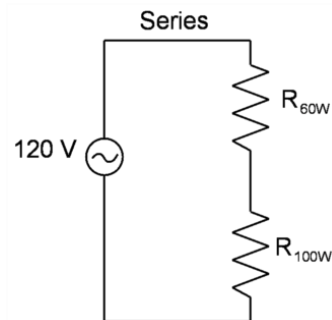
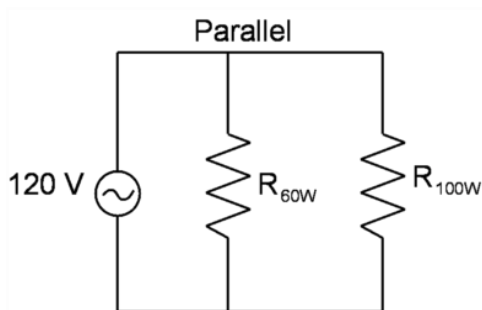
- The thing that is the same for resistors in parallel is voltage.

Use  $P = V^2 / R$  to compare power. Higher power corresponds lower resistance.

- The thing that is the same for resistors in series is current.

Use  $P = I^2 R$  to compare power. Higher resistance corresponds to higher power.

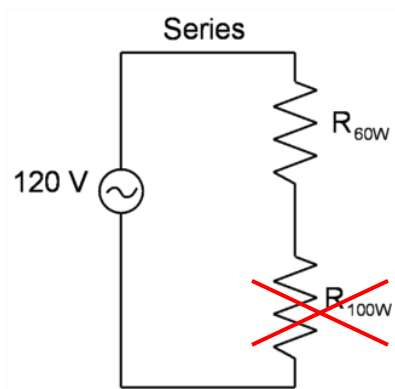
- In your house, Parallel is **always** used.



**Demonstration.** In Class Discussion Question.

If the bulbs are wired in series and the 100 W bulb is unscrewed, what will happen to the 60 W bulb?

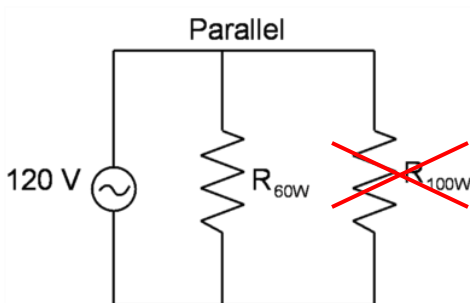
- A. It will light up.
- B. It will not light up.



**Demonstration.** In Class Discussion Question

If the bulbs are wired in parallel and the 100 W bulb is unscrewed, what will happen to the 60 W bulb?

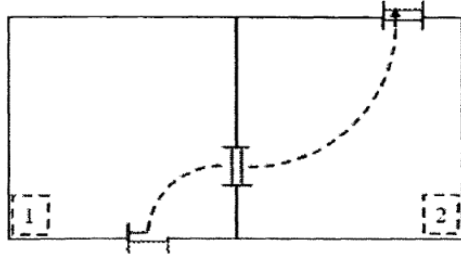
- A. It will light up.
- B. It will not light up.



From April 2014 Final Exam

**PART A [10 marks]**

A particle of charge  $q$  travels through two chambers as shown below. It enters Chamber 1 from below and exits Chamber 2 from above. The particle moves at a speed of 15.0 m/s. The magnetic field in Chamber 2 has a magnitude of 1.5 T.



Assuming the particle is negatively charged, what is the direction of the magnetic field in Chamber 1?

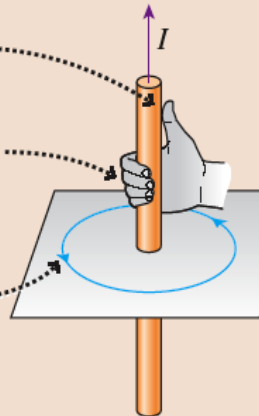


Let's review.

## Right-hand rule for magnetic fields

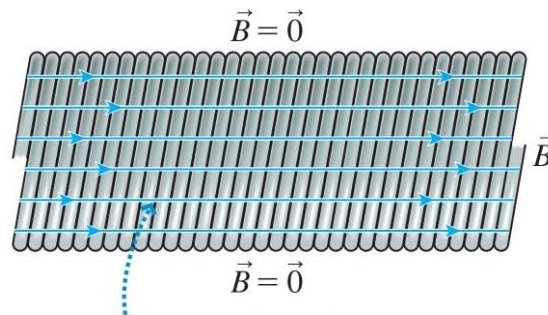
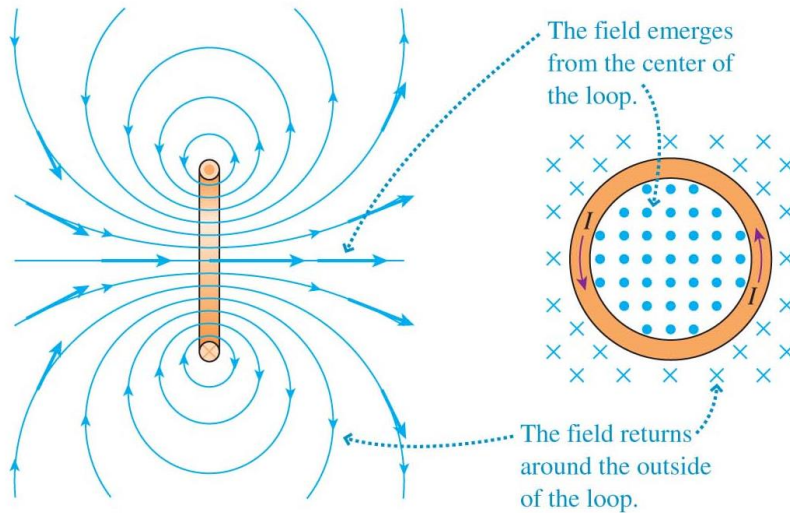
### Right-hand rule for fields

- 1 Point your *right* thumb in the direction of the current.
- 2 Curl your fingers around the wire to indicate a circle.
- 3 Your fingers point in the direction of the magnetic field lines around the wire.



**FIGURE 33.18** The magnetic field of a current loop.

(a) Cross section through the current loop (b) The current loop seen from the right



The magnetic field is uniform inside this section of an ideal, infinitely long solenoid. The magnetic field outside the solenoid is zero.

The strength of the uniform magnetic field inside a solenoid is

$$B_{\text{solenoid}} = \frac{\mu_0 N I}{l} = \mu_0 n I$$

where  $n = N/l$  is the number of turns per unit length.

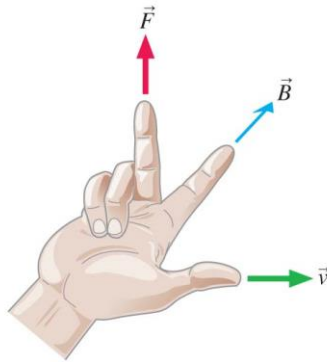
## The Magnetic Force on a Moving Charge

The magnetic force on a charge  $q$  as it moves through a magnetic field  $\mathbf{B}$  with velocity  $\mathbf{v}$  is

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B} = (qvB \sin \alpha, \text{ direction of right-hand rule})$$

where  $\alpha$  is the angle between  $\mathbf{v}$  and  $\mathbf{B}$ .

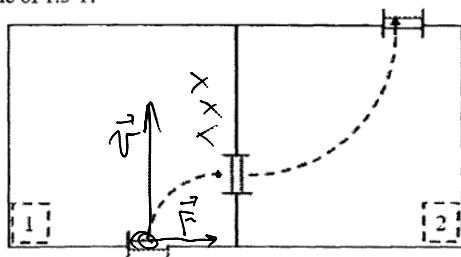
Right Hand Rule  
for Forces:



From April 2014 Final Exam

### PART A [10 marks]

A particle of charge  $q$  travels through two chambers as shown below. It enters Chamber 1 from below and exits Chamber 2 from above. The particle moves at a speed of 15.0 m/s. The magnetic field in Chamber 2 has a magnitude of 1.5 T.



$$\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$$

Assuming the particle is negatively charged, what is the direction of the magnetic field in Chamber 1?

if particle were +,

$$\vec{B} \odot$$

$$\Rightarrow \vec{B} \otimes$$

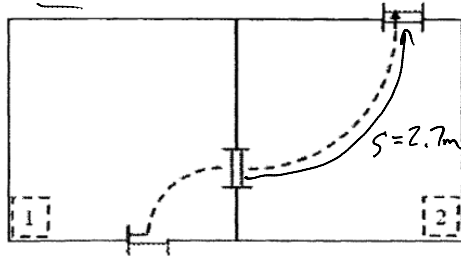
into page.



From April 2014 Final Exam

**PART A [10 marks]**

A particle of charge  $q$  travels through two chambers as shown below. It enters Chamber 1 from below and exits Chamber 2 from above. The particle moves at a speed of 15.0 m/s. The magnetic field in Chamber 2 has a magnitude of 1.5 T.



$$s = \frac{\text{circ}}{4} = 2.7\text{m} = \frac{2\pi r}{4}$$

$$\Rightarrow r = \frac{2(2.7\text{m})}{\pi}$$

The particle travels a path of length 2.7 meters through Chamber 2. If it has a mass of 3.2 grams, what is the magnitude of the charge of the particle?

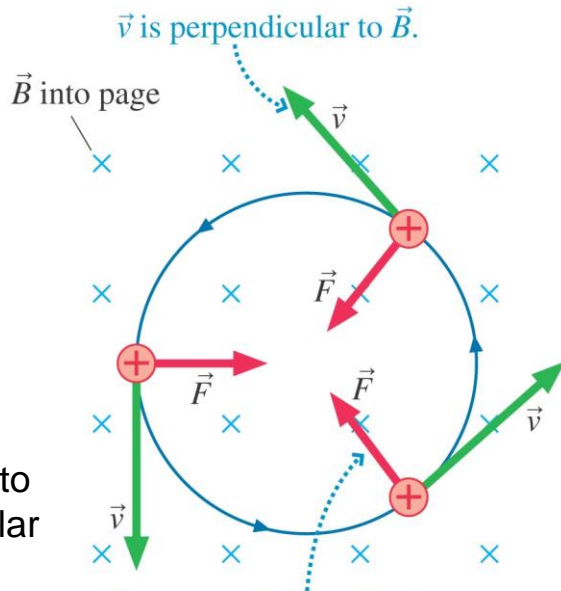
$$\vec{v} \perp \vec{B} \Rightarrow F = qvB = \frac{mv^2}{r}, \text{ solve for } q -$$

$$q = \frac{mv}{Br} = \frac{(3.2 \times 10^{-3})(15)}{1.5(2)2.7} \pi = 0.0186$$

$$\boxed{q = 0.019 \text{ C}}$$

Magnetic Force is important for fast moving electrons or positive ions in a vacuum.

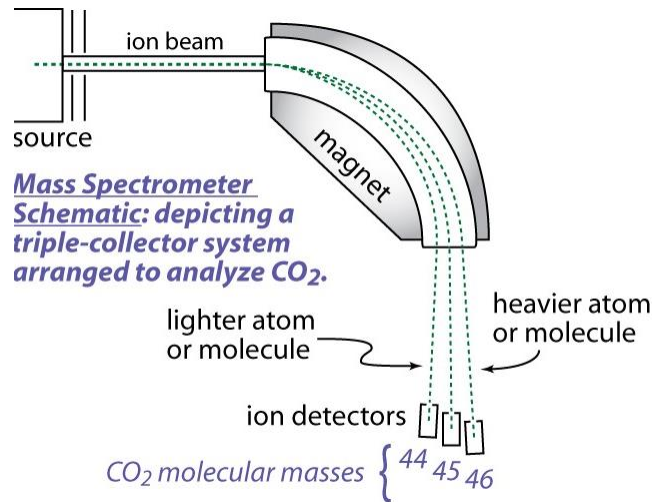
Since  $F$  tends to be perpendicular to  $v$ , it forms a good **centripetal** force.



The magnetic force is always perpendicular to  $\vec{v}$ , causing the particle to move in a circle.

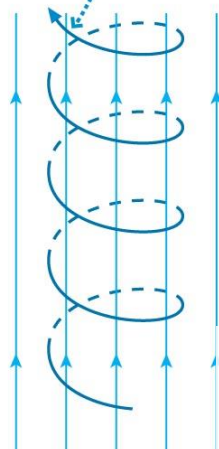
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**Mass Spectrometers** use the fact that the radius of circular trajectory in a magnetic field depends on the mass of the particles.

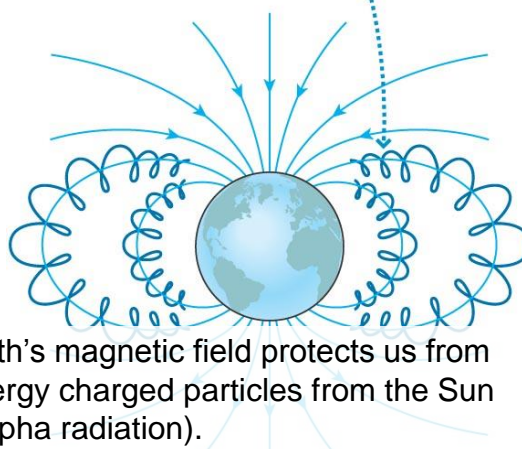


**Cyclotron Motion:** in 3D the motion of charged particles is not a circle but a spiral.

Charged particles spiral around the magnetic field lines.



The earth's magnetic field leads particles into the atmosphere near the poles, causing the aurora.



The Earth's magnetic field protects us from high energy charged particles from the Sun (beta, alpha radiation).

Aurora Borealis is natural light caused by charged particles accelerating in the Earth's magnetic field.



From April 2014 Final Exam

**Question 9**

A set of twins, Andrea and Courtney, are initially 10 years old. While Courtney remains on Earth, Andrea rides on a spaceship which travels away from Earth at a speed of  $0.60c$  for five years (as measured by Courtney), then immediately turns around and comes back at  $0.60c$ . When Andrea returns, Courtney is 20 years old. How old is Andrea upon her return?

(A) 10 y

(B) 12 y

(C) 15 y

(D) 18 y

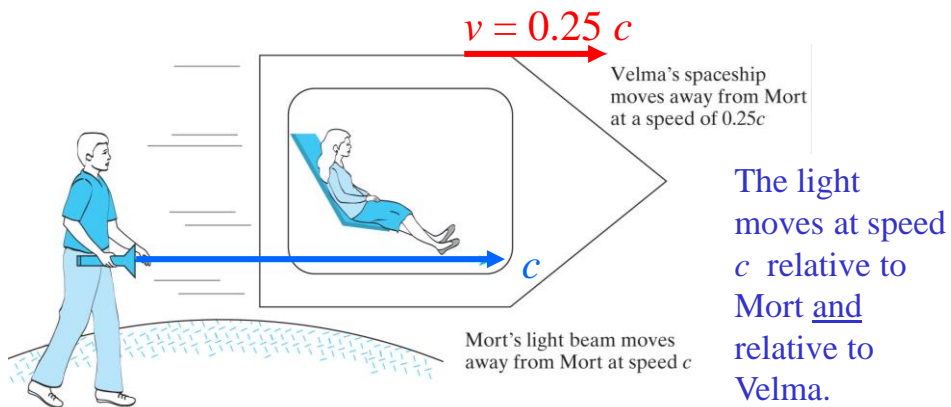
(E) 20 y



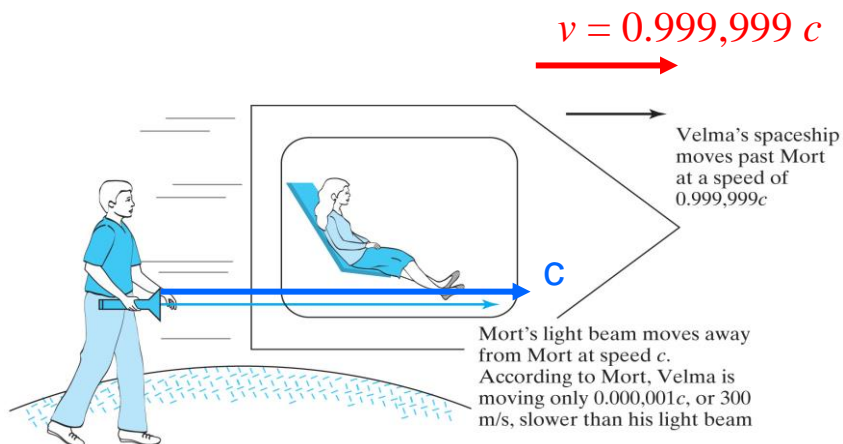
Let's review.

## Principle of Constancy of Lightspeed

The speed of light (and of other electromagnetic radiation) in empty space is the same for all observers, regardless of the motion of the light source or of the observer.



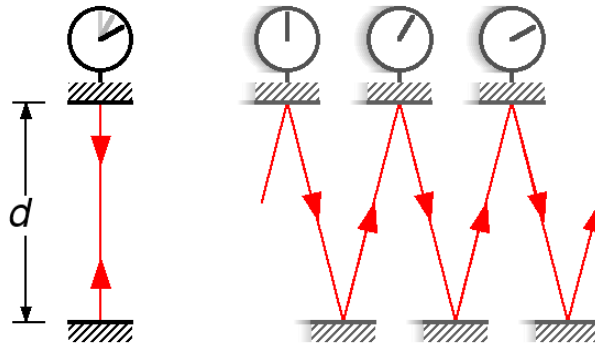
## Do you really believe this??



Even here, both Mort and Velma observe the speed of light to be  $c$ .

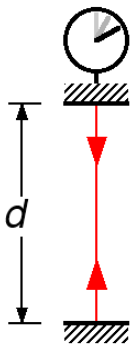
## Light Clocks

A “light clock” is made up of two parallel mirrors, separated by a vacuum and held at a fixed distance of  $d$ .



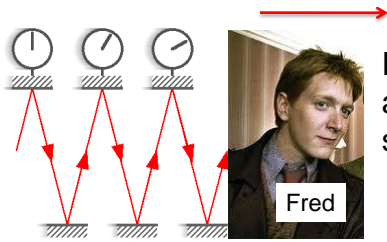
A short pulse of light bounces between the mirrors, “ticking” for each bounce.

## Twin Paradox



Fred and George are identical, and so have identical life-spans. They each have a light clock. This light clock “ticks” once every millisecond, so they both expect to observe  $2.5 \times 10^{12}$  ticks in their 80 year life-span.

## Twin Paradox



Fred flies on his broomstick to the right at 20% of the speed of light. George stays on the ground.

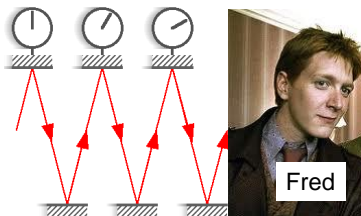
Over his life, George sees  $2.5 \times 10^{12}$  ticks of his stationary clock.

How many “ticks” of Fred’s clock does George observe?



- A. More than  $2.5 \times 10^{12}$
- B. Fewer than  $2.5 \times 10^{12}$
- C.  $2.5 \times 10^{12}$

## Twin Paradox



Fred flies on his broomstick to the right at 20% of the speed of light. George stays on the ground.

After George sees  $2.5 \times 10^{12}$  ticks of his stationary clock, he dies of old age. How do you expect his twin brother is doing?



- A. Fred will also probably die at this time.
- B. Fred has more life to live.
- C. Fred has already been dead for some time.

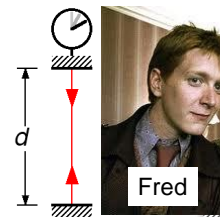
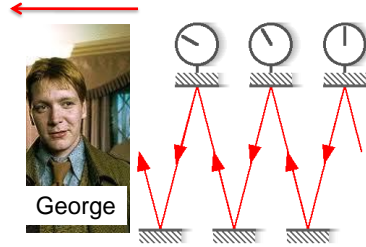
## Twin Paradox

Fred flies on his broomstick to the right at 20% of the speed of light. George stays on the ground.

According to Fred, in his reference frame, he is stationary, and his brother is moving **to the left** at 20% of the speed of light.

Over his life, Fred sees  $2.5 \times 10^{12}$  ticks of his clock, which is stationary relative to him. How many “ticks” of George’s clock does Fred observe?

- A. More than  $2.5 \times 10^{12}$
- B. Fewer than  $2.5 \times 10^{12}$
- C.  $2.5 \times 10^{12}$



## Time Dilation

The time interval between two events that occur at the *same position* is called the proper time  $\Delta\tau$ . In an inertial reference frame moving with velocity  $v = \beta c$  relative to the proper time frame, the time interval between the two events is

$$\Delta t = \frac{\Delta\tau}{\sqrt{1 - \beta^2}} \geq \Delta\tau \quad (\text{time dilation})$$

The “stretching out” of the time interval is called time dilation.

**Question 9**

A set of twins, Andrea and Courtney, are initially 10 years old. While Courtney remains on Earth, Andrea rides on a spaceship which travels away from Earth at a speed of  $0.60c$  for five years (as measured by Courtney), then immediately turns around and comes back at  $0.60c$ . When Andrea returns, Courtney is 20 years old. How old is Andrea upon her return?

- (A) 10 y      (B) 12 y      (C) 15 y      (D) 18 y      (E) 20 y

Andrea Not accelerating for 1st half of trip, nor 2nd half.

1st half: Leaving Earth, arriving way out in space are events at same place for Andrea: Andrea measures  $\Delta\tau$ . Courtney:  $\Delta t = \frac{\Delta\tau}{\sqrt{1 - v^2/c^2}}$

Earth frame (Courtney)  
 $\Delta t = 5 \text{ yr.}$   
 $\frac{v}{c} = 0.6$

$\Delta\tau = 5 \text{ yr} \sqrt{1 - 0.6^2}$   
 $\Delta\tau = 4 \text{ yr.}$

2nd half, again  $\Delta\tau = 4 \text{ yr}$   
 total  $\Delta\tau = 4 + 4 = 8 \text{ yrs}$



Good Luck!!

福

- I'll see you on Thursday at 2:00pm!!
- Then I hope to see you again in the future – please say hi if you see me around campus, and feel free to stop by my office any time you see my open door.
- It's been a lot of fun teaching you physics this year – have a fantastic rest of your life!!!!

Course	Last Name	Date	Time	Location
PHY132H1S	A - LE	THU 09 APR	PM 2:00 - 4:00	EX 100
PHY132H1S	LI - W	THU 09 APR	PM 2:00 - 4:00	EX 200
PHY132H1S	X - Z	THU 09 APR	PM 2:00 - 4:00	EX 320