**PART A [20 points]**

You are driving North along a highway at a speed $v_o$ when you hear the siren of a police car approaching you from behind at speed $v_s$, where $v_s > v_o$. The frequency the siren emits when at rest is $f_0$, and the frequency that you observe is $f_+$, where $f_+ > f_0$.

1. Write down a relation for $f_+$ in terms of $v_o$, $v_s$, $f_0$ and the speed of sound in air, $v$.

$S \xrightarrow{v_s} \quad O \xrightarrow{v_o}$

Source approaching: $f_1 = \dfrac{f_0}{1 - v_s/v}$

observe receding, shift $f_1$:

$$f_+ = f_1(1 - v_o/v) = \boxed{f_0 \frac{(1 - v_o/v)}{(1 - v_s/v)}}$$

or this

or:

$$\boxed{f_+ = f_0\left(\frac{v - v_o}{v - v_s}\right)} \quad ①$$

2/2 for a correct formula: No work needed.

You are relieved that the police car is in pursuit of a different speeder when he ~~continues past you. So~~ now the police car is in front of you, still traveling North at $v_s$, and you are following at speed $v_o$. The frequency that you observe is $f_-$, where $f_- < f_0$.

2. Write down a relation for $f_-$ in terms of $v_o$, $v_s$, $f_0$ and the speed of sound in air, $v$.

$O \xrightarrow{v_o} \quad S \xrightarrow{v_s}$

Source receding: $f_1 = f_0 / 1 + v_s/v$

observe approaching, shift $f_1$:

$$f_- = f_1(1 + v_o/v) = \boxed{f_0 \frac{(1 + v_o/v)}{(1 + v_s/v)}}$$

or this

or:

$$\boxed{f_- = f_0\left(\frac{v + v_o}{v + v_s}\right)} \quad ②$$

2/2 for a correct formula: No work needed.

3. If $v_o = 35$ m/s, $f_+ = 1310$ Hz and $f_- = 1240$ Hz, what is the speed of the police car, $v_s$, in m/s?

$② \Rightarrow f_0 = f_-\left(\dfrac{v + v_s}{v + v_o}\right) \qquad ① \Rightarrow f_0 = f_+\left(\dfrac{v - v_s}{v - v_o}\right)$

$$f_0 = f_0$$

$$f_-\left(\frac{v + v_s}{v + v_o}\right) = f_+\left(\frac{v - v_s}{v - v_o}\right), \text{ solve for } v_s.$$

$$f_-(v + v_s)(v - v_o) = f_+(v - v_s)(v + v_o)$$

$$f_- v^2 - f_- v v_o + f_- v v_s - f_- v_s v_o = f_+ v^2 + f_+ v v_o - f_+ v_s v - f_+ v_s v_o$$

$$(f_- v - f_- v_o + f_+ v + f_+ v_o) v_s = f_+ v^2 + f_+ v v_o - f_- v^2 + f_- v v_o$$

$$v_s = \frac{v\,(f_+(v + v_o) - f_-(v - v_o))}{f_+(v + v_o) + f_-(v - v_o)}$$

$$= 343\left[\frac{1310(378) - 1240(308)}{1310(378) + 1240(308)}\right] = 44.29$$

$$\boxed{v_s = 44 \text{ m/s}}$$

4/16 for good attempt that got garbled up and ended with wrong answer

+2 if it was looking similar to here (6/16)

# Notes

1. $f_+ = \dfrac{f_0}{1 - \left(\dfrac{v_s - v_0}{v}\right)}$ is not quite correct : 1/2

2. $f_- = \dfrac{f_0}{1 + \left(\dfrac{v_s - v_0}{v}\right)}$ is not quite correct : 1/2

these can be used, surprisingly, to get 44 m/s for part 3 : → 18/20
(16/16)

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1. $f_+ = f_0\left(\dfrac{v + v_0}{v - v_s}\right)$ 1/2

2. $f_- = f_0\left(\dfrac{v - v_0}{v + v_s}\right)$ 1/2

3. $\ldots \approx -25.7$ m/s ← ~~I think this is actually consistent with the wrong equations above!! 16/16~~ → ~~18/20~~

negative. doesn't make sense. +10/16

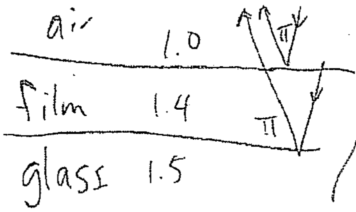
→ 12/20

NN

Part B1 : 10 Points.

1. Light is incident normally from air onto a liquid film that is on a glass plate. The liquid film is 300 nm thick, and the liquid has index of refraction 1.40. The glass has index of refraction $n = 1.50$. Calculate the longest visible wavelength (as measured in air) of the light for which there will be totally destructive interference between the rays reflected from the top and bottom surfaces of the film. (Assume that the visible spectrum lies between 400 and 700 nm.)

$t = 300\ nm$




$$\boxed{\lambda_D = \frac{2n_f t}{m - \frac{1}{2}}} \quad m = 1, 2, 3, \ldots$$

+2 for close but wrong eq. +3 for correct start, but didn't get it. (+5)

$m = 1: \ \lambda_D = \frac{2(1.4)(300)}{0.5} = 1680\ nm$ ↑ infrared.

$m = 2: \ \lambda_D = \frac{2(1.4)(300)}{1.5} = 560\ nm$ ✓

$\lambda_{max} = 560\ nm$

Use wrong equation: $\boxed{\lambda_D = \frac{2nt}{m + \frac{1}{2}}}$, $m = 1$ → 5/10 for accidentally obtaining correct answer.

"m = 2" comes with no justification. Why?

"Why m = 2?" 8/10

$$\lambda_D = \frac{2n_f t}{m + \frac{1}{2}} \quad (m = 0 \ldots)$$

600 nm +3

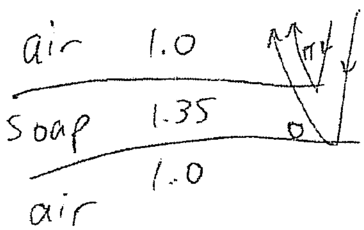
$$\lambda_D = \frac{2n_f t}{m - \frac{1}{2}} \quad (m = 1 \ldots)$$

2.8 × 3 = 8.4

2.5 √ 8.4, 7.5, 9

Part B2.

2. A soap bubble, when illuminated with light of frequency $5.11 \times 10^{14}$ Hz, appears to be especially reflective. If it is surrounded by air and if its index of refraction is 1.35, what is the thinnest thickness the soap film can be?




Use: Constructive $\boxed{\lambda_c = \frac{2 n_s t}{m - \frac{1}{2}}}$ ← 5/10 for correctly figuring this out.

+2 for choosing $\boxed{\frac{2nt}{m}}$ +1 for showing a numerical answer

$t = \frac{\lambda_c (m - \frac{1}{2})}{2 n_s}$

thinnest will be for $m = 1$.

$v = \lambda f = c$

$\boxed{\lambda = \frac{c}{f}}$

$\lambda = 587 \text{ nm}$

$t = \frac{c(0.5)}{2 f n_s} = \frac{3 \times 10^8 (0.5)}{2(5.11 \times 10^{14})(1.35)}$

$= 1.09 \times 10^{-7}$ m

$t_{min} =$ 109 nm

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Special cases.

$t_{min} = 217$ nm used $\boxed{\lambda_c = \frac{2 n_s t}{m}}$ × wrong eq. +3

+2 for finding $\lambda$ correctly.

5/10

Using $\lambda_c = \frac{2nt}{m}$, $m = 1$, but getting some random answer ≠ 217nm because $\lambda$ is wrong.

3/10