HARLOW ANSWERS.

MULTIPLE CHOICE [5 points per question × 12 questions = 60 points total]

Possibly helpful information for this test:
$\pi = 3.14159$ is the ratio of the circumference to the diameter of a circle
$g = 9.80\ \text{m/s}^2$ is the acceleration due to gravity near the Earth's surface.
$c = 3.00 \times 10^8\ \text{m/s}$ is the speed of light in a vacuum.
The speed of sound in air may be assumed to be 343 m/s, unless otherwise stated.
The index of refraction of water is $n = 1.33$.
Common Prefixes: k = "kilo-" = 10^3 c = "centi-" = 10^{-2} m = "milli-" = 10^{-3}
μ = "micro-" = 10^{-6} n = "nano-" = 10^{-9}

Question 1
Which one of the following statements is true regarding the sound intensity level, β, in dB, and the intensity of a sound wave, I, in W/m^2?
A) β obeys an inverse-square distance law, but I does not.
B) Both β and I obey inverse-square distance laws.
C) Both β and I can be negative.
D) Both β and I can never be negative.
(E) I can never be negative, but β can be negative. if $I < I_0$, β is negative.

Question 2
A heavy stone of mass m is hung from the ceiling by a thin 8.0 g wire that is 65 cm long. When you gently pluck the upper end of the wire, a pulse travels down the wire and returns 7.8 ms later, having reflected off the lower end. The stone is heavy enough to prevent the lower end of the wire from moving. What is the mass m of the stone?
A) 0.21 kg
B) 2.5 kg
C) 7.8 kg
(D) 35 kg
E) 340 kg

$\mu = 8\times10^{-3}\ \text{kg} / 0.65\ \text{m} = 1.23\times10^{-2}\ \text{kg/m}$
$v = \frac{d}{t} = \frac{2(0.65)}{7.8\times10^{-3}} = 166.67\ \text{m/s}$
$v = \sqrt{\frac{T_s}{\mu}} \Rightarrow T_s = v^2\mu = 166.67^2(1.23\times10^{-2}) = 341.9\ \text{N}$
$T_s = mg \Rightarrow m = \frac{T_s}{g} = 34.9\ \text{kg}$

Question 3
Why does the intensity of waves from a small source decrease with the square of the distance from the source? (Choose the best answer.)
A) The waves run out of energy as they travel.
(B) The waves spread out as they travel.
C) The medium through which the waves travel absorbs the energy of the waves.
D) The waves slow down as they travel away from the source.
E) The frequency of the waves decreases as they get farther from the source.

$I = \frac{P}{4\pi r^2}$ ← area waves are spread over.

Question 4

Two strings of identical material and radius are stretched with the same tension with their ends fixed, but one string is 8.0 mm longer than the other. Waves on these strings propagate at 420 m/s. The fundamental frequency of the longer string is 528 Hz. What is the beat frequency when each string is vibrating at its fundamental frequency?

A) 5.5 Hz
B) 11 Hz
C) 16 Hz
D) 22 Hz
E) 27 Hz

$f_1 = \frac{v}{2L}$

Longer: $L = \frac{v}{2f} = \frac{420}{2(528)} = 0.3977\ m$

Shorter $f = \frac{v}{2L} = \frac{420}{2(0.3977 - 8\times10^{-3})} = 538.838$

$f_{beat} = 538.83 - 528 = 10.8\ Hz$

Question 5

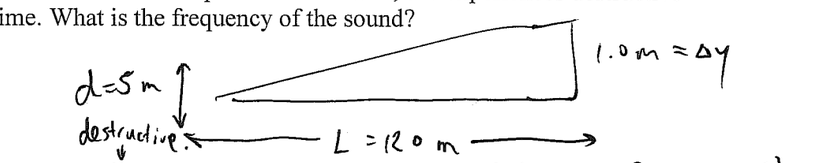
Consider the standing wave on a guitar string and the sound wave generated by the string as a result of this vibration. What must these two waves have in common?

A) They have the same amplitude.
B) They have the same frequency.
C) They have the same speed.
D) They have the same wave number.
E) They have the same wavelength.

Question 6

Two in-phase loudspeakers that emit sound with the same frequency are placed along a wall and are separated by a distance of 5.0 m. A person is standing 120 m away from the wall, equidistant from the loudspeakers. When the person moves 1.0 m parallel to the wall, she experiences destructive interference for the first time. What is the frequency of the sound?

A) 2600 Hz
B) 4100 Hz
C) 5100 Hz
D) 6700 Hz
E) 8200 Hz



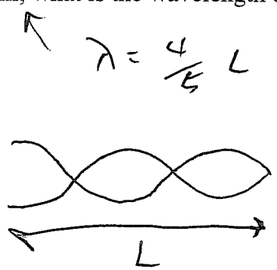
$\Delta r = \frac{\lambda}{2} = d\sin\theta \approx \frac{d\Delta y}{L}$ $\lambda = \frac{2d\Delta y}{L}$ $f = \frac{v}{\lambda} = \frac{vL}{2d\Delta y}$

Question 7

A pipe is 0.90 m long and is open at one end but closed at the other end. If it resonates with a tone whose wavelength is 0.72 m, what is the wavelength of the next higher overtone in this pipe?

A) 0.36 m
B) 0.40 m
C) 0.45 m
D) 0.51 m
E) 0.58 m

$\lambda = \frac{4}{5}L$



this must be the m=5 mode.

Next higher is m=7 mode.

$\lambda_m = \frac{4L}{m} = \frac{4(0.9)}{7} = 0.514\ m$

$f = \frac{343(120)}{2(5)(1)}$

$f = 4116\ Hz$

Question 8

Light of wavelength 687 nm is incident on a single slit 0.75 mm wide. At what distance from the slit should a screen be placed if the second dark fringe in the diffraction pattern is to be 1.7 mm from the center of the diffraction pattern?

A) 0.39 m
B) 0.47 m
C) 0.93 m
D) 1.1 m
E) 1.9 m

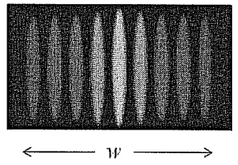
$y_2 = \frac{2\lambda L}{a}$

$L = \frac{y_2 a}{2\lambda} = \frac{1.7\times10^{-3}(0.75\times10^{-3})}{2(687\times10^{-9})}$

$L = 0.928\ m$

Question 9

In a double-slit experiment, the distance between the slits, *d*, is 5 times greater than the width of either slit, *a*. On the viewing screen, you observe a central maximum of width, *w*, and count 9 narrow bright fringes within this maximum. If the slit separation, *d*, is increased, but the widths of the slits, *a*, stays the same, which of the following happens to the interference pattern shown on the screen?



A) *w* stays the same, but there are more closely spaced narrow fringes within it.
B) *w* stays the same, but there are fewer closely spaced narrow fringes within it.
C) *w* decreases, but the spacing between the narrow bright fringes stays the same.
D) *w* decreases, but the spacing between the narrow bright fringes increases.
E) *w* decreases, and the spacing between the narrow bright fringes also decreases.

Question 10 Not for Marks.

A 4.0-cm tall object is placed 60 cm away from a converging lens of focal length 30 cm. What are the nature and location of the image? The image is

A) real, 2.5 cm tall, and 30 cm from the lens on the same side as the object.
B) virtual, 2.5 cm tall, and 30 cm from the lens on the side opposite the object.
C) virtual, 2.0 cm tall, and 15 cm from the lens on the side opposite the object.
D) virtual, 4.0 cm tall, and 60 cm from the lens on the same side as the object.
E) real, 4.0 cm tall, and 60 cm from the lens on the side opposite the object.

$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1}$

$= \left(\frac{1}{30} - \frac{1}{60}\right)^{-1}$

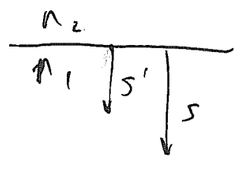
$s' = 60\ cm$

$\frac{h'}{h} = \frac{-s'}{s} = -1$ real, inverted.

Question 11

A fish appears to be 2.0 m below the surface of a pond when viewed almost directly above by a fisherman. What is the actual depth of the fish?

A) 0.38 m
B) 0.66 m
C) 1.5 m
D) 2.0 m
E) 2.7 m



$s' = \frac{n_2}{n_1} s$

$s = \frac{n_1}{n_2} s' = \frac{1.33}{1.0}(2) = 2.66$

Question 12

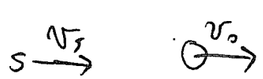
As you walk away from a vertical plane mirror, your image in the mirror

A) decreases in height.
B) stays the same height.
C) increases in height.

FREE-FORM PART (40 points total)
Clearly show your reasoning and work as some part marks may be awarded. Write your final answers in the boxes provided.

PART A [20 points]
You are driving North along a highway at a speed v_o when you hear the siren of a police car approaching you from behind at speed v_s, where $v_s > v_o$. The frequency the siren emits when at rest is f_0, and the frequency that you observe is f_+, where $f_+ > f_0$.

1. Write down a relation for f_+ in terms of v_o, v_s, f_0 and the speed of sound in air, v.



Source approaching: $f_1 = \dfrac{f_0}{1 - v_s/v}$

observer receding, shift f_1:

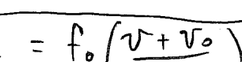
$f_+ = f_1(1 - v_o/v) = f_0 \dfrac{(1 - v_o/v)}{(1 - v_s/v)}$

or:

$$\boxed{f_+ = f_0\left(\frac{v - v_o}{v - v_s}\right)} \quad (1)$$

You are relieved that the police car is in pursuit of a different speeder when he continues past you. So now the police car is in front of you, still traveling North at v_s, and you are following at speed v_o. The frequency that you observe is f_-, where $f_- < f_0$.

2. Write down a relation for f_- in terms of v_o, v_s, f_0 and the speed of sound in air, v.



Source receding: $f_1 = f_0 / 1 + v_s/v$

observer approaching, shift f_1:

$f_- = f_1(1 + v_o/v) = f_0 \dfrac{(1 + v_o/v)}{(1 + v_s/v)}$

or:

$$\boxed{f_- = f_0\left(\frac{v + v_o}{v + v_s}\right)} \quad (2)$$

3. If $v_o = 35$ m/s, $f_+ = 1310$ Hz and $f_- = 1240$ Hz, what is the speed of the police car, v_s, in m/s?

$(2) \Rightarrow f_0 = f_-\left(\dfrac{v + v_s}{v + v_o}\right)$ $\quad (1) \Rightarrow f_0 = f_+\left(\dfrac{v - v_s}{v - v_o}\right)$

$f_0 = f_0$

$f_-\left(\dfrac{v + v_s}{v + v_o}\right) = f_+\left(\dfrac{v - v_s}{v - v_o}\right)$, solve for v_s.

$f_-(v + v_s)(v - v_o) = f_+(v - v_s)(v + v_o)$

$f_- v^2 - f_- v v_o + f_- v v_s - f_- v_s v_o = f_+ v^2 + f_+ v v_o - f_+ v_s v - f_+ v_s v_o$

$(f_- v - f_- v_o + f_+ v + f_+ v_o) v_s = f_+ v^2 + f_+ v v_o - f_- v^2 + f_- v v_o$

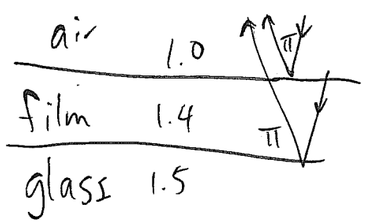
$$v_s = \frac{v(f_+(v + v_o) - f_-(v - v_o))}{f_+(v + v_o) + f_-(v - v_o)}$$

$= 343\left[\dfrac{1310(378) - 1240(308)}{1310(378) + 1240(308)}\right] = 44.29$

$\boxed{v_s = 44 \text{ m/s}}$

PART B [20 points]

1. Light is incident normally from air onto a liquid film that is on a glass plate. The liquid film is 300 nm thick, and the liquid has index of refraction 1.40. The glass has index of refraction $n = 1.50$. Calculate the longest visible wavelength (as measured in air) of the light for which there will be totally destructive interference between the rays reflected from the top and bottom surfaces of the film. (Assume that the visible spectrum lies between 400 and 700 nm.)



$t = 300$ nm

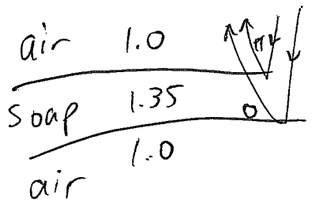
$\lambda_D = \dfrac{2 n_f t}{m - \frac{1}{2}}$ $\quad m = 1, 2, 3, \ldots$

$m = 1$: $\lambda_D = \dfrac{2(1.4)(300)}{0.5} = 1680$ nm ↑ infrared.

$m = 2$: $\lambda_D = \dfrac{2(1.4)(300)}{1.5} = 560$ nm ✓

$\lambda_{max} = 560$ nm

2. A soap bubble, when illuminated with light of frequency 5.11×10^{14} Hz, appears to be especially reflective. If it is surrounded by air and if its index of refraction is 1.35, what is the thinnest thickness the soap film can be?



Use: $\lambda_c = \dfrac{2 n_s t}{m - \frac{1}{2}}$

$t = \dfrac{\lambda_c (m - \frac{1}{2})}{2 n_s}$ thinnest will be for $m = 1$.

$v = \lambda f = c$

$\lambda = \dfrac{c}{f}$

$t = \dfrac{c(0.5)}{2 f n_s} = \dfrac{3 \times 10^8 (0.5)}{2(5.11 \times 10^{14})(1.35)}$

$= 1.09 \times 10^{-7}$ m

$t_{min} = 109$ nm