PHY138 Waves Test Fall 2005 - Solutions

Multiple Choice, Version A

Question 1 A simple harmonic oscillator begins at the equilibrium position with non-zero speed. At a time when the magnitude of the displacement is ¼ of its maximum value, through what phase angle has the oscillator moved?

(A) $(1.32 \times N\pi)$ radians (N any integer)
* **(B) $(N\pi \pm 0.253)$ radians (N any integer)**
(C) 75.5 degrees
(D) $(14.7 \times N\, 180)$ degrees (N any integer)
(E) $(2N\pi + 1.32)$ radians (N any integer)

Reasoning: If the oscillator begins at equilibrium, $x=0$, then we can write $x=A\sin(\omega t)$. We are not sure if it is initially moving up or down, so we don't know if A is positive or negative. Set $x=A/4$, solve for ωt, which will be the phase angle the oscillator has moved through (since $\omega t=0$ at the beginning). $A/4=\pm A\sin(\omega t)$, $\omega t=\sin^{-1}(\pm 0.25)$. The answer on your calculator is 0.253 radians, but a look at the $\sin(\theta)$ curve shows that $N\pi \pm 0.253$ for $N=0, \pm 1, \pm 2$, etc also is a solution to $\sin^{-1}(\pm 0.25)$.

Question 2 What is the magnitude of the phase difference, in radians, between the displacement wave and the corresponding pressure wave in a sinusoidal sound wave?

(A) 0 (B) $\pi/6$ (C) $\pi/4$ ***(D) $\pi/2$** (E) π

Reasoning: As discussed in class and the text, pressure is at a maximum when longitudinal displacement is zero. This represents a phase difference of $\pi/2$.

Question 3 Radar involves using an electromagnetic wave to detect an object by observing the energy reflected from that object. Assume the target object is a perfectly reflecting circular disk of area, A. The receiving antenna has a collecting area of $2A$. Both the emitting and receiving antennae are at a distance, d, from the target object. What is the ratio of the received power to the power originally radiated by the emitting antenna?

(A) $\dfrac{A^2}{16\pi^2 d^4}$ (B) $\dfrac{1}{16\pi d^2}$ (C) $\dfrac{A}{4\pi d^2}$ ***(D)** $\dfrac{\mathrm{A}^2}{8\pi^2 \mathrm{d}^4}$ (E) $\dfrac{A}{2\pi d^2}$

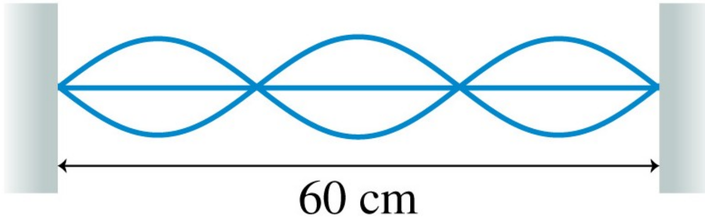
Reasoning: Set the power emitted by the emitting antenna to be P. It spreads out in all directions, so the intensity a distance d away is $I_1=P/(4\pi d^2)$. This is the intensity at the circular disk. The power that is reflected by the circular disk is $P_1=I_1(A)$, where A is the area of the disk. Assuming it spreads out in all directions, the intensity a distance d away is $I_2=P_1/(4\pi d^2)=\mathrm{PA}/(16\pi^2 d^4)$. This is the intensity back at the receiving antenna. The power that is collected by the receiving antenna is $P_2=I_2(2A)$, where $2A$ is the collecting area of the receiving antenna. So $P_2=PA(2A)/(16\pi^2 d^4)= PA^2/(8\pi^2 d^4)$, and the ratio $P_2/P= A^2/(8\pi^2 d^4)$,

Question 4 A trumpet player stands on a rolling platform, which moves at a constant velocity towards a large stationary brick wall. He blows into the trumpet, producing a sound with constant frequency of f_0 = 256 Hz, as measured when the trumpet and an observer are both at rest. The trumpet player hears the emitted sound as well as the echo of the sound which is reflected from the wall. He hears a beat frequency of f_{beat} = 3.0 Hz. What is the speed of the trumpet player?

(A) 0.25 m/s (B) 0.50 m/s ***(C) 2.0 m/s** (D) 4.0 m/s (E) 330 m/s

Reasoning: Call the speed the trumpet player moves v_t, the beat frequency he hears is f_b, and the speed of sound v=343m/s. We need to solve for v_t. The wall is stationary and the trumpet is moving, so the frequency the wall receives is found from the Doppler equation for an approaching source: $f_1=f_0/(1-v_t/v)$. The wall reflects this sound, so it acts like a source which is stationary and emitting with frequency f_1. The trumpet player is moving toward this source, so the frequency he hears is found from the Doppler equation for an approaching observer: $f_2=f_1(1+v_t/v)=f_0(1+v_t/v)/(1-v_t/v)$. This is slightly higher than f_0, so the beat frequency heard by the trumpet player is $f_b=f_2-f_0$. Or, $f_b= f_0(1+v_t/v)/(1-v_t/v)-f_0$. Everything in this equation is known except for v_t. Solving for v_t takes a bit of algebra, and the answer is $v_t=vf_b/(2f0+fb)=(343)(3)/(2\times 256+3)=2.0$ m/s.

Question 5 A standing wave is oscillating at 950 Hz on a string, as shown in the figure. What is the wave speed?



(A) 190 m/s (B) 290 m/s (C) 343 m/s ***(D) 380 m/s** (E) 570 m/s

Reasoning: The wavelength of a standing wave is twice the distance between adjacent nodes. In this case, it can be found from the diagram to be $\lambda(2/3)\times60$ cm = 0.4 m. The speed is found from $v=\lambda f$=(0.4)(950)=380 m/s.

Question 6 A form of sound-proofing is a fine wire mesh which is held at a fixed distance from a flat wall. When sounds waves are normally incident on the wall, they first encounter the mesh. About half of the sound intensity is reflected, and half is transmitted. The transmitted sound waves can then travel the distance, d, reflect off the wall, travel the distance d again, and then combine with the original reflected sound from the wire mesh. If the two sound waves are exactly out of phase at this point, they will destructively interfere, reducing the total reflected sound intensity. If d = 2.54 cm (one "inch"), what is the minimum frequency for which the sound-proofing will work properly?

(A) 135 Hz (B) 141 Hz ***(C) 3380 Hz** (D) 6750 Hz (E) 13,500 Hz

Reasoning: The reflected wave from the wall travels a total extra distance of $2d$ further than the reflected wave from the mesh. (This is similar to what happens in thin-film interference.) The phase delay will be $kx=\pi$ for the first destructive interference, where $x=2d$ is the path difference. Solving for k we have $k=\pi/2d$. The wavelength is $\lambda=2\pi/k=4d$. Recall that the speed of sound waves is $v=\lambda f$. Solving for frequency, $f=v/\lambda=v/4d$=(343 m/s)/(4×0.0254 m) = 3375 Hz. (This question is similar to the suggested Knight Problem 21.65.)

Question 7 A fish tank whose bottom is a mirror is filled with water to a depth, d. A small fish floats motionless, a distance y under the surface of the water. The index of refraction of the water is n. What is the apparent depth of the reflection of the fish in the bottom of the tank when viewed at normal incidence?

(A) $\frac{d-y}{n}$ (B) $\frac{y}{n}$ (C) $\frac{2y-d}{n}$ (D) $\frac{d-y}{2n}$ ***(E)** $\frac{2d-y}{n}$

Reasoning: In order to observe the reflection of the fish, the light rays must first reflect from the mirror's surface, then pass through the water/air surface of the tank. First surface: The object is the fish. It is a distance $s=d-y$ from the mirror, so the image is a distance $s'=s=d-y$ below the mirror. Second surface: the object is the image from the first surface, which is $(d-y)$ below the bottom of the tank. So $s=d+(d-y)=2d-y$. To find the apparent depth we assume the object is immersed in water with $n_1=n$, and the observer is in air with $n_2=1$. The apparent depth is $s'=(n_2/n_1)s=(2d-y)/n$. (This question is similar to an assign masteringphysics problem, "How Deep Is the Goldfish" from the Chapter 23 Problem Set.)

Question 8 A ray of light is incident at angle θ_i=32.00° on the surface of a liquid from below. The refractive index of the liquid depends on the wavelength of the light, and is given by $n_1 = 2.500\times10^{-3}\,\lambda$, where λ is in nanometers (nm). Air with index of refraction n_2 = 1.000 exists above the liquid. What is the minimum wavelength for which total internal reflection can occur?

(A) 377.4 nm (B) 471.7 nm (C) 515.4 nm (D) 653.4 nm ***(E) 754.8 nm**

Reasoning: Let's set $C=2.500\times10^{-3}$, so that $n_1=C\lambda$. For the minimum wavelength, we will want the ray to be exactly at the critical angle, so that θ_c=32.00°. The total internal reflection equation is $\sin\theta_c=n_2/n_1=1/C\lambda$. Solving for λ gives $\lambda=1/(C\sin32)$=754.8 nm.

Multiple Choice, Version B

Question 1 What is the magnitude of the phase difference, in radians, between the displacement wave and the corresponding pressure wave in a sinusoidal sound wave?

(B) $\pi/6$ (B) 0 (C) π (D) $\pi/4$ ***(E) $\pi/2$**

Reasoning: As discussed in class and the text, pressure is at a maximum when longitudinal displacement is zero. This represents a phase difference of $\pi/2$.

Question 2 A simple harmonic oscillator begins at the equilibrium position with non-zero speed. At a time when the magnitude of the displacement is ¼ of its maximum value, through what phase angle has the oscillator moved?

***(A) ($N\pi \pm 0.253$) radians (N any integer)** (B) ($1.32 \times N\pi$) radians (N any integer)
(C) ($14.7 \times N$ 180) degrees (N any integer) (D) 75.5 degrees
(E) ($2N\pi + 1.32$) radians (N any integer)

Reasoning: If the oscillator begins at equilibrium, x=0, then we can write $x=A\sin(\omega t)$. We are not sure if it is initially moving up or down, so we don't know if A is positive or negative. Set $x=A/4$, solve for ωt, which will be the phase angle the oscillator has moved through (since ωt=0 at the beginning). $A/4=\pm A\sin(\omega t)$, $\omega t=\sin^{-1}(\pm 0.25)$. The answer on your calculator is 0.253 radians, but a look at the $\sin(\theta)$ curve shows that $N\pi \pm 0.253$ for N=0, ±1, ±2, etc also is a solution to $\sin^{-1}(\pm 0.25)$.

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(A) 0.50 m/s (B) 0.25 m/s (C) 330 m/s ***(D) 2.0 m/s** (E) 4.0 m/s

Reasoning: Call the speed the trumpet player moves v_t, the beat frequency he hears is f_b, and the speed of sound v=343m/s. We need to solve for v_t. The wall is stationary and the trumpet is moving, so the frequency the wall receives is found from the Doppler equation for an approaching source: $f_1=f_0/(1-v_t/v)$. The wall reflects this sound, so it acts like a source which is stationary and emitting with frequency f_1. The trumpet player is moving toward this source, so the frequency he hears is found from the Doppler equation for an approaching observer: $f_2=f_1(1+v_t/v)=f_0(1+v_t/v)/(1-v_t/v)$. This is slightly higher than f_0, so the beat frequency heard by the trumpet player is $f_b=f_2-f_0$. Or, $f_b= f_0(1+v_t/v)/(1-v_t/v)-f_0$. Everything in this equation is known except for v_t. Solving for v_t takes a bit of algebra, and the answer is v_t=vf$_b$/(2f0+fb)=(343)(3)/(2×256+3)=2.0 m/s.

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(B) $\dfrac{1}{16\pi d^2}$ (B) $\dfrac{A^2}{16\pi^2 d^4}$ ***(C)** $\dfrac{\mathrm{A}^2}{8\pi^2 \mathrm{d}^4}$ (D) $\dfrac{A}{4\pi d^2}$ (E) $\dfrac{A}{2\pi d^2}$

Reasoning: Set the power emitted by the emitting antenna to be P. It spreads out in all directions, so the intensity a distance d away is $I_1=P/(4\pi d^2)$. This is the intensity at the circular disk. The power that is reflected by the circular disk is $P_1=I_1(A)$, where A is the area of the disk. Assuming it spreads out in all directions, the intensity a distance d away is $I_2=P_1/(4\pi d^2)$=PA/($16\pi^2 d^4$). This is the intensity back at the receiving antenna. The power that is collected by the receiving antenna is $P_2=I_2(2A)$, where $2A$ is the collecting area of the receiving antenna. So $P_2=PA(2A)/(16\pi^2 d^4)= PA^2/(8\pi^2 d^4)$, and the ratio $P_2/P= A^2/(8\pi^2 d^4)$,

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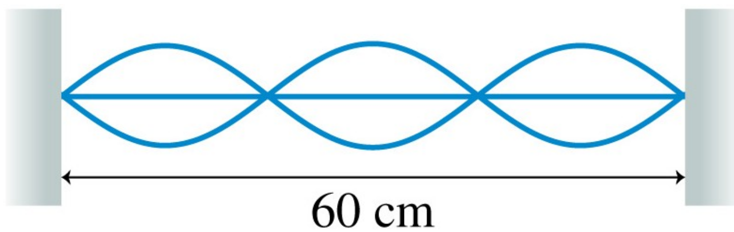

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Reasoning: Let's set $C=2.500\times10^{-3}$, so that $n_1=C\lambda$. For the minimum wavelength, we will want the ray to be exactly at the critical angle, so that $\theta_c=32.00°$. The total internal reflection equation is $\sin\theta_c=n_2/n_1=1/C\lambda$. Solving for λ gives $\lambda=1/(C\sin 32)=754.8$ nm.

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Long Answer Question

Any wrong result used correctly in subsequent parts incurs no further penalty. Any correct method giving the correct answer gets full marks. The question is worth **36 points**.

- **PART A (4 points)**

While he is dangling, set the upward force of the bungee equal to the weight of the man.

The weight of the man is Mg.

The upward force of the bungee is found from Hooke's Law: $F=-kx$. In this case x is downward, and is equal in magnitude to the new length, L_2 minus the unstretched equilibrium length, L. Define positive to be upward, so $x=-(L_2-L)$, so the force is $F=k(L_2-L)$.

Setting the forces equal:

$$|Mg|=|kx|$$

$$Mg=k(L_2-L)$$

$$Mg=kL_2-kL$$

$$kL_2=kL+Mg$$

$$\boxed{L_2 = L + Mg/k}$$

- **Part B (10 points)**

The constant speed of the pulse is $v_{pulse} = \sqrt{\frac{T_s}{\mu}}$. The length of the bungee is L_2, so the time is found from $v_{pulse}=L_2/t_{pulse}$, so $t_{pulse}=L_2/v_{pulse}$.

The mass density of the bungee is its total mass divided by its current length: $\mu=m/L_2$.

The tension in the bungee is Mg

$$t_{pulse} = \frac{L_2}{v_{pulse}} = L_2\sqrt{\frac{\mu}{T_s}} = L_2\sqrt{\frac{m}{MgL_2}}$$

$$\boxed{t_{pulse} = \sqrt{\frac{mL_2}{Mg}}}$$

Note: any reasonable simplification of the above form for the final answer gets the full two points.

- **Part C (10 points)**

There are several ways to solve this problem. Any correct method giving the correct answer gets full marks. Here are two methods:

Method 1. Conservation of Energy

At the top of the motion, his kinetic energy is zero, and the bungee is unstretched, so his total energy is just his gravitational potential energy. Let's set height=0 to be the point when the bungee cord is fully extended. So at the top of the motion, height=L_{max}. $E_{top} = mgL_{max}$

At the bottom of the motion, his kinetic energy is also zero, and his height is also zero as we have defined it. All of the energy of the system is contained in the stretched bungee, which is a distance $L_{max}-L$ away from its equilibrium. So $E_{bottom} = \frac{1}{2} k (L_{max}-L)^2$. By conservation of energy:

$$E_{top} = E_{bottom}$$
$$mgL_{max} = \tfrac{1}{2} k (L_{max}-L)^2$$
$$2MgL_{max} = k L_{max}^2 - 2kL_{max} L + kL^2$$
$$kL_{max}^2 - (2Mg + 2kL) L_{max} + kL^2 = 0$$

Use the quadratic equation to solve for L_{max}:

$$L_{max} = \frac{2Mg + 2kL \pm \sqrt{(2Mg+2kL)^2 - 4L^2k^2}}{2k}$$

$$L_{max} = \frac{Mg}{k} + L \pm \sqrt{\frac{M^2g^2}{k^2} + \frac{2MgL}{k} + L^2 - L^2}$$

$$L_{max} = L_2 \pm \sqrt{\frac{Mg}{k}\left(\frac{Mg}{k} + 2L\right)}$$

Numerically,

$$L_{max} = 27.25 \pm 22.75 = 50 \text{ m or } 4.5 \text{ m.}$$

Physically, L_{max} must be longer than 15 m, so the 50 m solution is the only one that makes sense. This should have two significant figures, so students may either answer 50 m or 5.0×10^1 m. Either way to display significant digits is acceptable.

$$\boxed{L_{max} = 50 \text{ m}}$$

Method 2. Simple Harmonic Motion with specified initial conditions

We know from Part A that $L_2 = L + Mg/k = 15 + (85)(9.8)/68 = 27.25$ m. When the bungee cord is extended, the motion will be Simple Harmonic Motion (S.H.M.) of a vertical oscillator, $y = A\cos(\omega t + \phi_0)$ with the equilibrium, y=0, when the bungee has a length L_2. Let's define y to be positive upward. The angular frequency will be $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{68}{85}} = 0.8944$ rad/s.

If we can determine the position, y_1, and velocity, v_1, at the beginning of the S.H.M. motion, we should be able to solve for the amplitude, A, and then determine the maximum length of the bungee cord. **(3 points)**

At the beginning of S.H.M., y_1=27.25 – 15 = 12.25 m, the bungee has just begun to stretch. Above that, the man has been free-falling a distance of 15 m, starting from rest up on the bridge. From kinematics of constant acceleration, we know that $v^2=v_0^2+2gd$, so $v_1 = \pm\sqrt{2gd} = \pm\sqrt{2(9.8)(15)} = \pm 17.1464$ m/s. We choose the negative solution, because we have defined $+y$ to be upward, and we know the velocity is downward.

So now we have two equations:

$$y_1 = A\cos\phi_1$$
$$v_1 = -A\omega\sin\phi_1$$

with two unknowns, A and $\phi_1 = (\omega t + \phi_0)$. Let's combine the equations to eliminate ϕ_1 and solve for A. Use the fact that $\cos\phi_1 = \sqrt{1-\sin^2\phi_1}$:

$$y_1 = A\sqrt{1-\sin^2\phi_1}$$

where, from the velocity equation, we have $\sin\phi_1 = \frac{-v_1}{A\omega}$. Plugging this in, we have:

$$y_1 = A\sqrt{1-\frac{v_1^2}{A^2\omega^2}}$$

$$y_1^2 = A^2\left(1-\frac{v_1^2}{A^2\omega^2}\right)$$

$$y_1^2 = A^2 - \frac{v_1^2}{\omega^2}$$

Solving for A:

$$A^2 = \pm\sqrt{y_1^2 + \frac{v_1^2}{\omega^2}} = \pm\sqrt{12.25^2 + \frac{17.1464^2}{0.89443^2}} = \pm 22.75\text{ m}$$

Choose the positive solution since A is positive. The maximum length of the bungee cord will be the equilibrium length plus the amplitude of S.H.M.: $L_{max}=L_2+A$=27.25+22.75 = 50 m. This should have two significant figures, so students may either answer 50 m or 5.0×10^1 m.

L_{max} = 50 m

- **Part D (12 points = 4 points for each solution)**

Amplitude

The amplitude of S.H.M. will be the maximum length of the bungee found from Part C minus the equilibrium length found from Part A: $A = L_{max} - L_2$ = 50 – 27.25 = 22.75 m. This should have two significant figures, so we should report finally that A=23 m.

Angular Frequency

The angular frequency will be $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{68}{85}} = 0.8944$ radians per second. This should have two significant figures, so we should report finally that ω=0.89 rad/s.

Phase Constant

When t=0, the length of the bungee is 15 m. Its equilibrium at y=0 is when $L=L_2$=27.25 m, so the value of y at t=0 is y=12.25 m. We also know the velocity is downward, or negative at this instant.

$$y = A\cos(\omega t + \phi_0) = A\cos\phi_0$$

$$\phi_0 = \cos^{-1}\left(\frac{y}{A}\right) = \cos^{-1}\left(\frac{12.25}{22.75}\right)$$

$\phi_0 = \pm 1.002$ radians. The velocity at t=0, $v = -A\omega\sin\phi_0$ must be negative, therefore $\sin\phi_0$ must be positive, so only the +1.002 positive solution is possible. This should have two significant figures, so we should report finally that ϕ_0 =1.0 rad.

Note that any phase angle that is different than this value by N(2π) is also a correct solution. For examples, ϕ_0 =−11.6, −5.3, 7.3 or 13.6 radians are all also correct.

A = 23 m

ω = 0.89 rad/s

ϕ_0 =1.0 rad

(or 1.0 + N(2 π)
where N=any integer)