PHY151H1: MIDTERM 2014
WEDNESDAY, 22 OCT 2014, 6:15 PM-7:45 PM (90 minutes)

- **Answer all 5 questions. Each question has equal weight; all parts of questions have equal weight.**
- **You may use any of the formulas in the aid sheet without derivation.**
- **Other permitted aids: an electronic calculator, language dictionary.**
- **Please include your name, student number, and your tutorial section on each exam book.**

1. The acceleration of a particle restricted to motion in a straight line is given by $a(t) = 2t + b$ where b is some constant. Its velocity at $t=0$ seconds is v=1 m/s. If $x(0)=0$, for what constant b would the position of the particle at $t = 1$s be 2m?

2. A cart of mass m_1 and velocity v_1 collides elastically with a cart of mass m_2, initially moving with velocity -0.5 v_1. What is the speed of each cart after the collision?

3. Alice rides her bike at constant velocity to the east along a straight road. Bob is sitting on a park bench beside the road and determines that Alice travels 25m east in 5s. Charlie is walking at constant velocity along the road and determines that Alice travels 20m East in 5s. Dave is driving at a constant velocity along the road and determines that Alice travels 25m west in 5s relative to him.
   (i) How fast is Charlie walking relative to Bob?
   (ii) How fast is Dave driving relative to Charlie?
   (iii) How long does it take Charlie to travel 3m in Dave's reference frame?

4. A space ship, traveling away from earth at 0.999 c, reaches its destination after 1.2×10^6 s (measured in its own reference frame).
   (a) How far has it traveled in its own reference frame?
   (b) How far has it traveled in the reference frame of earth?
   (c) Suppose the ship launches a probe forward at a velocity of 0.5c in its own reference frame; what is the speed of the probe in the reference frame of earth?

5. (i) In practicals you use the cart launcher on an inclined track to launch the cart up the track. Each time you launch the cart, as it travels up the incline it eventually slows to a stop. You repeat the experiment three times, and each time record the distance the cart travelled. The distances you record are: 167.5 cm, 167.0 cm, and 165.5 cm.
   (a) What is the mean distance the cart travelled?
   (b) What is the variance of the three distances you measured?
   (c) What is the standard deviation of the distances you measured?

(ii) In practicals you use the digital scale at the back of the room to measure the mass of a cart. The reading on the scale is 603.5 g.
   (d) What is the reading uncertainty $u_{Reading}$ in the measurement of mass?
   (e) According to the technical specifications of the digital scale, the instrumental accuracy uncertainty is $u_{Accuracy} = 0.05$ g. What is the uncertainty in your measurement?
   (f) Using the correct rules for significant figures when using uncertainties, how should you report the mass of the cart?

PHY151 Midterm Aid Sheet (you may detach this if you like)

Derivatives & Integrals: $f(t)$ and $g(t)$ are functions of time t, and b, c and n are constants,

$$f(t) = c(t-b)^n \Rightarrow \frac{df}{dt} = cn(t-b)^{n-1}$$

$$\frac{d(f+g)}{dt} = \frac{df}{dt} + \frac{dg}{dt} \qquad \frac{d(fg)}{dt} = \frac{df}{dt}g + f\frac{dg}{dt} \qquad \frac{d}{dt}f(g) = \left(\frac{df}{dg}\right)\left(\frac{dg}{dt}\right) \text{ (Chain rule)}$$

$$\int_{t_1}^{t_2} c(t-b)^n\,dt = \left[\frac{c(t-b)^{n+1}}{n+1}\right]_{t_1}^{t_2} = \frac{c(t_2-b)^{n+1}}{n+1} - \frac{c(t_1-b)^{n+1}}{n+1} \quad (n \neq -1)$$

• Kinematics: $v(t_f) = v(t_i) + \int_{t_i}^{t_f} a(t)\,dt$; $x(t_f) = x(t_i) + \int_{t_i}^{t_f} v(t)\,dt$

for constant acceleration a_0: the velocity at time t

$$v(t) = v(t_i) + (t - t_i)a_0 \,; \qquad v(t)^2 = v(t_i)^2 + 2(x - x_i)a_0$$

and the position at time t: $x(t) = x(t_i) + (t-t_i)v(t_i) + \frac{1}{2}(t-t_i)^2 a_0$

projectiles: $y_{\max} = y(t_i) + v_0^2 \sin^2\theta_0 / 2g \qquad x_{\max} = x(t_i) + v_0^2 \sin 2\theta_0 / g$

• Galilean Relativity:

$$x' = x - vt \qquad t' = t \qquad u' = u - v$$

• Special Relativity:

$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - vx/c^2\right) \qquad u' = \frac{u-v}{1 - uv/c^2} \qquad \gamma = 1\Big/\sqrt{1 - v^2/c^2}$$

For any two events at space-time points (x_1, t_1) and (x_2, t_2) the quantity $\Delta s^2 = (x_2 - x_1)^2 - c^2(t_2 - t_1)^2$ is the same in all inertial frames. If $\Delta s^2 > 0$ the two events are space-like separated, and there is an inertial frame in which they occur simultaneously; if $\Delta s^2 < 0$ the two events are time-like separated, and there is an inertial frame in which they occur at the same point.

• Conservation laws:

Momentum $P = \sum_{i=1}^{N} m_i v_i$; $\Delta P \equiv P_{after} - P_{before} = 0$ when no external forces are applied;

Kinetic Energy: $K = \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2$; $\Delta K = 0$ in a *perfectly elastic* collision.

For **two bodies**: $K = P^2 / 2(m_1 + m_2) + \mu(v_1 - v_2)^2 / 2$, where $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass.

• Statistics and Error Analysis:

For repeated measurements $x_1, x_2, \ldots, x_N$,

mean (or average): $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ variance: $var = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$

standard deviation: $\sigma = \sqrt{var}$

Variance of a digital reading in which half the last digit is a: $var = a^2/3$.