PHY151 Midterm 2014 : Solutions

Q.1 $a(t) = 2t + b$ $\qquad b = \text{constant}$

$v(0) = 1\ \text{m/s}. \qquad x(0) = 0.$

$x(1) = 2\ \text{m} \qquad$ find b.

$$v(t) = v(0) + \int_0^t a(t')\,dt'$$

$$= v(0) + \int_0^t (2t' + b)\,dt'$$

$$= v(0) + \Big[(t')^2 + bt'\Big]_0^t$$

$$= v(0) + t^2 + bt \qquad v(0) = 1\ \text{m/s}$$

$$\therefore\ v(t) = t^2 + bt + 1$$

$$x(t) = x(0) + \int_0^t v(t')\,dt'$$

$$= x(0) + \int_0^t (t'^2 + bt' + 1)\,dt'$$

$$= x(0) + \left[\frac{t'^3}{3} + \frac{bt'^2}{2} + t'\right]_0^t$$

$$= x(0) + \frac{t^3}{3} + \frac{bt^2}{2} + t \qquad x(0) = 0$$

$$\therefore\ x(t) = \frac{t^3}{3} + \frac{bt^2}{2} + t$$

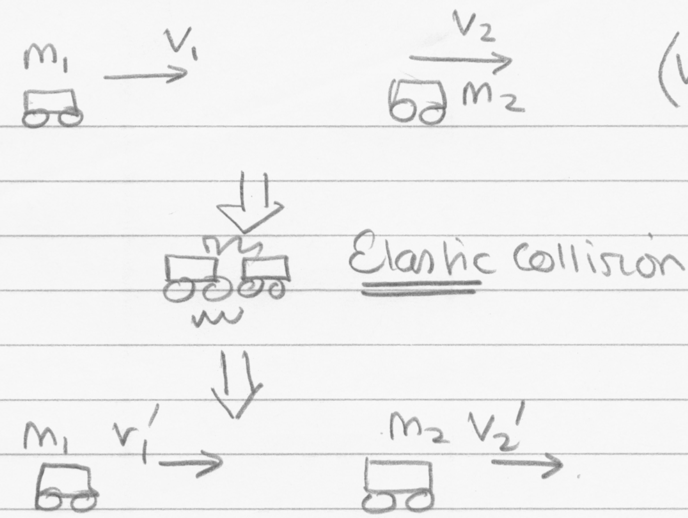
$$x(1) = \frac{1}{3} + \frac{b}{2} + 1 = 2.$$

$$\Rightarrow b = 2\left(2 - 1 - \frac{1}{3}\right) = \frac{2}{3}(6 - 3 - 1) = 4/3$$

$$\therefore\ \boxed{b = \frac{4}{3}\ \text{ms}^{-2}}$$

units matter!

2. Before $\qquad (V_2 = -V_1/2)$



After

Elastic collision $\Rightarrow$ $\quad P_{before} = P_{after} \qquad (1)$

$$K_{before} = K_{after}. \qquad (2)$$

since, for 2-bodies, $k = \dfrac{1}{2(m_1+m_2)} P^2 + \dfrac{\mu}{2}(v_1 - v_2)^2$,

Eq. (2) implies

$$(V_1 - V_2) = -(v_1' - v_2') \qquad (3)$$

writing out (1):

$$m_1 v_1 - m_2 \frac{v_1}{2} = m_1 v_1' + m_2 v_2'$$

$$\left(m_1 - \frac{m_2}{2}\right) v_1 = m_1 v_1' + m_2 v_2' \qquad (4)$$

writing out (3):

$$v_1 - \left(\frac{-1}{2} v_1\right) = v_2' - v_1'$$

$$\therefore \quad \frac{3}{2} v_1 = v_2' - v_1'$$

$$V_2' = \frac{3}{2} v_1 + v_1' \qquad (5)$$

(5) $\rightarrow$ (4) $\quad \left(m_1 - \dfrac{m_2}{2}\right) v_1 = m_1 v_1' + \dfrac{3}{2} m_2 v_1 + m_2 v_1'$

$$\left(m_1 - \frac{m_2}{2} - \frac{3}{2} m_2\right) v_1 = (m_1 + m_2) v_1'$$

$$\boxed{v_1' = \left(\frac{m_1 - 2m_2}{m_1 + m_2}\right) v_1} \qquad (6)$$

(6) → (5)

$$v_2' = \frac{3}{2} v_1 + \left(\frac{m_1 - 2m_2}{m_1 + m_2}\right) v_1$$

$$= \left(\frac{3m_1 + 3m_2 + 2m_1 - 4m_2}{2(m_1 + m_2)}\right) v_1$$

$$\boxed{v_2' = \frac{(5m_1 - m_2)}{2(m_1 + m_2)} v_1}$$

3.) Gallilean Relativity

Here is a systematic way to approach this problem:
In general, consider three points X, Y and Z all in relative motion. From standard vector relations, we know

$$\overrightarrow{XY} = \overrightarrow{XZ} + \overrightarrow{ZY}$$

Differentiating with respect to time, we define the velocity of Y with respect to X as:

$$\vec{V}_Y^{(X)} = \frac{d\,\overrightarrow{XY}}{dt} = -\vec{V}_X^{(Y)}$$

then

$$\vec{V}_Y^{(X)} = \vec{V}_Z^{(X)} + \vec{V}_Y^{(Z)}$$

In this problem, all velocities are along a single east-west line, thus unit vector along this direction cancels out and we can write:

$$v_y^{(x)} = v_z^{(x)} + v_y^{(z)} \qquad (1)$$

$$v_y^{(x)} = -v_x^{(y)} \qquad (2).$$

we are given:

$$v_A^{(B)} = 25/5 = 5 \text{ m/s}$$
$$v_A^{(C)} = 20/5 = 4 \text{ m/s}$$
$$v_A^{(D)} = -25/5 = -5 \text{ m/s}$$

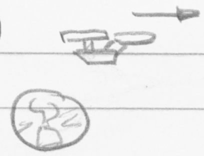
i) $v_C^{(B)} = v_A^{(B)} + v_C^{(A)}$ by (1)
$= v_A^{(B)} - v_A^{(C)}$ by (2)
$= 5 - 4 = \boxed{1 \text{ m/s}}$

ii) $v_D^{(C)} = v_A^{(C)} + v_D^{(A)}$ by (1)
$= v_A^{(C)} - v_A^{(D)}$ by (2)
$= 4 - (-5) = \boxed{+9 \text{ m/s}}$

iii) In charlie's frame, Dave's velocity is +9 m/s
hence time taken to travel 3 m is $\boxed{0.333 \text{ sec}}$

$v = 0.999c$.

4.)



Event #1

Ship leaves earth

$x_1^{earth} = 0$

$t_1^{earth} = 0$

Event #2

Ship arrives

$x_2^{earth} = d$ (unknown)

$t_2^{earth} = d/v$

$\therefore x_1^{ship} = 0$

$t_1^{ship} = 0$

a) $x_2^{(ship)} = \gamma\left(x_2^{(earth)} - v\, t_2^{(earth)}\right)$

$$= \gamma\left(d - \cancel{v}\,\frac{d}{\cancel{v}}\right) = 0.$$

$\boxed{\text{- ship in at rest in its own frame}}$

b) $t_2^{(ship)} = \gamma\left(t_2^{earth} - \frac{v}{c^2}x_2^{earth}\right)$

$$= \gamma\left(\frac{d}{v} - \frac{v}{c^2}d\right) = \frac{d}{v}\gamma\left(1 - \frac{v^2}{c^2}\right)$$

$$= \frac{d}{v}\frac{(1 - v^2/c^2)}{\sqrt{(1 - v^2/c^2)}}$$

$$= \frac{d}{v}\sqrt{1 - v^2/c^2} = \frac{d}{v}\frac{1}{\gamma}.$$

we are given $t_2^{(ship)} = 1.2 \times 10^6\,s$ (about 2 weeks)

$v = 0.999c \Rightarrow \gamma = \frac{1}{\sqrt{1 - 0.999^2}} \doteq 22.4$

$\therefore d = \gamma v\, t_2^{(ship)} = 22.4 \times 0.999 \times 3\, 10^8 \times 1.2 \times 10^6$

$= \boxed{8.06 \times 10^{15}\,m}$ (≈ 0.85 light years)

c) Relativistic addition of velocities

$$v' = \frac{v + u}{1 + \frac{vu}{c^2}} = \frac{(0.999 + 0.5)c}{1 + 0.999 \times 0.5}$$

$$= \frac{1.499}{1.4995} = \boxed{0.9997c}$$

PHY151 Midterm Fall 2014

Solution to Qu. 5 on Error Analysis.

5. (i) $x_1 = 167.5$ cm $\quad N = 3$

out of 6 $\quad x_2 = 167.0$ cm

$x_3 = 165.5$ cm

(a) $\bar{x}$ = Mean = $\boxed{166.7 \text{ cm}}$ 2

(b) $\text{Variance} = \frac{1}{N-1} \sum (x_i - \bar{x})^2$

$= \frac{1}{2}\left[(167.5 - 166.667)^2 + (167 - 166.667)^2 + (165.5 - 166.667)^2\right]$

$\text{var} = \boxed{1.08 \text{ cm}^2}$ 2

(c) $\sigma = \frac{1}{\sqrt{\text{var}}} = \boxed{1.04 \text{ cm}}$ 2

[−1 for part (i) if units are missing or wrong.]

5 (ii) (a) Scale says: $m = 603.5$ g

out of 6

a = "half last digit" = 0.05 g

$\text{var} = \frac{a^2}{3}$

$u_{Reading} = \sqrt{\text{var}} = \frac{a}{\sqrt{3}} = \boxed{0.029 \text{ g}}$ 2

(b) $u_{Total} = \sqrt{u_{Reading}^2 + u_{Accuracy}^2}$

$= \sqrt{0.029^2 + 0.05^2} = \boxed{0.058 \text{ g}}$ 2

(c) Two ways: $m = (603.50 \pm 0.06)$ g

or: $m = (603.500 \pm 0.058)$ g 2