NOTE: Today's activities must be done in teams of **one** or **two**. There are now twice as many computers in the room, and twice as many TERM booklets. Please split up into pairs or find a computer and booklet to work alone; *no teams of 3 or 4 today!* As usual, don't forget to write your name or names on the front of your booklet before you start. Also put the date and number your pages.

Today's Textbook Reference to review before Practical:
"Principles & Practice of Physics" 1st Edition by Eric Mazur ©2015
Chapter 5 from "Principles", particularly Sections 5.5 and 5.6. Example 5.5 in particular is vital to understand.

In this exercise we will investigate elastic and inelastic collisions.

- There are questions asked of you throughout this assignment, labeled **Question 1**, **Question 2**, etc. Write down the answers in the TERM booklet (to be handed in at the end of the Practical).
- At the end of the Practical, hand in your TERM booklet, along with a printed copy of your code for the final exercise (lab6exC.py). Please staple the code into the booklet.
- If you are working with a partner, the two of you must reach a consensus about an answer before moving on to the next part. Feel free to ask the TA for help if a consensus cannot be reached.
- All quantities are in SI units; distance in meters (m), time in seconds (s), velocity in m/s, acceleration in m/s^2, and mass in kg.

If we consider a system of 2 particles with no external forces acting, then the momentum of the system must be conserved. If the particles collide, then the total momentum of the system after the collision must equal the total momentum of the system before the collision.

All collisions will conserve momentum. *Elastic* collisions also conserve kinetic energy. *Totally inelastic* collisions lose the maximum possible kinetic energy. Most collisions are *inelastic*, where some kinetic energy is lost, but not the maximum possible.

Which type of collision occurs depends on the material of the colliding objects and other conditions. For example, two billiard balls colliding will be very different from two balls of Velcro.

Exercise A: Totally Inelastic Collisions

In "Totally inelastic collisions" the particles completely stick together after the collision. This means their velocities after the collision are the same.

Question 1: For a totally inelastic collision, if the masses are m_1 and m_2 and their velocities before the collision are $\vec{v}_1$ and $\vec{v}_2$, what is the velocity of both masses after the collision?

On the Practicals section of the course website under week 6 is the program "lab6exA.py". Download this program and save it on your computer. This program is not complete, so if you try to run it you will get a "syntax error" – see the steps below to fix it. The goal of the program is to create and display 2 balls with masses m1 and m2 and initial constant velocities v1 and v2. The positions of the balls are updated in the while loop. When the

balls collide, the velocities need to be changed. This is done in the "if" conditional which is supposed to check that the balls have collided, and if so, changes the velocities of both balls.

Question 2: What condition would indicate that balls have collided? Write this conditional in place of the `???Q2???` in the if statement. Hint: the balls collide when the edge of each is 'inside' the other one. What is the relationship between the distance between the balls' centres and their radii when this happens? Your condition should be an inequality involving the position and radius properties of both balls (ball1.pos, ball2.pos, ball1.radius and ball2.radius). You will want to use the mag(…) function, which calculates the magnitude of a vector. Keep in mind that the position is a vector, but the radius is a scalar.

Question 3: Fill in the `???Q3???` field with the value of ball1's velocity after the collision (use your result from question 1). Notice we have set ball2's velocity to equal ball1's velocity after the collision. Now you should be able to save and run the program without getting a syntax error. The program prints out the velocities, momenta, and kinetic energies before and after the collision. Is momentum conserved? Is kinetic energy conserved or some of it lost? Is this what you expected?

Question 4: By keeping the initial velocities as they are, and changing the masses of the particles in the program, what can you say about the final velocity if: (i) m1 = m2, (ii) m1 > m2, (iii) m1 < m2, (iv) m1 >> m2, (v) m2 >> m1?

Exercise B: Elastic Collisions

The other extreme in collisions is an "elastic collision". In this case, kinetic energy is conserved. We can't use the constraint that $\vec{v}_2 = \vec{v}_1$ after the collision so we need another piece of information to find the velocities of the 2 balls after the collision.

Question 5: Using the facts that both momentum and kinetic energy are conserved, you will have two equations. Use these two equations to find expressions for the final velocities v_{1f} and v_{2f} in terms of the initial velocities v_{1i} and v_{2i} and the masses m_1 and m_2. [Hint: It is helpful as intermediate step if you show that the two equations taken together imply that velocity of approach is equal to velocity of escape: $v_{2i} - v_{1i} = -(v_{2f} - v_{1f})$. This is equation 5.4 from page 112 of Mazur's white Principles book. This elastic collision equation can be combined with the momentum conservation equation (5.5) to find the expressions we are looking for.]

Save a new copy of the .py file you had at the end of Exercise A, call it "lab6exB.py". Replace the two lines under the comment "#Update the post-collision velocities" with the following:

```
v1final= ??
v2final= ??
v1=v1final
v2=v2final
```

Note that we calculated the final velocities first before updating v1 and v2 values.

Question 6: Replace the two ?? in the lines you inserted in your code with the python equivalent for the expressions you found in Question 5. Note that if you are multiplying by an integer, add a .0 to make it a float rather than an integer (e.g. use 2.0 instead of 2).

Run your program.

Question 7: Are momentum and kinetic energy conserved? Is this what you expected?

Question 8: Now change the initial velocity of the second ball, v2, to zero. What can you say about the final velocities of both masses if: (i) m1=m2, (ii) m1 > m2, (iii) m1 < m2, (iv) m1 >> m2, (v) m1 << m2? (run some vpython programs to confirm).

Exercise C: General Inelastic Collisions: Coefficient of restitution

In the past two exercises, we looked at totally inelastic collisions where the maximum kinetic energy is lost, and elastic collisions, where none of the kinetic energy is lost.

We now look at intermediate cases: *inelastic collisions*. We begin by defining the coefficient of restitution of the collision, which is the ratio of the final relative speed to the initial relative speed. For a collision between two bodies on the same line, coefficient of restitution is given by equation 5.19 of Mazur:

$$e = -\frac{v_{2f} - v_{1f}}{v_{2i} - v_{1i}}$$

where the velocities are scalars along the direction of motion. The coefficient of restitution of an elastic collision is 1, since the velocity of escape in this case is equal to the velocity of approach. The coefficient of restitution of a totally inelastic collision is 0, since there final velocities of both balls are the same and the numerator is zero. For other collisions, $0 < e < 1$.

Question 9: Now for the interesting part! Suppose we have a collision where we know the coefficient of restitution e is some fixed value, and momentum is conserved.
Combine equations 5.19 and 5.5 to find expressions for the final velocities v_{1f} and v_{2f} in terms of the initial velocities v_{1i} and v_{2i} and the masses m_1 and m_2, and the coefficient of restitution e.

Now take the python file from the end of the last exercise and save a new copy of it under the name "lab6exC.py". Set the initial values back to their original values: m1=1.0, m2=4.0, v1=(2,0,0), v2=(-2,0,0)

Somewhere before the while loop, declare the coefficient of restitution variable and set it to 1.0:

```
e = 1.0
```

Question 10: Change the formulas for v1final and v2final after the comment "`#Update the post-collision velocities`" to reflect the expressions you found in Question 9 above

Try various values of e between 0 and 1 to see what happens.

Question 11: Try $e < 0$, what happens?

Question 12: Try $e > 1$, what happens? What happens to the kinetic energy from before to after the collision? How would we do this with real objects? (Mazur calls this "explosive separation"!)

Question 13: Try each value of coefficient of restitution e in the following list: 0, $\sqrt{0.2}$, $\sqrt{0.4}$, $\sqrt{0.6}$, $\sqrt{0.8}$, 1, $\sqrt{1.2}$. (you may want to make use of the `sqrt(..)` function in python). For each value write down the kinetic energy before the collision, after collision, and the change in kinetic energy, put your results in a table. Does there seem to be a pattern between e^2 and change in kinetic energy?

Bonus Question : (If You Have Time)
Find an expression for the change in kinetic energy during the collision only in terms of the initial velocities v_{1i} and v_{2i} and the masses m_1 and m_2, and the coefficient of restitution e. Warning: lots of algebra! But the end result is a simple expression involving the square of the coefficient of restitution e^2, the reduced mass $m_1m_2/(m_1+m_2)$ and the relative initial velocity $v_{1i} - v_{2i}$.

Comment on the equation you get in light of the results of Question 13.