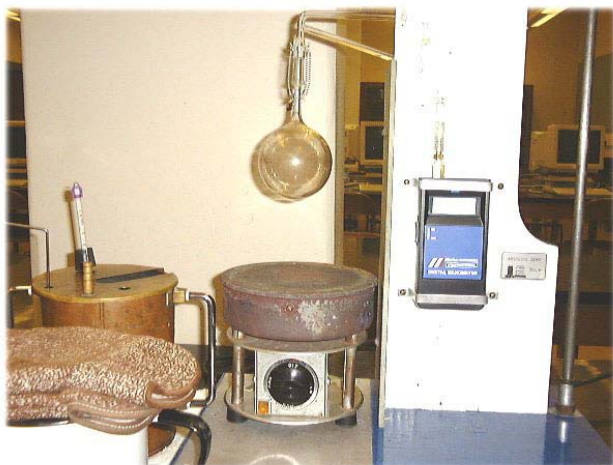


ABSOLUTE ZERO

INTRODUCTION



As you may know, for an ideal gas at constant volume, the relation between its pressure p and its temperature t , measured in $^{\circ}C$, is given by:

$$t = m p + b \quad (1)$$

Thus a plot of t versus p will be a straight line with slope m and intercept b .

This relation, called *Charles' Law*, is approximately true for all gases provided the pressure is reasonably small.

Different gases will have different values for the slope. However, the extrapolated value of the temperature when the pressure is zero, the intercept, turns out to be the same for all gases. Since the pressure cannot have a value less than zero, this temperature is the minimum value any gas (or any object) may have, and is called *Absolute Zero*. This in turn defines an absolute temperature scale, the *Kelvin*.

$$Kelvin \equiv t(^{\circ}C) + |Absolute\ Zero| \quad (2)$$

In this experiment you will find the value of Absolute Zero by taking temperature, pressure data for a fixed quantity of gas held at constant volume in a Pyrex glass bulb.

In everything that follows, temperatures measured in Kelvin are given the symbol T , while temperatures in Celsius are indicated by t .

THE EXPERIMENT

You will measure the pressure and temperature of the gas in the bulb for temperatures from $0^{\circ}C$ to about $100^{\circ}C$. Note that the volume of the gas is at least approximately constant; the constancy of the volume is explored further in the **ANALYSIS** section below.

The bulb is placed in a bath of water, whose temperature is controlled by adding ice to the bath or heating it with the hot plate. The temperature of the bath is measured with the supplied thermometer.

The thermometer is a *Partial Immersion* type, which means that it will give accurate readings only if it is immersed in the bath to the line indicated on the stem near the bulb, with the stem remaining at room temperature. Pay particular attention to the measured temperatures when the bath is a mixture of water and melting ice and when the water in the bath is boiling.

The manometer you will use to measure the pressure of the gas in the glass bulb has a specified accuracy of “ $\pm 0.25\%$ of the reading ± 5 digits”. The last part of the specification means that the last digit read is uncertain by ± 5 . For example, if the manometer reads:

737.6 mm of mercury (Torr)

then 0.25% of the reading is 1.8 Torr. Thus the total error of accuracy is $1.8 + 0.5$ Torr, and the pressure is:

737.6 ± 2.3 Torr

The most difficult part of the experimental procedure is manipulating the hot plate controls so that the temperature of the bath remains constant long enough for the temperature of the gas in the bulb to equal the temperature of the bath as measured by the thermometer. When equilibrium is achieved the pressure will be constant in time. The difficulty of achieving this condition will be one factor in determining the errors you should assign to your measurements.

There are three data points where thermal equilibrium is easy to achieve. One is when the bath is at room temperature, a second is when the bath is a mixture of water and melting ice, and the third is when the water in the bath is boiling.

Measure atmospheric pressure with the barometer in MP125 or MP126 when you begin to take data and again when you are finished. Note the room temperature with the thermometer beside the barometer for both measurements.

ANALYSIS

The first, most simple analysis of the data will involve fitting the temperature versus pressure data to a straight line. The intercept of the fit is your experimental value of Absolute Zero. **As a first term *Core* experiment you should also calibrate the thermometer as described below.**

As a second term experiment, *the other calibrations* described below may also be applied to your data.

Calibration of the Thermometer

You may calibrate the thermometer using the measured temperatures for boiling water and for a mixture of water and melting ice.

The boiling point of water goes down as atmospheric pressure decreases. Thus boiling water on a mountaintop has a lower temperature than boiling water at sea level. The boiling point t_B , in Celsius, at a given atmospheric pressure H_C is given by:

$$t_B = 100.0 + 0.037(H_C - 760) \quad (3)$$

H_C is measured in mm of mercury (Torr) at 0 Celsius. This is the reading you obtained from the barometer adjusted to 0 Celsius using the table pasted alongside the barometer. The factor 0.037 is an experimentally determined number for the change in the boiling point of water per Torr difference in pressure between the observed pressure and the Standard Pressure.

You may assume that the actual temperature of the water and melting ice is 0 Celsius provided equilibrium has been achieved.

Is there an appreciable difference between the expected values of the boiling point and the freezing point of water, and the values you actually read on the thermometer? If so, you may need to correct your temperature values. It is reasonable to assume a linear correction and you should devise one that gives you the correct values at both calibration points.

Volume Measurement

In the experiment you will have noticed that not all of the trapped air is heated when you heat the water around the bulb. The air in the tubes connecting the bulb to the manometer presumably stays close to room temperature. To take this effect into account we do the following calculation.

Suppose there are n moles of trapped air, n_1 being in the heated region itself and n_2 being in the connecting tube, most of which is approximately at room temperature. As the air is heated, it becomes less dense in the bulb relative to the tube, and therefore n_1 decreases and n_2 increases, but n remains constant:

$$n = n_1 + n_2 \quad (4)$$

For the bulb, with volume V_1 :

$$pV_1 = n_1RT \quad (5)$$

whereas for the connecting volume V_2 :

$$pV_2 = n_2RT_R \quad (6)$$

Here T_R is the room temperature in Kelvin. We eliminate n_1 and n_2 :

$$pV_1 + \frac{pV_2T}{T_R} = nRT \quad (7)$$

This looks like the normal ideal gas law except for the second term on the left hand side, which should be smaller than the other two terms.

Rearranging, we obtain:

$$\frac{1}{T} = \frac{nR}{V_1} \frac{1}{p} - \frac{V_2}{V_1T_R} \quad (8)$$

However we must consider one more correction.

Expansion of the Pyrex Glass Bulb

Given the volume V_R at room temperature T_R , the coefficient of volume expansion β gives the volume V at temperature T according to:

$$V = V_R[1 + \beta(T - T_R)] \quad (9)$$

Using this to correct for the volume of the bulb V_1 , Equation 8 becomes

$$\frac{1}{T} = \frac{nR}{V_{1R}(1 - \beta T_R)} \frac{1}{p} - \frac{V_2 + \beta V_{1R} T_R}{V_{1R} T_R (1 - \beta T_R)} \quad (10)$$

Thus a plot of $1/T$ versus $1/p$ will be a straight line if the ideal gas law holds over the temperature range explored in this experiment.

Further, the general form Equation 10 is:

$$\frac{1}{T} = \alpha \frac{1}{p} - \delta \quad (11)$$

where:

$$\alpha = \frac{nR}{V_{1R}(1 - \beta T_R)} \quad (12a)$$

$$\delta = \frac{V_2 + \beta V_{1R} T_R}{V_{1R} T_R (1 - \beta T_R)} \quad (12b)$$

This may be rewritten by rearranging and using a binomial expansion to get:

$$T = \frac{p}{\alpha} \left[\frac{1}{1 - \delta p / \alpha} \right] = \frac{p}{\alpha} \left[1 + \frac{\delta}{\alpha} p + \left(\frac{\delta}{\alpha} p \right)^2 + \left(\frac{\delta}{\alpha} p \right)^3 + \dots \right] \quad (13)$$

If $\delta \ll \alpha$ this becomes:

$$T \cong \frac{1}{\alpha} p + \frac{\delta}{\alpha^2} p^2 \quad (14)$$

Since T , the absolute temperature, is the temperature t in Celsius minus the value of Absolute Zero, this equation can be written as:

$$t = \frac{1}{\alpha} p + \frac{\delta}{\alpha^2} p^2 + b \quad (15a)$$

or:

$$b = t - \left(\frac{1}{\alpha} p + \frac{\delta}{\alpha^2} p^2 \right) \quad (15b)$$

where b is the value of Absolute Zero.

Fitting t versus p to a second order polynomial and using Equation 15a to find Absolute Zero is unlikely to give a good result. This is in part because the fitter will *fit* to values of $1/\alpha$ and α/δ^2 , although they are not free parameters but instead are numbers that you may calculate.

Thus, you should use Equation 15b to determine the value of b by:

- Estimating V_1 , V_2 , and n and calculating α and δ . Note that this will use your first, simple, determination for Absolute Zero in order to find T_R . You should be very careful about units for the pressure, the gas constant R , etc.
- Calculating b for each of your data points.
- Averaging the result of each calculation of b to give a final value of Absolute Zero.

CONSTANTS

Density of mercury at 0 °C = 13.5951 g/cm³

Coefficient of volume expansion of Pyrex glass $\beta = 3.3 \times 10^{-6}$ deg⁻¹

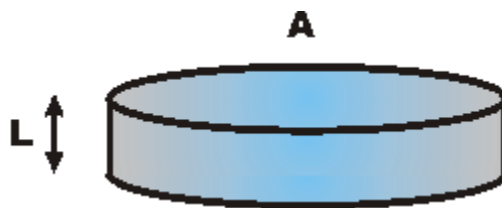
1 mole of any gas occupies 22.4 liters at STP (Standard Temperature and Pressure).

Universal gas constant $R = 8.31$ J/(mole K)

1 mole = 1 gram molecular weight = 10⁻³ kg molecular weight

PREPARATORY QUESTIONS

1. Above we refer to pressures in units of Torr, which is the pressure exerted by a cylinder of mercury, as shown to the right, of height $L = 1$ mm. Assume the area A of the cylinder is 1 square meter. What is that pressure in units of Pascals = Newton/meter² exerted by the column of mercury?



2. Does your answer to Question 1 depend on the area A of the column? Why? Does it depend on the column being circular? Why?
3. At what depth below the surface of a lake will you experience a pressure twice the pressure you experience on the surface?
4. If you are on top of a mountain where atmospheric pressure is 740 Torr, what is the boiling point of water? The answer to this question explains why you can't make a good cup of tea on top of a mountain.
5. The temperature of a gas is a measure of the average kinetic energy of the molecules of the gas. What is the average kinetic energy of the gas molecules when the temperature of the gas is 0 Kelvin?
6. Is it meaningful to talk about the temperature of a perfect vacuum? This is potentially a somewhat subtle question.

REFERENCE

Almost any first year university level physics textbook.

AUTHORS

This Guide Sheet was written by David M. Harrison, June 2001. The part on calibrations and corrections is based on material written by Derek A.L. in 1990. Milton From and Tony Key contributed to previous versions of the Guide Sheet in 1995 and 1996, 1998 respectively.