Chapter 14—Oscillatory Motion (SHM)

Background: Many things that move periodically (mass-spring systems, bungee jumpers, etc.) move according to Simple Harmonic Motion (SHM), where acceleration varies linearly with displacement (i.e. the further a bungee jumper has fallen, the “faster” he slows down). Using this equation (or pieces of it), we can find things like position, velocity, acceleration, period, frequency, and energy of the system. Pendulums are a special case, in that they do not move according to simple harmonic motion, but can be modeled to do so at very small displacements (less than 15° or so) from the vertical.

Summary of the Most Important Equations:

$$x(t) = A\cos(\omega t + \phi)$$

this equation is the definition of SHM, where A is amplitude, ω is angular frequency, t is time, and ϕ is just a phase constant

$$v(t) = -\omega A\sin(\omega t + \phi)$$

velocity in SHM, the derivative of the position function

$$a(t) = -\omega^2 A\cos(\omega t + \phi)$$

acceleration in SHM, the second derivative of the position function

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

where T is the period of the system, in seconds, and f is its frequency, in Hertz

More equations (can be derived from the above)

$$v_{\max} = \pm\omega A$$

because the sine function is never greater than 1 or less than -1

$$a_{\max} = \pm\omega^2 A$$

same idea as above

Specific to Mass-Spring Systems:

$$\omega_{mass-spring} = \sqrt{\frac{k}{m}}$$

where k is the spring constant and m is the mass…this equation only works for mass-spring systems

$$T_{mass-spring} = 2\pi\sqrt{\frac{m}{k}}, \text{ so } f_{mass-spring} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

based on $T = \frac{2\pi}{\omega} = \frac{1}{f}$

Total $E_{mass-spring} = \frac{1}{2}kA^2$

this is a constant…while in motion, the systems energy is continuously converted from spring potential energy—(1/2)kx^2—to kinetic energy—(1/2)mv^2—and back

Specific to Simple Pendulums:

$a = -g\sin(\theta)$

note that this does not follow the form (acceleration equals constant times position) so pendulums can't be modeled using SHM (except when θ is small)

$\omega = \sqrt{\frac{g}{l}}$, so $T = 2\pi\sqrt{\frac{l}{g}}$

where l is the pendulum's length

Specific to Physical Pendulums:

$\alpha = \frac{rmg}{I}\sin(\theta)$

where I is moment of inertia and r is distance from pivot point to center of mass

$\omega = \sqrt{\frac{rmg}{I}}$

Terminology of SHM

Simple Harmonic Motion: motion exhibited by mass-spring systems and pendulums (with small angular displacements) described by $x(t) = A\cos(\omega t + !)$.

Period: the amount of time a system takes to complete one full cycle of its motion (usually expressed in seconds)

Frequency: the number of complete cycles a system makes per unit time (usually expressed in Hz—cycles per second)

Amplitude: the greatest distance from the equilibrium point the system achieves while oscillating…at one amplitude away from the equilibrium point, v=0 and a is at a max

Angular frequency: the "speed" at which the system is oscillating (expressed in radians per second and related to regular frequency by $\omega = 2\pi f$

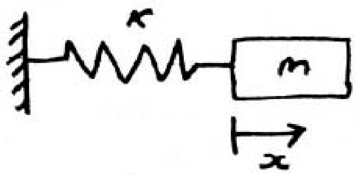
Simple pendulum: a pendulum with all the mass concentrated in one spot (i.e. a mass at the end of a string)

Physical pendulum: a pendulum with the mass distributed all along itself (i.e. a piece of wood swinging back and forth)

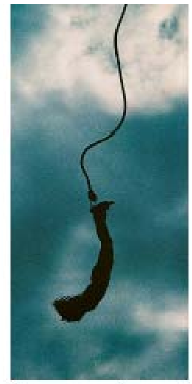
Applications:

simple pendulum

physical pendulum



mass-spring system

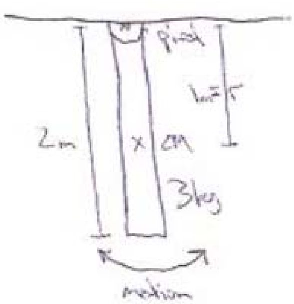


bungee jumper

almost every oscillatory motion problem out there uses one of these four scenarios

Practice Problems:

1. A physical pendulum of length 2m, mass 3kg, and uniform mass density pivots about one of its ends. How long does it take to swing back and forth twice?

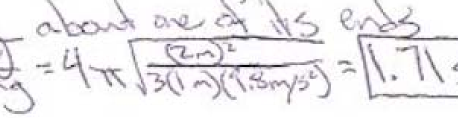


The question is asking for 2T

... We know that $\omega=\sqrt{\frac{r(m)g}{I}}$ for a physical pendulum and that $T=\frac{2\pi}{\omega}$ $\left(2T=\frac{4\pi}{\omega}\right)$

$$2T=\frac{4\pi}{\omega}=\frac{4\pi}{\sqrt{\frac{rmg}{I}}}=4\pi\sqrt{\frac{I}{rmg}}$$

we know r, m, and g from the problem, and remembering back to chapter 10, $I=\frac{1}{3}m\ell^2$ for a rod rotating about one of its ends



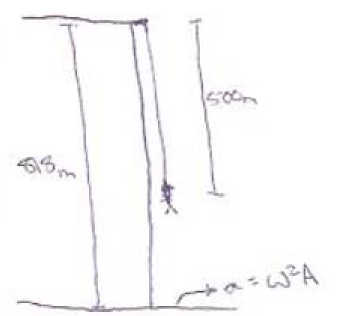
Therefore, $2T=4\pi\sqrt{\frac{\cancel{m}\ell^2}{3r\cancel{m}g}}=4\pi\sqrt{\frac{(2m)^2}{3(1m)(9.8m/s^2)}}=\boxed{1.71s}$

2. A bungee jumper of mass 70kg jumps off of the tallest building in the world (h=818m) with a 500m long bungee. He notes that just at the bottom of his oscillation, as his feet barely brush the ground, he experiences about 2g of acceleration. What is his bungee's spring constant?

$\omega=\sqrt{\frac{k}{m}}$, so $k=m\omega^2$ ... we have m, need ω

at bottom of oscillation, $a=\omega^2 A$, so $\omega=\sqrt{\frac{a}{A}}$ ... we have $a=2g$, we can get A

2.



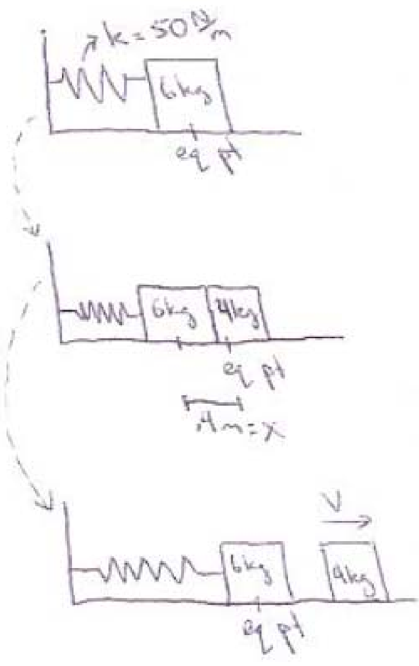
On ground, bungee is fully stretched ($v=0$)
so $A = 818 - 500 = 318m$

$$\omega=\sqrt{\frac{2(9.8m/s^2)}{318m}} = .248s^{-1}$$

... plugging in ...

$$k = m\omega^2 = (70kg)(.248s^{-1})^2 = \boxed{4.3\frac{N}{m}}$$

3. A 6kg mass is connected to a spring with k=50N/m which is connected to a wall. A 4kg mass (not connected to anything) is then pushed against the 6kg mass, compressing the spring by 0.4m. The system is then released, and at the equilibrium point the 6kg mass begins to decelerate while the 4kg mass continues onward along the frictionless surface with speed v. What is v?



v will equal the speed of the 2 blocks directly at the equilibrium point

we can look at this as a conservation of energy problem where

$$U_s = K$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$v=\sqrt{\frac{kx^2}{m}} = \sqrt{\frac{50\frac{N}{m}(.4m)^2}{(6kg+4kg)}} = \boxed{.89m/s}$$