PHY 132 Day 2 Disc. Questions 1, 2

4A. Consider a transverse wave moving from left to right along a rope. At time t particle P on the rope is at the position shown on the diagram. Particle P has

(A) zero velocity and maximum downward acceleration
(B) zero velocity and maximum upward acceleration
(C) maximum downward velocity and zero acceleration
(D) maximum upward velocity and zero acceleration
(E) none of the above

Each particle on the rope undergoes simple harmonic motion in the vertical direction.
Since particle P is passing through the equilibrium position at time t, it has maximum speed and zero acceleration.
Since the wave is moving to the right, the figure shows that particle P is moving up.
At time t the figure implies that P has a maximum upward velocity and zero acceleration.

4B. Suppose that the tension in the rope above is quadrupled (increases 4-fold) while the frequency remains the same. It follows that the new wavelength λ will be

(A) one quarter of its original value
(B) one half of its original value
(C) equal to its original value
(D) double its original value
(E) four times its original value

Since $v = \lambda f$, $\lambda = \frac{v}{f}$ where the speed v of the wave on the rope is given by $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \lambda = \frac{1}{f}\sqrt{\frac{T}{\mu}}$$

Now the frequency f and the length density μ remain the same while the new tension $T' = 4T$

$$\Rightarrow \lambda' = \frac{1}{f}\sqrt{\frac{T'}{\mu}} = \frac{1}{f}\sqrt{\frac{4T}{\mu}} = 2\left(\frac{1}{f}\sqrt{\frac{T}{\mu}}\right) = 2\lambda$$

Day 2
PHY132 Discussion Question 3

5A. Consider a sound wave that generates a pressure fluctuation ΔP of amplitude P_0 and a density fluctuation $\Delta\rho$ of amplitude ρ_0 in the air through which it passes. The pressure and density fluctuations are best represented as function of position and time via

(A) $\Delta P = P_0 \sin(kx - \omega t)$ $\quad \Delta\rho = \rho_0 \sin(kx - \omega t)$
(B) $\Delta P = P_0 \sin(kx - \omega t)$ $\quad \Delta\rho = \rho_0 \sin(kx + \omega t)$
(C) $\Delta P = P_0 \sin(kx - \omega t)$ $\quad \Delta\rho = \rho_0 \cos(kx - \omega t)$
(D) $\Delta P = P_0 \sin(kx - \omega t)$ $\quad \Delta\rho = \rho_0 \cos(kx + \omega t)$
(E) $\Delta P = P_0 \cos(kx - \omega t)$ $\quad \Delta\rho = \rho_0 \cos(kx + \omega t)$

Since pressure P and density ρ are directly related to each other in that they increase and decrease in phase, the pressure fluctuations ΔP and the density fluctuations are in phase.
Both fluctuations must have the same mathematical description, i.e the same trigonometric functions, representing waves moving in the same direction.
Only choice (A) satisfies these conditions.
[In choices B, D and E the two wave representations move in opposite directions.
Choices C and D correspond to different phases (mix of cos and sin)]

PHY132 Day 2, Discussion Question 4

5B. Two identical but oppositely moving sound waves pass each other. If the air density due to the first wave is $\rho_1 = 1.29\ kg/m^3 + 0.01\ kg/m^3\ (\cos kx - \omega t)$ and the air density due to the second wave is $\rho_2 = 1.29\ kg/m^3 + 0.01\ kg/m^3\ (\cos kx + \omega t)$, then the maximum air density is

(A) $2.60\ kg/m^3$ (B) $2.58\ kg/^3$ (C) $1.31\ kg/m^3$ (D) $1.30\ kg/m^3$
(E) There is not enough information to choose one of the above

Sound waves correspond to the air density fluctuating about the average air density, here $1.29\ kg/m^3$.
The pressure fluctuation can add to give a maximum increase in air density over and above the average air density

$$\rho_{max} = 0.01\ kg/m^3 + 0.01\ kg/m^3 + 1.29\ kg/m^3$$
$$\rho_{max} = 1.31\ kg/m^3$$

Or since $\rho = \rho_{av} + A\cos(kx - \omega t)$

$$\Delta\rho = \rho - \rho_{av} = A\cos(kx - \omega t) = 0.01\ kg/m^3 \cos(kx - \omega t)$$
$$\Delta\rho_{total} = \Delta\rho_1 + \Delta\rho_2 = 0.01\ kg/m^3 \cos(kx - \omega t) + 0.01\ kg/m^3 \cos(kx - \omega t)$$
$$\Delta\rho_{max} = 0.02\ kg/m^3$$
$$\rho_{max} = \rho_{av} + \Delta\rho_{max} = 1.29\ kg/m^3 + 0.02\ kg/m^3 = 1.31\ kg/m^3$$